

Regular paper

**A comparative study on the performance of modern function estimation tools for estimation of non-linear functions in high voltage system**

*To safeguard the electric power system, protecting the overhead transmission line from lightning stroke is an important task. For evaluating the lightning performance of transmission line several algorithms were used. Choosing a proper learning algorithm to train the network is very important for better estimation. Gradient based function estimation tools like Radial Basis Function (RBF) and back propagation Artificial Neural Network (ANN) are mainly used for this purpose. In this paper an attempt is made to compare the performance of modern function estimation tools, viz. Particle Swarm optimization (PSO) aided Wavelet neural Network (WNN), gradient based WNN, and back propagation ANN for estimation of such non-linear functions. The simulated input, output data of lightning impulse voltage, non-sinusoidal current and chopped impulse voltage were used for training and testing data of developed network for estimation. Results are presented for modern function estimation tools for learning these non-linear functions and showed that the PSO aided network estimates function more accurately than gradient based algorithms. The PSO aided WNN method can be used by electric power utilities as a useful tool for the design of electric power systems, alternative to the conventional analytical method.*

**Keywords:** PSO aided WNN, WNN, function estimation, impulse voltage, non-sinusoidal current

## I. INTRODUCTION

Developing models from observed data, is a fundamental need in many fields, viz. signal processing, forecasting, statistical data analysis and classification. This is frequently referred as function estimation and it involves estimating the underlying relationship from a given finite input-output data set. Estimation of general functions by non-linear networks such as multilayer perceptron and radial basis function networks are important tools for system modelling and identification. For more than a decade, neural networks have received considerable attention due to its self-learning, massive parallelism, and ability to self-adapt. It finds many applications particularly in the field of signal processing. The feed forward multi-layer neural networks, along with the back-propagation (BP) training algorithm, have been widely used for function estimation since they provide a generic, non-linear, black box for function representation. Besides this, the wavelet decomposition, being a powerful tool for signal processing, has also emerged as a new tool for function estimation. Because of the similarity between wavelet decomposition and one-hidden-layer neural networks, the idea

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of combining both wavelets and neural networks has been proposed. Combining wavelets and feed forward neural networks can hopefully remedy the limitations of each other, resulting in networks with efficient constructive methods and capable of handling problems of moderately large dimensions [1]. Several authors have reported the connection of wavelet theory and neural network [2] [3] and combined them in closed or open type. In close combined type the wavelet and neural network merge directly with the wavelet function as the activation function of neuron. The Wavelet Neural Network (WNN) is one such network that has gained great interest because of its advantages over radial basis function networks. Stochastic gradient based algorithm is used to optimize the properly initialized parameters of the WNN, viz. weights, translation and scale [4]. Since, the WNN is universal approximator with faster convergence. WNN has been used in many areas of electrical engineering applications viz. detection and classification of transients [5], transformer condition assessment [6], fault diagnosis [7], system identification [8], estimation of non-linear functions [9], etc.

Moreover, to improve the performance of WNN, global search training algorithm may be used instead of gradient based algorithms. Recently developed Particle Swarm Optimization (PSO) [10] is one such algorithm, which is based on the simulation of the behavior of a flock of birds or school of fish. The PSO aided algorithm has gained more attraction since no gradient informations are required for implementation and its fast convergence speed [11]. Researchers have reported PSO algorithm for training the neural network [12]. The PSO aided WNN has many applications, particularly in the field of speech recognition [13], fault diagnosis [14], etc.

In this paper the performance of modern function estimation tools has been compared for the estimation of non-linear functions. Commonly encountered non-linear functions in high voltage system like, lightning, switching impulses and non-sinusoidal current function have been selected for this purpose. The comparisons are made between a WNN, developed with different mother wavelets using stochastic gradient algorithm, PSO algorithm, and the typical feed forward network with back propagation algorithm. The accuracy with which the developed network estimates the different high voltage signals is considered as the parameter for comparison.

## II. MODERN FUNCTION ESTIMATION TOOLS

### A. Wavelet Neural Network (WNN)

Wavelet Neural Network (WNN) has been introduced as a feed forward network supported by the wavelet theory, and is emerging as an efficient tool in function estimation. It combines the properties of wavelet transform in analyzing non-stationary signal and the classification capabilities of artificial neural network. The wavelet network can be considered as an expanded perceptron in which the neurons are replaced by the wavelons. The one input and one output structure of the WNN used for this study is shown in Figure.1. The hidden nodes have wavelet activation function of different resolution. Output of the WNN is given by (1)

$$g(x) = \sum_{i=1}^m v_i \psi [D_i (w_i x - t_i)] \quad (1)$$

where,  $t_i$  are translation vector,  $D_i (=1/s_i)$  are dilation vector,  $w_i$  are the connection weights between input and hidden layer,  $v_i$  are the weights between the hidden layer and output and  $m$  is the number of hidden nodes. The  $\psi[D_i(w_i x - t_i)]$  is mother wavelet function and also the prototype of window function. There are many mother wavelets such as Haar, Daubechies, Symlet, Biorthogonal, Gaussian, Mexican Hat, Morlet, Shannon, etc.

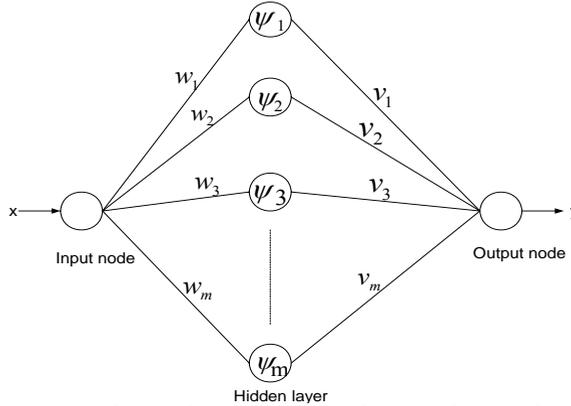


Figure.1. Structure of Wavelet Neural Network

A stochastic gradient algorithm is used for adjusting the parameters of the network in training phase. The learning is based on a sample of input-output pairs  $\{x_k, y_k\}$ , where  $y_k$  is the non-linear function to be estimated.

All the parameters of network  $w_i, v_i, t_i$  and  $D_i (=1/s_i)$ , are accumulated in a vector  $\theta$  and the output  $g_\theta(x)$  is calculated for that vector  $\theta$  using equation (1).  $y_k$  is actual value of the output. The objective function to be minimized is

$$c(\theta, x_k, y_k) = \frac{1}{2} [g_\theta(x_k) - y_k]^2 \quad (2)$$

During the training phase the algorithm optimizes the network weights, translation and dilation of the WNN. The training error is measured in terms of Root Mean Square Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (g(x_i) - y_i)^2} \quad (3)$$

where, N-number of training data.

The parameters of the vector  $\theta$  are adjusted after the presentation of each observation  $(x_k, y_k)$  in the backward direction. The modified values of the parameters are obtained by (4)

$$\theta_{k+1} = \theta_k - \eta \text{grad}(\theta_k, x_k, y_k) + \mu \Delta\theta_k$$

$$w_{k+1} = w_k - \eta \frac{\partial c}{\partial w_k} + \mu \Delta w_k$$

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$$v_{k+1} = v_k - \eta \frac{\partial c}{\partial v_k} + \mu \Delta v_k \quad (4)$$

$$t_{k+1} = t_k - \eta \frac{\partial c}{\partial t_k} + \mu \Delta t_k$$

$$s_{k+1} = s_k - \eta \frac{\partial c}{\partial s_k} + \mu \Delta s_k$$

where,  $\eta$  and  $\mu$  are the learning rate and momentum rate respectively.  $k$  is the iteration number.

The performance of developed WNN is assessed by estimating the non-linear functions at different uniformly distributed test points in the entire time span. These test error is also presented in RMSE. The network which gives the minimum value of training and test errors has been considered as the optimal network. The stochastic gradient based training algorithm is given in Figure. 2. The WNN code has been written in MATLAB and was run in Pentium-IV machine on windows platform.

## B. Particle Swarm Optimization

Particle Swarm Optimization (PSO) is an evolutionary optimization technique introduced by Eberhart and Kennedy in 1995. It mimics the way birds find food by the co-operation and competition among the entire population. PSO algorithm starts with the random initialization of a population of individuals (particles) in the search space. The particles evaluate their positions relative to a goal (fitness) at every iteration, and particles in a local neighborhood share memories of their “best” positions. Then particles use those memories to adjust their own velocities, and thus subsequent positions. In the whole procedure, each particle memorizes its own best position found so far, which is denoted by  $pbest$ . Similarly, the position of the best individual of the whole swarm found so far is denoted as  $gbest$ . The velocity and position of each particle is updated by (5)

$$v_i^{k+1} = \omega_i * v_i^k + c_1 * Rand() * (pbest_i - p_i^k) + c_2 * rand() * (gbest_i - p_i^k) \quad (5)$$

$$p_i^{k+1} = p_i^k + v_i^{k+1}$$

where,  $p_i^k$  and  $v_i^k$  are the position and velocity of the  $i^{\text{th}}$  particle at  $k^{\text{th}}$  iteration. The parameters  $c_1$  and  $c_2$  are acceleration coefficient.  $Rand()$  and  $rand()$  are two independent random numbers distributed in the range of (0, 1). The inertia weight  $\omega_i$  is introduced to improve PSO performance. The inertia weight  $\omega_i$  is assigned using (6)

$$\omega_i = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \quad (6)$$

where,  $\omega_{\max}$  and  $\omega_{\min}$  are the initial weight and the final weight,  $iter_{\max}$  is the maximum iteration number.

Researchers [15] [16] have applied PSO technique to optimize neural networks and have found that it is comparable to training of a network with gradient based algorithm. Hence, the PSO technique has been implemented in this work to train the developed WNN. The connection weights  $w$ ,  $v$ , wavelet coefficients of hidden nodes in the hidden layer,  $t$  and  $s$  are considered as the particles of a population. PSO generates possible solutions for these particles in the  $N$  dimensional search space. Each new population of weights and coefficients are transferred to the WNN for evaluation, during the forward propagation and the value of the error function for each particle in the swarm is obtained. This error value is used as particles fitness function and is optimized by updating the particle position, weights and wavelet coefficients. New generation of particles will be created and then the fitness value is calculated again. This process is repeated until the pre-defined error goal is achieved or the maximum number of generations has reached.

The network model has been developed for the estimation of complex high voltage signals using PSO training algorithm in MATLAB and was run in Pentium-IV machine on windows platform. The corresponding training algorithm is given in Figure.3.

### III. SIMULATION OF NON-LINEAR FUNCTIONS:

The developed WNN and PSO-WNN are used for function estimation and their performances have been assessed for estimation of three non-linear functions in high voltage systems, viz. lightning impulse voltage, non-sinusoidal current and chopped impulse voltage. The formulae of these waveforms are given in (7), (8) and (9) respectively.

$$\begin{aligned} V(t) &= 101.9 \exp(-0.00355t) \sin[0.01384t+1.815] \\ &+ 115 \exp(-0.977t) \sin[1.636t+4.177] \\ t &= 1 \text{ to } 100\text{s} \end{aligned} \quad (7)$$

$$\begin{aligned} i(t) &= 200 \exp(-100t) \sin 4\omega t \quad \text{for } 0 \leq t \leq 0.01; \\ &= 100 \exp(-60t_1) \sin 2\omega t_1 \quad \text{for } 0.01 \leq t \leq 0.03; \\ &\quad \& t_1 = t - 0.01; \\ &= 40 \exp(-40t_2) \sin \omega t_2 \quad \text{for } 0.03 \leq t \leq 0.07; \\ &\quad \& t_2 = t - 0.03; \\ \omega &= 2\pi f; \quad f = 50\text{Hz}; \end{aligned} \quad (8)$$

$$\begin{aligned} V(t) &= 1.037[\exp(-0.0147t) - \exp(-2.47t)] \quad \text{for } 0 \leq t \leq 6.0 \\ &= 1.037[\exp(-0.0147t) - \exp(-2.47t)] \\ &\quad - 0.9496[\exp(-0.0147t_1) - \exp(-86.8367 t_1)] \\ &\quad \text{for } 6.0 \leq t \leq 6.1 \ \& \ t_1 = t - 6.0; \\ &= -0.3 \exp(-2 t_2) \sin \omega t_2 \\ &\quad \text{for } 6.1 \leq t \leq 7.3 \ \& \\ &\quad \quad \quad t_2 = t - 6.1; \quad \omega = 2\pi f; \quad f = 2.5 \text{ Hz}; \end{aligned} \quad (9)$$

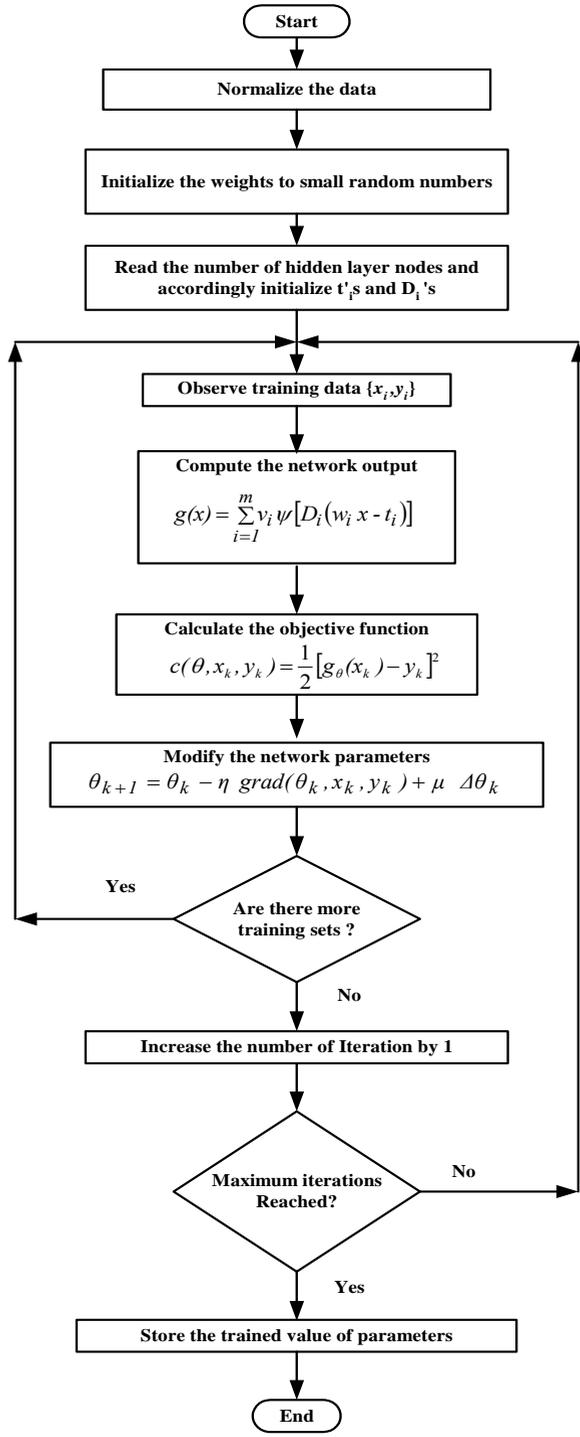


Figure.2 WNN Training Algorithm

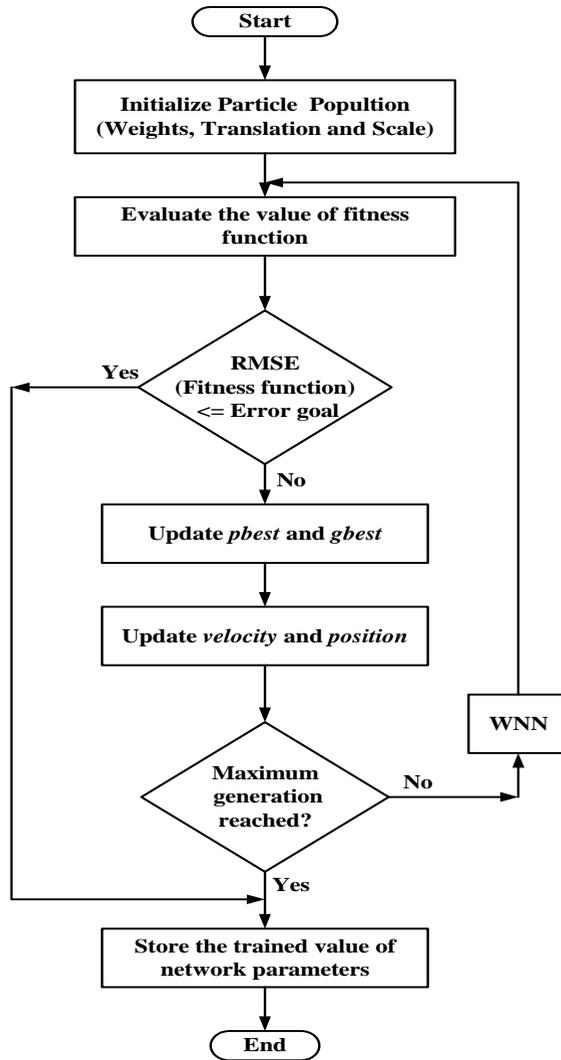


Figure.3 PSO aided WNN Training Algorithm

## IV. RESULTS AND DISCUSSIONS

### A. Wavelet Neural Network

The developed WNN has been studied with different mother wavelets, viz. Gaussian, Mexican Hat, Morlet and Shannon, to obtain an efficient network for the estimation of these non-linear functions. The network which gives acceptable values of training RMSE and testing RMSE is considered as an efficient network. It is worth mentioning here that, one may arrive at some other network structure as the efficient one if the limit of acceptable value of error is varied. The WNN is trained separately for the estimation of lightning impulse voltage, non-sinusoidal current and chopped impulse voltage. The networks will consist of one input, viz. time ( $t$ ) and one output, viz. voltage,  $V(t)$  or current,  $i(t)$ , as the case may be. However, the number of hidden nodes and particle size may vary depending upon the nature of waveform being estimated. To improve the

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convergence and also the accuracy of the learning process the training and testing data are normalized.

Investigations shows that the Mexican Hat mother wavelet has estimated the lightning impulse voltage more efficiently compared to other three mother wavelets using 116 numbers of training data. Thus the WNN is suitably designed with Mexican Hat mother wavelet. For testing the WNN another 58 data points equally distributed in the entire time span are used. To study the effect of different network parameters in details the developed WNN is trained using different values of learning rate ( $\eta$ ) and momentum rate ( $\mu$ ) with various numbers of hidden nodes. Table I shows the effect of learning rate on training and test errors for estimation of lightning impulse voltage. The results are obtained using 15 hidden nodes after 1000 iterations with  $\mu$  as 0.2. From Table I it is evident that  $\eta$  as 0.01 has given minimum value of learning and test RMSE as 0.01 and 0.1012, respectively. Hence, 0.01 has considered as the best value of learning rate. Similarly, Table II depicts the influence of momentum rate ( $\mu$ ) on the estimation of lightning impulse voltage. The results were given with the best value of  $\eta$  which was obtained from Table I, with 15 hidden nodes after 1000 number of iterations. It shows that  $\mu$  as 0.3 has given minimum testing and training error without any oscillations in the error convergence, and it has been considered as the best value of momentum rate ( $\mu$ ) of designed network. With best value of  $\mu$  and  $\eta$  obtained from Table I and II, the network was trained with different number of hidden nodes and its effect on the performance of estimation is shown in Table III. It is clear from Table III that 25 numbers of hidden nodes has given minimum values of training and testing errors after 1000 iterations. Thus the WNN with Mexican Hat mother wavelet, 25 numbers of hidden nodes with  $\eta$  and  $\mu$  as 0.01 and 0.3 has been considered as the efficient network for the estimation of lightning impulse voltage. Table IV shows that the values of training and test RMSE errors are 0.00017 and 0.0174, respectively, obtained from this efficient network after 3000 iterations, which is acceptable for all practical purposes. Comparison between estimated and analytical value of lightning impulse voltage at different test points are shown in Figure.4, where the solid line shows the analytical voltage and the dotted one represents the corresponding estimated value. The same sets of data are processed with a one input-output node with 25 hidden nodes (1-25-1) feed forward network to compare its performance with WNN. The error function and all other relevant parameters of ANN are kept same as WNN.

Similarly for non-sinusoidal current estimation, the performance of WNN is analyzed with different mother wavelets using 183 numbers of training data at different time instants. Result shows that the Morlet mother wavelet function is more suitable for this estimation compared to other mother wavelets. Sensitivity analysis has been carried out in details to find the best values of learning rate, momentum rate and number of hidden nodes. The Table IV shows that the developed network with  $\eta$  as 0.0002,  $\mu$  as 0.2 using 17 numbers of hidden nodes using Morlet mother wavelet has given acceptable value of training and test errors after 3500 iterations. The estimated values of non-sinusoidal current using this network for another 92 number of test points distributed in the entire time span are graphically shown in Figure.5 as dotted line, where as the solid lines show the respective analytical value. To compare the estimation capability of WNN with feed forward network, the same data sets are trained and tested with (1-17-1) feed forward network.

The analysis shows that the network developed with Mexican Hat mother wavelet using 25 numbers of hidden nodes after 5000 iterations with  $\eta$  and  $\mu$  as 0.0007 and 0.2 has estimated the chopped impulse voltage more efficiently compared to other mother wavelets

at 198 data points. Table IV shows the test and training error of developed network for this voltage estimation. Figure. 6 compares the analytical and estimated value of chopped impulse voltage at another 98 data point, where the solid line shows the analytical value and the dotted line represents that of estimated value. Using (1-25-1) BP network the performance of WNN has been compared using the same set of data points.

Table I. Effect of  $\eta$  on the performance of WNN for estimation of lightning impulse voltage function

$\eta$	0.04	0.02	<b>0.01</b>	0.009	0.008
<b>Training RMSE</b>	0.0126	0.0103	<b>0.0100</b>	0.0119	0.0125
<b>Test RMSE</b>	0.1335	0.1028	<b>0.1012</b>	0.1017	0.1072

Table II. Effect of  $\mu$  on the performance of WNN for estimation of lightning impulse voltage function

$\mu$	0.2	0.25	<b>0.3</b>	0.35	0.4
<b>Training RMSE</b>	0.0100	0.0096	<b>0.0093</b>	0.0126	0.0134
<b>Test RMSE</b>	0.0972	0.0954	<b>0.0932</b>	0.0997	0.1307

Table III. Effect of number of hidden layer nodes on the performance of WNN for estimation of lightning impulse voltage function

No. of hidden layer nodes	21	23	<b>25</b>	27	29
<b>Training RMSE</b>	0.0119	0.0092	<b>0.0086</b>	0.0120	0.0110
<b>Test RMSE</b>	0.0984	0.0973	<b>0.0863</b>	0.1056	0.0981

Table IV. Best values of WNN parameters and their performance in non-linear function estimation

S. No	Waveform	Type of Mother Wavelet	$\eta$	$\mu$	No. of Hidden layer nodes	No. of Iterations	Training RMSE	Test RMSE
1	Lightning Impulse voltage	Mexican Hat	0.01	0.3	25	3000	0.00017	0.0174
2	Non-sinusoidal Current	Morlet	0.0002	0.2	17	3500	0.00082	0.0360
3	Chopped impulse voltage	Mexican Hat	0.0007	0.2	25	5000	0.00079	0.0913

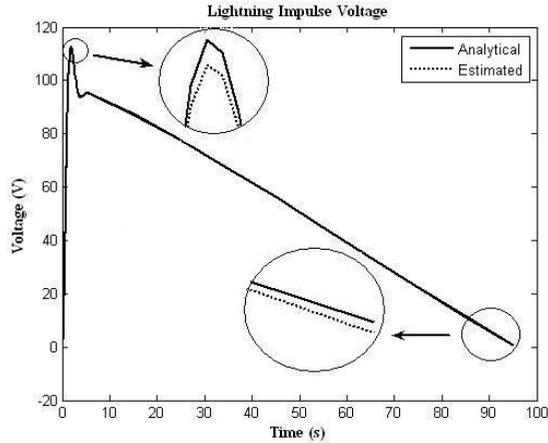


Figure. 4 Comparison of analytical and estimated values of lightning impulse voltage using WNN

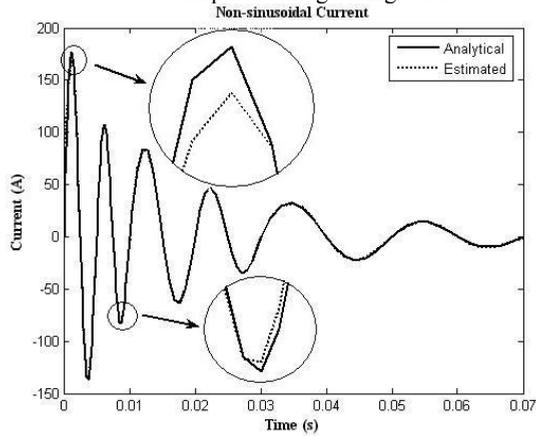


Figure. 5 Comparison of analytical and estimated values of non-sinusoidal current using WNN

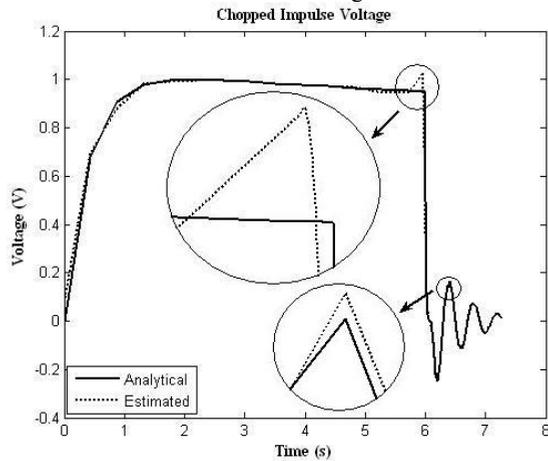


Figure. 6 Comparison of analytical and estimated values of chopped impulse voltage using WNN

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## A.II. PSO-WNN

To obtain an efficient network for the estimation of non-linear function in high voltage system, the developed PSO aided WNN has been studied with different mother wavelets, viz., Mexican Hat, Gaussian, Morlet and Shannon. These non-linear functions are estimated using three different networks, which utilizes the time as input and the voltage or current as output depending on the nature of the function.

The network developed with Mexican Hat mother wavelet is efficient compared to others for lightning impulse voltage estimation. During training PSO algorithm adjusts the parameters of the network to minimize the error function. Training and test RMSE values of the network are more sensitive to initial value of the network parameters. Since, the network weights are randomly initialized, the error values are averaged over 25 runs. To study the effect of number of hidden layer nodes, the network is trained with various number hidden nodes using 200 swarm size with 500 iterations and the value of error is given in Table V. It shows that 15 numbers of hidden nodes has given minimum value of training and test errors, and hence, it is considered as the best value of number of nodes. With the best value of number of hidden layer nodes, the network is trained with different swarm size in the span of 50 to 1000 to obtain best swarm size. The effect of swarm size on the error convergence of network after 500 iterations is shown in Table VI. It is evident that the least value of error is obtained with 500 swarm size and hence, it has been considered as the best value. Thus the network developed with Mexican Hat wavelet using 15 hidden layer nodes with 500 swarm size has been considered as the best one for estimation of lightning impulse voltage and it has given 0.00012 and 0.0103 as training and test error. Figure.7 shows that the comparison of analytical and estimated value of lighting impulse voltages at 58 different test points distributed in the entire time span as solid and dotted lines, respectively.

Similar analysis has been carried out to find the best value of network parameters for estimation of non-sinusoidal current and chopped impulse voltage. Thus the network designed using Morlet mother wavelet with 12 hidden layer nodes and 500 swarm size after 1500 iterations has been found suitable for estimation of non-sinusoidal current, and the performance of developed network for this estimation is shown in Figure. 8. Further, the network developed with 18 hidden layer nodes with 1000 swarm size after 4000 iteration has estimated the chopped impulse voltage more effectively compared to other mother wavelets at 198 data points. Comparison of analytical and estimated values chopped impulse voltage at another 92 test points are given in Figure. 9 as solid and dotted lines respectively.

Table V Performance of PSO-WNN for different number of hidden layer nodes for estimation of lightning impulse voltage

No. of hidden layer nodes	13	14	<b>15</b>	16	17
Training RMSE	0.00784	0.00572	<b>0.00365</b>	0.00476	0.00459
Test RMSE	0.1195	0.0941	<b>0.0916</b>	0.0925	0.0937

Table VI Effect of swarm size on PSO-WNN for estimation of lightning impulse voltage function

Swarm Size	300	400	500	600	700
Training RMSE	0.00173	0.00087	<b>0.00081</b>	0.00092	0.00097
Test RMSE	0.0715	0.0643	<b>0.0536</b>	0.0591	0.0621

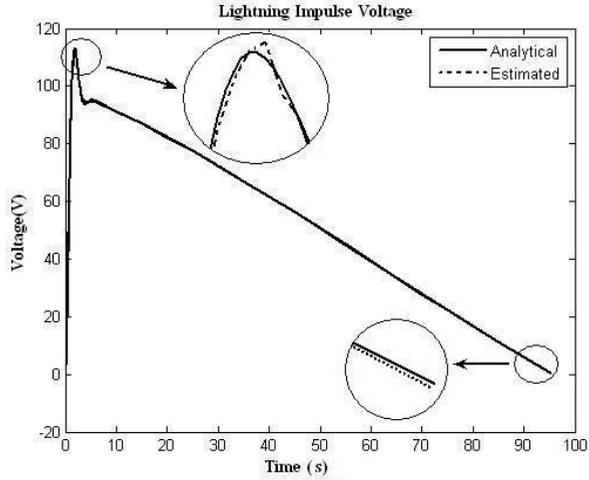


Figure. 7 Comparison of analytical and estimated values of lightning impulse voltage using PSO - WNN

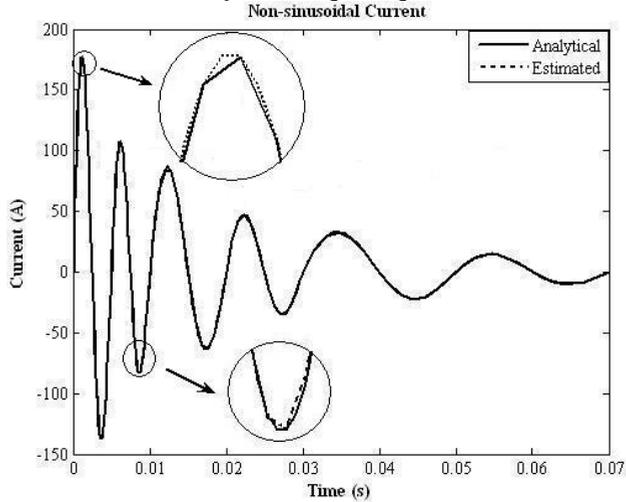


Figure. 8 Comparison of analytical and estimated values of non-sinusoidal current using PSO - WNN

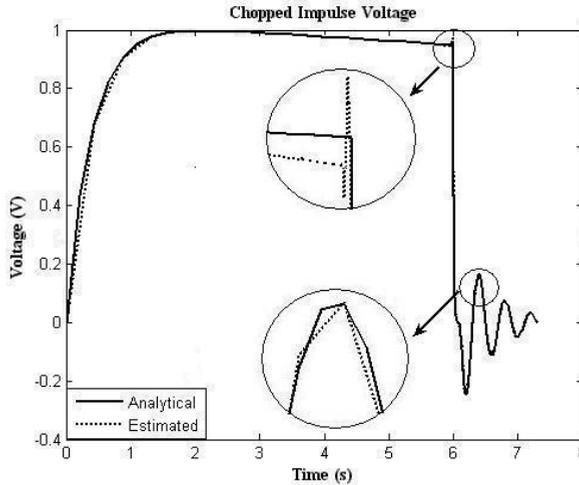


Figure. 9 Comparison of analytical and estimated values of chopped impulse voltage using PSO – WNN

### B. Comparison of WNN and PSO – WNN

A comparative study on the performance of WNN with gradient based and PSO training algorithms has been done. Further to demonstrate the efficacy of the developed WNN in function estimation, results of classical back-propagation trained feed-forward neural network (ANN) is also presented. The term ‘unit’ is used to refer neurons and hidden layer nodes. All the networks and their learning algorithms are implemented in MATLAB programming environment on Pentium IV processor. Table VII - IX provides the comparative RMSE results for estimation of impulse voltage, non-sinusoidal current and chopped impulse voltage respectively.

Table VII. Comparison of ANN, WNN and PSO-WNN performance on estimation of lightning impulse voltage

Network Type	No. of Units	No. of Iterations	Training RMSE	Test RMSE
ANN	40	5000	0.00028	0.0190
WNN	25	3000	0.00017	0.0174
PSO-WNN	15	1000	0.00012	0.0103

Table VIII. Comparison of ANN, WNN and PSO-WNN performance on estimation of Non-sinusoidal current

Network Type	No. of Units	No. of Iterations	Training RMSE	Test RMSE
ANN	55	5000	0.000937	0.03942
WNN	17	3500	0.000820	0.03604
PSO-WNN	12	1500	0.000184	0.02060

Table IX. Comparison of ANN, WNN and PSO-WNN performance

on estimation of Chopped Impulse Voltage

Network Type	No. of Units	No. of Iterations	Training RMSE	Test RMSE
ANN	62	6000	0.00074	0.0920
WNN	25	5000	0.00069	0.0913
PSO-WNN	18	4000	0.00034	0.0891

The comparison reveals that the PSO-WNN provide better performance, since, it offers a parallel search, which can overcome local minima. Table VII shows that for estimation of lightning impulse voltage the PSO-WNN yields minimum RMSE with reduced number of units and lesser iterations. Moreover, the WNN has better estimation accuracy compared to that of ANN. Table VIII depicts that the WNN requires only 17 units with 3500 iterations to obtain an accuracy that the ANN has achieved using 55 units after 5000 iterations. From Tables VII – IX, it is evident that the PSO aided WNN has higher estimation accuracy compared to both WNN and ANN.

#### IV. CONCLUSIONS

This paper describes in details the design of modern tools to estimate typical non-linear functions, viz. non-sinusoidal current, lightning and chopped impulse voltages that are practically experienced in high voltage system. It has been found that the PSO-WNN required small number of hidden layer nodes, for all the non-linear waveforms presented in this work. However, this greater accuracy has been achieved at the cost of increased execution time during training. The parameters of these trained networks may be used as features of the respective waveforms for the purpose of classification in high voltage system that lies within the scope of future work.

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