J. Electrical Systems 5-1 (2009)

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Regular paper

Use of Cooperative Game Theory Concepts for Loss Allocation in Multiple Transaction Electricity Markets

The paper presents a cooperative game theory application for analyzing the allocation of the transactional transmission losses in Multiple-Transaction Electricity Markets. This paper identifies a set of approaches documented in literature in areas of application of Cooperative Game Theory (CGT) and reviews their applicability to allocations of losses in Multi-Transaction Electricity Markets. The allocation methods can be broadly classified into three categories, i.e. Classical, Existing, and Variants of Nucleolus. In this paper, two points are focused during the study i.e. the structure of the game and the concept of fairness behind it. The allocation concept of these methods is systematically analyzed, compared and calculations have been performed on standard IEEE 14 and IEEE 118 bus systems. The Proportional Nucleolus is shown to be among the most plausible concept.

A power flow (PF) procedure is applied to calculate power losses for each transaction. Various methods available in the literature of CGT applied to loss allocation in the Multi-Transaction are also compared with the convectional approach and the results are tabulated accordingly.

Keywords: Multiple-transaction Model, Cooperative game theory, Deregulated Electricity market, loss allocation, Shapley, Core, Nucleolus, Proportional Nucleolus, Disruptive Nucleolus.

1. INTRODUCTION

There has been a lot of change in many aspects of the electricity business, including the pricing of electricity by introduction of the new concept of deregulation in the electricity market. In this deregulated market, participants must require a fair and equitable pricing structure that reflects both the share of power Generated/Consumed in the network and the cost of power loss caused by users. These allocations influence on decision-making of the electricity market participants for their financial commitments and their profits, i.e., these changes have resulted in a radical shift in the way losses are distributed among market participants and have gained financial significance. The Market participants must be charged for the losses in a way, which reflects their use of the network. Many different loss allocation schemes have been proposed in this area, but no method has gained universal acceptance. The problem of allocating the transmission losses among the power system users has become more important with the increase in the competition level in electricity markets.

The evaluation of the loss allocations by proportional sharing procedures has been widely discussed in many papers [1]-[11]. Reviews of Cost Based Transmission Losses Allocation Methods are presented in [12]. These methods have been suggested to allocate the system losses to generators and loads in a pool market or to individual transactions in a bi-lateral contracts market. They are mainly classified into four categories as follows: (i)

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Pro rata method allocates the transmission system losses to the generators and loads proportional to their active power generation and load consumption. The main disadvantage in this approach is not taking topology of network into account. It is not fair as it allocates the same amount of losses when two identical loads are considered in which one is located near to generators and other is far away from generators.
(ii) Incremental transmission loss (ITL) methods are utilizing the sensitivities with respect to nodal injections to allocate the losses to generators and loads. The ITL methods depend on the selection of the slack bus and there is no allocation of losses to the slack bus. (iii) Proportional sharing procedures allocate the system losses by using the tracing techniques. These methods compute the losses at each branch of transmission network and then the electrical tracing (power flow between each generator and each demand) is computed the responsibilities in the line losses without considering generator or demand. (iv)Transactional losses allocation methods are formulated according to bilateral contracts and Multi-Transaction contracts in competitive markets.

Traditionally, agreement over allocation of losses is reached through discussions between participants and these losses can be divided without any assistance from knowledge of game theory. It remains a challenge to electricity business to select allocation concepts or methods for total losses and distribute it among the participants in Multi-Transaction Electricity Markets. This is because the participants do not have knowledge about the fair allocation concepts to be accepted.

Game theory looks at rational behavior when each decision maker's well being depends on the decisions of others as well as his or her own. Game theory comprises of *cooperative* and *non-cooperative* game theory. In cooperative game, it is assumed that the decision makers (or, "players") are able to sign legally enforceable contracts with each other. Non-cooperative game theory does not make this assumption. It is a discipline that is used to analyze problems of conflict among interacting decision makers. It may be considered as a generalization of decision theory to include multiple players or decision makers.

This paper focused on the allocation of losses using CGT where several transactions jointly maintain in Multi-Transaction Electricity Market environment. An extension of the Core Solution Concept is presented for handling the empty-core situations in [13]. The advantage of extended core is presented in this paper. If core is nonempty, the concept of extended core always coincides with core. The unique allocation belongs to this extended core, i.e. Proportional Nucleolus is proposed as selection device. The supportive mathematical proofs are presented in this paper. How the Proportional Nucleolus allocation is better than other allocations such as Shapley and Nucleolus etc is dealt in [14]. The comparison of nucleolus approach with Shapley is presented in [15]. The Shapley value is unstable from the perspective of the core or negotiation standpoint. The various methods in CGT and the application of testing the convexity and additivity functionalities of allocation methods in CGT are extensively dealt in [16]. A cooperative theory model by representing equivalent bilateral exchanges as players for selecting a unique loss allocation, the core, is presented in [17]. The other solution concepts such as the Shapley value, the Bilateral Shapley value and the kernel are also explored. It is also dealt why is not possible to find an optimal solution for allocating the cost of losses to the users of network. The application of CGT methods applied to the deregulated environment is presented in [18]-[19]. This paper presents the transactional losses to be allocated among traders using cooperative game theory methods

This paper suggests that CGT application is an effective approach for the solution of the problem. First, it shows that the loss allocation problem is identical to the problem of computing the value of an n-person cooperative game. A case study shows why some allocation methods, failing to satisfy the mandatory properties, have to be rejected. The cooperative game theoretic ideas are attentively implemented for loss allocation i this paper. Then, the paper proposes *Proportional Nucleolus method* as it has unique solution even if the core is empty and compares the variants of Nucleolus and Shapley from cooperative game theory.

2. REVIEW OF CGT CONCEPTS AND ALLOCATION METHODS

Cooperative games have the following ingredients

i) A set of players

Let N= {1, 2…n} be the finite set of players and let i, where i, is from 1 to n, index the different numbers of N.

ii) A characteristic function, specifying the value created by different subsets of the players in the game, is denoted by ν. The Characteristic function is a function expressed as a number and is associated with every subset S of N, denoted by ν(S). The number ν(S) is interpreted as the value created when the members of S come together and interact. Into to, a cooperative game is a pair (N,ν), where N is a finite set and ν is a function mapping subsets of N to members of the game.

iii) Imputation:

For a Given Cooperative Game (N,ν), an allocation $X = (x_1, x_2, x_3 ....x_n)$ is called as an imputation.

iv) A key concept in cooperative game theory is the *core* of the game. The Core is defined as a set of imputations satisfying the following conditions.

$$x_i \geq \nu(\{i\}) \quad \text{(Individual rationality)} \tag{1}$$

$$\sum_{i \in N} x_i \geq \nu(S) \quad \text{(Coalition rationality)} \tag{2}$$

$$\sum_{i \in N} x_i = \nu(N) \quad \text{(Collective Rationality)} \tag{3}$$

For a game is in coalition form (N, ν) the core is denoted as $C(N, \nu)$.

2.1. Formulation of Classical Cost Allocation methods

The mathematical formulation of these methods is given in [14].

The separable cost for player i (SC) and non-separable cost for player i (NSC) and the marginal cost for player i in S (MC (S, S- {i})) are defined as follows.

$$SC(i) = \nu(N) - \nu(N \setminus \{i\}) \tag{4}$$

$$NSC = \nu(N) - SC(i) \tag{5}$$

$$MC(S, S - \{i\}) = \nu(S) - \nu(S - \{i\}) \tag{6}$$

The separable cost for player i (SC_i) represent an incremental (marginal) cost when i participate in coalition N- {i}. The separable cost is considered the minimum cost, which should be allocated when player i is assumed to participate in the grand coalition at the last moment. NSC is remaining costs after SC_i is allocated to the each player.

Based on the above definitions and standard nomenclature, the following models are formulated [14].

2.1.1 Equal Repartition of the Total Gain (ERTG)

$$x_i = \nu(i) - \frac{1}{N}\left[\sum_j \nu(j) - \nu(N)\right] \tag{7}$$

2.1.2 *Proportional Repartition of the Total Gain or Morianty's Method* (PRTG)

$$x_i = v(i) - \frac{v(i)}{\sum_j v(j)}\left[\sum_j v(j) - v(N)\right] \\ = \frac{v(i)}{\sum_j v(j)} v(N) \tag{8}$$

2.1.3 *Equal Repartition of the Non-Marginal Costs* (ERNMC)

$$x_i = SC(i) - \frac{1}{N}\left[v(N) - \sum_j SC(j)\right] \tag{9}$$

2.1.4 *Proportional Repartition of the Non-Marginal Costs* (PRNMC)

$$x_i = SC(i) - \frac{SC(i)}{\sum_j SC(j)}\left[v(N) - \sum_j SC(j)\right] \\ = \frac{SC(i)}{\sum_j SC(j)} v(N) \tag{10}$$

2.1.5 Practical Cost allocation Method

This method is called as Separable Cost Remaining Benefits (SCRB). It allocates SC (i) to player i as the players minimum burden. Then, the NSC is allocated to the player i in proportional to the remaining benefits [16].

$$x_i = SC(i) + \frac{v(\{i\}) - SC(i)}{\sum_{j \in N} v(\{j\}) - SC(j)} NSC \tag{11}$$

2.1.6 Egalitarian –Non Separable Cost allocation Method (ENSC)

This method is called as Egalitarian- Non Separable Cost Method (ENSC). It allocates SC (i) to player i as the players minimum burden. Then, the NSC is allocated equally to all the players [14].

$$x_i = SC(i) + \frac{1}{N} NSC \tag{12}$$

2.2. The Existing Methods

2.2.1 Shapley

The Shapley value is a solution concept that predicts a unique expected allocation for every given value in the coalition. The rule for the Shapley Value allocation is that each player should be awarded his average marginal contribution to the coalition, if one considers all possible sequences for forming the full coalition.

For a given game in coalitional form (N,ν), the Shapley value is denoted by $\phi(v)$

$$\phi(v) = (\phi_1(v), \phi_2(v) \ldots\ldots\ldots\ldots\ldots \phi_n(v)) \tag{13}$$

where

$$\phi_i(\nu) = \frac{1}{n!}\sum_S (s-1)!(n-s)!\left[\nu(s) - \nu(S \setminus \{i\})\right] \tag{14}$$

This formula can be interpreted as follows: suppose n –players participate one after the other into the coalition that will eventually form the grand coalition. Consider all possible sequential participation of n players. Suppose that any sequence occurs with a probability $\frac{1}{n!}$. If player i participates and finds Coalition (S- {i}) already in the coalition and the player's contribution to the coalition is ($\nu(S) - \nu(S \setminus \{i\})$). The Shapley value is the expected value of the contribution of player, i.e. $\phi_i(\nu)$. That is, the Shapley value awards to each player the average of his marginal contributions to each coalition. While taking this average, all orders of the players should be considered equally. It is a fair way to distribute the total gains to the players assuming that they form coalitions.

Shapley has proved that there exists one and only one allocation that satisfies the following four axioms.

Efficiency: $\sum_{i \in N} \phi_i(\nu) = \nu(N)$, this is a collective rationality that the total value of the players is the grand coalition.

Symmetry: If i and j are such that $\nu(S \cup \{i\}) = \nu(S \cup \{j\})$

Dummy Axiom: If i is such that $\nu(S) = \nu(S \cup \{i\})$ for every coalition S not containing i such that $\phi_i(\nu) = 0$

Additive: If u and v are characteristic functions, then $\phi(u+v) = \phi(u) + \phi(v)$

2.2.2 Nucleolus

All the allocated benefit x satisfying three properties stated in the equations (1), (2), (3) respectively is a Core solution, which is generally not unique. To decide a unique benefit allocation from a Core solution, the Nucleolus is introduced. It is based on the concept of coalition satisfaction. For a given allocation x, the complaint or excess of coalition S is defined as $e(X : S) = \nu(S) - \sum_{i \in S} x_i$.

From the equation (2), it is understood that an imputation X is in the core if and only if all of its excesses are negative. Then, the Nucleolus is a maximum lexicographical solution for all coalition excesses vectors.

The Nucleolus can be calculated by using linear programming, i.e. the objective is to minimize function of the maximum excesses (dissatisfaction) vector over the non-empty set of imputations, represented as

$\varepsilon = \max_S e(X : S)$. It is also called as prenucleolus. Whenever the prenucloelus satisfies the individual rationality, the imputation coincides with the nucleolus

$$\begin{aligned} &\text{Min } \varepsilon \\ &\sum_{i \in S} x_i - \nu(S) \le \varepsilon \qquad (\forall S \subset N) \\ &\sum_{i \in N} x_i = \nu(N) \end{aligned} \tag{15}$$

2.3. Variants of Nucleolus

The variants of the Nucleolus are defined by modifying the excesses (dissatisfaction) function as follows

2.3.1 Proportional Nucleolus (Prop-Nu)

An extended core concept is introduced, as a solution concept for cooperative games for the empty core environment .The main characteristic of extended core is always nonempty unlike core. This solution concept coincides in the cases where core is nonempty. It is an important characteristic of extended core solution concept. The question is how to handle these empty-core situations. It gains greater importance, as there are considerable numbers of games in which the core cannot be applied. As the extended core is a multiple valued concept, it is important to find a unique solution among its imputations. The proportional nucleolus always chooses an imputation from the extended core in a similar way as the concept of nucleolus can be used to select a particular imputation from the core. The nucleolus formalizes the idea of a fair distribution of output in the sense of choosing the imputations that minimizes the biggest excess by any coalition as illustrated above. The proportional nucleolus differs from the original nucleolus in the definition of excess concerned with coalitions that suffer the biggest proportional excess of their worth. It is defined as

$$e(X:S)=\frac{\sum_{i\in S} x_i - v(S)}{v(S)} \tag{16}$$

If $\mathrm{N}=\{S_1.S_2........S_{2^n}\}$=Set of all possible coalitions,

the proportional nucleolus $\hat{\mathrm{N}}(N,v)$ of a strictly positive game satisfies the following properties.

$\hat{\mathrm{N}}(N,v)$ is non-empty.

$\hat{\mathrm{N}}(N,v)$ is a single-valued.

$\hat{\mathrm{N}}(N,v)$ always belongs to the extended core.

If the core $C(N,v)$ is nonempty, $\hat{\mathrm{N}}(N,v)$ belongs to the core.

The proportional Nucleolus can expand the core to obtain a unique solution in both cases of empty core and large core. *Thus, the proportional nucleolus is a better solution to both the extended core and core selection problem. This ability of Proportional Nucleolus to select an imputation is another advantage of the extended core as a solution concept.*

The solution approach for Proportional Nucleolus is to solve a linear program as the following formation

$$\text{Min } \varepsilon$$

$$\frac{\sum_{i\in S} x_i - v(S)}{v(S)} \quad \text{and} \quad \sum_{i\in N} x_i = v(N) \tag{17}$$

2.3.2 Weak Nucleolus or Average Nucleolus (W-Nu)

The weak Nucleolus concept replaces the excess with the average excess $\frac{\sum_{i\in S} x_i - v(S)}{|S|}$ where $|S|$ is the cardinality of coalition S.

The solution approach is given as follows

$$\text{Min } \varepsilon$$
$$\frac{\sum_{i\in S} x_i - v(S)}{v(S)} \le \varepsilon \quad \text{and} \quad \sum_{i\in N} x_i = v(N) \tag{18}$$

2.3.3 Disruption Nucleolus or Propensity to Disrupt (Dis-Nu)

Propensity to disrupt is an approach similar to weak-Nucleolus; only the excess e *(X: S)* is replaced by *d* (X: *S)*, where d (X: *S)* is a ratio of excess for coalition S to excess for coalition *{N-S}*. and given as $\dfrac{\sum_{i\in N\setminus S} x_i - v(N\setminus S)}{\sum_{i\in S} x_i - v(S)}$

The solution approach is given as

$$\text{Min } \varepsilon$$
$$\frac{\sum_{i\in N\setminus S} x_i - v(N\setminus S)}{\sum_{i\in S} x_i - v(S)} \le \varepsilon \quad \text{and} \quad \sum_{i\in N} x_i = v(N) \tag{19}$$

As a necessary condition for the Propensity to Disrupt to have a solution, group rationality must hold [16].

2.3.4 Average Disruption Nucleolus (AVG-DIS-NU)

$$\text{Min } \varepsilon$$
$$\frac{\sum_{i\in N\setminus S} x_i - v(N\setminus S)}{|N\setminus S|} - \frac{\sum_{i\in S} x_i - v(S)}{|S|} \le \varepsilon \quad \text{and} \sum_{i\in N} x_i = v(N) \tag{20}$$

3. Comparison of Methods for the Problem of Loss Allocation and Selection of Suitable Approach

3.1. Conventional Methods

The ERNMC has equivalent solution with ENSC in the conventional allocation methods [16]. Practical Cost allocation method (SCRB) is useful in terms of simplicity of its calculation and it holds the fairness of Disruption Nucleolus and Average Disruption Nucleolus. But it is given in [20] that the SCRB is not a good allocation method in Conventional Method.

3.2. Shapley Allocation

The Shapley value is monotonic and its allocation may not lie in the core. As the Shapley value has been subjected to satisfy some axioms, mainly focusing on the additive axiom, it does not seem to be good solution for loss allocation. *The Coalition of losses never satisfies the additive property.* Under deregulation environment of power market, all the transactions are being taken place through utilizing the same network with cooperative manner. The transmission losses caused by active power contract cannot be contemplated in advance in contracted quantity of power. These transaction losses can be considered as a common platform that should be allocated among players. Owing to quadratic form of the loss expression, *the sum of the transmission losses caused by individual transaction is not equal to the system losses caused by transactions simultaneously in coalition. The losses allocation using CGT do not satisfy the additivity property. The Shapley is unstable from* *the perspective of core or the negotiation standpoint [15].*

3.3. Nucleolus and its Variants

The advantage of Nucleolus is always being contained in the core and drawback is, sometimes it is not monotonic and the variants of Nucleolus are having the similar properties except Proportional Nucleolus. i.e. Proportional Nucleolus is monotonic. The Nucleolus loss allocation method favors to some players when compared with the other allocation methods. This is another drawback of Nucleolus Method. Both Disruption Nucleolus and Average Disruption Nucleolus give nearly identical allocation. These concepts belong to the same class in terms of consideration of both the players, own and opponent's dissatisfaction.

The proportional Nucleolus has greater importance as its application encourages the empty core situations. As Proportional Nucleolus consists of a single point, it provides an alternative to the Shapley value. Among these methods, one is considered preferable by the authors, while among the CGT ones the Proportional Nucleolus seems to be the best, perhaps the inherent property of Proportional Nucleolus that it finds a unique solution in the extended core and the extended core coincides with the core when it is non empty. This is precisely the way in which the Proportional Nucleolus works. *This paper proposes the approach of Proportional Nucleolus as the best Method suitable to loss allocations in the* Multiple-Transaction Electricity Markets.

4. MULTIPLE TRANSACTION FRAME WORK FOR LOSS ALLOCATION

The problem of loss allocation can be solved by using multiple transactions framework with cooperative game theory. The multiple transaction frameworks for transmission loss allocation definitions are given in [1]-[2] for the system with n buses and m transactions. A triplet can characterize every transaction m. The elements of this triplet are $t^{(m)}$, the transaction amount in MW and $S^{(m)}$,$B^{(m)}$, the selling and buying entities respectively.
The selling $S^{(m)}$ is the collection of 2-tuples of selling entities.

$$S^{(m)} = \left\{ \left(s_i^{(m)}, \alpha_i^{(m)} \right), \mathrm{i} = 1,2,3\ldots..\mathrm{N}_\mathrm{s}^{(\mathrm{m})} \right\} \tag{21}$$

with the selling bus $s_i^{(m)}$ supplying $\alpha_i^{(m)}.t_i^{(m)}$ MW of the transaction amount. The fraction of $\alpha_i^{(m)}$ must satisfy the conditions $\sum_{i=1}^{N_s^{(m)}} \alpha_i^{(m)} = 1$ and $\alpha_i^{(m)} \in [0,1]$, i=1,2,3… $N_S^{(m)}$

Similarly $B_i^{(m)}$ is the collection of 2-tuples of buying entities.

$$B^{(m)} = \left\{ \left(b_i^{(m)}, \beta_i^{(m)} \right), \mathrm{i} = 1,2,3\ldots..\mathrm{N}_\mathrm{b}^{(\mathrm{m})} \right\} \tag{22}$$

where the buying bus $b_i^{(m)}$ receives $\beta_i^{(m)}.t_i^{(m)}$ MW of the transaction amount. The fraction of $\beta_i^{(m)}$ must satisfy the conditions $\sum_{i=1}^{N_b^{(m)}} \beta_i^{(m)} = 1$ and $\beta_i^{(m)} \in [0,1]$, i=1,2,3…$N_b^{(m)}$

This paper considers each multiple transaction as one player in the cooperative game theory and all transactions happening simultaneously as coalition. Various methods available in the literature of CGT is applied to loss allocation in the Multi-Transaction and also compared with the approach presented in Transmission Loss Allocation in a Multiple-Transaction Frame work [2]. The method is abbreviated as TLAMTF.

5. ALGORITHM FOR THE PROPOSED APPROACH FOR THE PROBLEM OF LOSS ALLOCATION

1 Read Power Flow Data of System.
2 Read the Number of Transactions as Players in the Game.
3 Start with player i=1.
4 Set the status of Transaction, S for individual player i, ON and transaction state is put into operation.
5 Run the Power flow to compute the losses corresponding to S and form the elements of $\nu(S)$ for individual transaction.
6 Formation of losses is completed for individual transactions? If no, choose the next individual player (i=i+1) and set the transaction state related to the player, if no, go to step 4. If yes, go to 7.
7 Set the status of Transaction, $(S \cup \{i\})$ for coalition ON and corresponding coalition Transaction state is put into service.
8 Run the power flow to compute the losses related to the coalition $\nu(S \cup \{i\})$ including Grand Coalition.
9 All coalition elements are formed? If no, choose the next combination of coalition $(S \cup \{i\})$ of transactions, If no, go to step 7.If yes, go to 10.
10 Form the characteristic function $\nu(S)$.
11 Find the maximum dissatisfaction (proportional) using $\varepsilon = \max_S \frac{e(X:S)}{V(S)}$ by using linear Programming
12 Minimize the maximum dissatisfaction (Proportional) subject to $\sum_{i \in S} x_i \le \nu(S)(1-\varepsilon)$ and $\sum_{i \in N} x_i = \nu(N)$
13 Compute the loss allocation vector X
14 Stop.

6. CASE STUDY

Evaluation of loss allocation by different methods is performed onIEEE-14 bus test system and six bus test systems

6.1. Case Study 1: IEEE 14 Bus System

Standard IEEE 14 Bus System is shown in Fig 1 and is used to illustrate and examine the methods described above.
The system is shown in Fig.1 that has 14 nodes, 20 branches and 3 generators.
The total trading of system is 259 MW. This system is partitioned with specified three transactions and each transaction may involve multiple generation sources and multiple load centers. It is assumed that the total load at buying node is considered as it is in the transaction without partitioning. Table 1 summarizes the transaction profiles.
In this model, three transactions exist in the market, which are interacting simultaneously and are treated as three players. The situation of allocation of the loss for transactions is considered as cooperative game model.

TABLE 1 TRANSACTION DATA FOR IEEE-14 BUS TEST SYSTEM

Transaction M	$t^{(m)}$ MW	$S^{(m)}$	$B^{(m)}$ Participation of Buying Nodes in the Transaction

1	83	(1,80%), (2,20%)	2,4,13
2	115	(1,80%), (2,10%), (3,10%)	3,6,11,12
3	61	(1,66.5%), (2,19.5%), (3,14%)	5,9,10,14
Total	259		

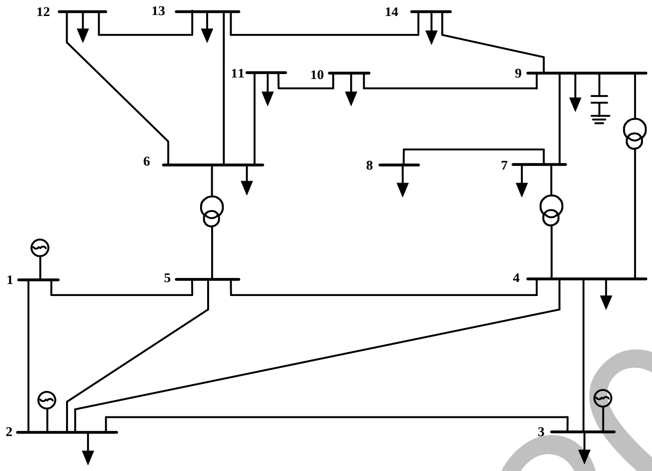

Fig .1 IEEE 14-bus Test system

6.1.1 Formation of Game Structure

While defining the characteristic function of transactional losses, the first problem in this case study is to identify the independent players in the system. Three independent transactions according to the considerations are involved. The possible (2^3-1) coalition structures for losses of the above 3-player game, are introduced as a vector with all consequent possibilities, grouped together into seven. They are $v(1), v(2), v(3), v(12), v(13), v(23), v(123)$ (for simplicity $v(\{1,2\})$ is denoted as $v(12)$ …. etc) respectively. A power flow procedure is applied to calculate power losses for the each transaction model and the results are shown below. In this paper the active power losses (MW) are considered for the analysis. It is found that

$v(1)$ =1.275 MW $v(2)$ =3.471 MW $v(3)$ =1.466 MW

$v(12)$ =7.005 MW $v(13)$ =4.081 MW $v(23)$ =5.672 MW

$$v(123) = 11.21 \tag{23}$$

The core is evaluated as follows.

$$\begin{aligned} &1.2750 \le x_1 \le 5.5380 \\ &3.4710 \le x_2 \le 7.1290 \text{ MW} \\ &1.466 \le x_3 \le 4.2050 \end{aligned} \tag{24}$$

where x_1, x_2, x_3 are the losses allocated to transactions 1, 2,3 respectively.

6.1.2 Concept of Fairness

The methods proposed for cost of loss allocation are originating from game theory. Now, the question is which of them should be selected? The study of the following analytical properties helps leading to the choice for loss allocation. The results of the methods indicate widely different allocations. They are compared by their natural properties. In order for a method to be fair, it certainly has to satisfy the three natural properties defined in the equations (1), (2), (3) respectively. It is observed that from the Table 2

6.1.2.1 Individual rationality

The loss allocation of each transaction is greater than the loss created by individual transaction. It is desirable property.

$$x_1 \geq v(\{1\}) \tag{25}$$

$$x_2 \geq v(\{2\}) \tag{26}$$

$$x_3 \geq v(\{3\}) \tag{27}$$

TABLE 2 ALLOCATION RESULTS OF DIFFERENT METHODS FOR IEEE-14 BUS SYSTEM

Type of Method	x_1	x_2	x_3	$x_1 + x_2 + x_3$
ERTG	2.941	5.137	3.132	11.21
PRTG	2.300829	6.263669	2.645502	11.21
ERNMCG	3.650667	5.241667	2.317666	11.21
PRNMC	3.679527	4.73661	2.79863	11.21
SCRB	3.679527	4.73661	2.793863	11.21
ENSC	3.650667	5.241667	2.317666	11.21
Shapley	3.2958	5.1893	2.7249	11.21
Nucleolus	3.1764	5.1981	2.8355	11.21
W-Nu	3.1458	5.2287	2.8355	11.21
Prop-Nu	3.6218	5.6482	1.94	11.21
Dis-Nu	3.2737	5.1861	2.7502	11.21
AVG-DIS-NU	3.1184	5.1632	2.9284	11.21
TLAMTF	3.717	5.146	2.349	11.21

From Table 2, for instance, the Nucleolus allocation is x = (3.1764, 5.1981, and 2.8355) MW. The above equality is tested as

$$x_1 = 3.1764 > v(\{1\}) = 1.275 \text{ MW} \tag{28}$$

$$x_2 = 5.1981 > v(\{2\}) = 3.471 \text{ MW} \tag{29}$$

$$x_3 = 2.8355 > v(\{3\}) = 1.466 \text{ MW} \tag{30}$$

It is observed from Table 2 that all allocation methods satisfy individual rationality.

For example, consider also the Shapley allocation for which x = (3.2958, 5.1893, and 2.7249),

where

$$x_1 = 3.2958 > v(\{1\}) = 1.275 \text{ MW} \tag{31}$$

$$x_2 = 5.1893 > v(\{2\}) = 3.471 \text{ MW} \tag{32}$$

$$x_3 = 2.7248 > v(\{3\}) = 1.466 \text{ MW} \quad (33)$$

This means that loss allocation by Shapley is inside the core.

6.1.2.2 Coalition Rationality

The sum of losses of each transaction in coalition is more than the losses caused by the coalition

$$x_1 + x_2 > v(12)$$
$$x_1 + x_3 > v(13) \quad (34)$$
$$x_2 + x3 > v(23)$$

From Table 2, for instance, the Nucleolus allocation is x = (3.1764, 5.1981, 2.8355) MW. The above criterion is tested as

$$x_1 + x_2 = 3.1764 + 5.1981 = 8.3745 > v(12) = 7.005 MW$$
$$x_1 + x_3 = 3.1764 + 2.8355 = 6{,}0119 > v(13) = 4.0811 MW \quad (35)$$
$$x_2 + x3 = 5.1981 + 2.8355 = 8.0336 > v(23) = 5.672 MW$$

It is inferred from the loss allocation Table 2 that all the loss allocation methods satisfy the Coalition Rationality condition.

6.1.2.3 Collective Rationality

$$x_1 + x_2 + x_3 = v(123) \quad (36)$$

From the Table 2, for instance, the Nucleolus allocation is x = (3.1764, 5.1981, 2.8355) MW. The above equality is tested as

$$x_1 + x_2 + x_3 = 3.1764 + 5.1981 + 2.8355$$
$$= 11.21 MW = v(123) \quad (37)$$

It is observed that from the loss allocation Table 2 that all the loss allocation methods satisfy the Collective Rationality condition.

6.1.3 Properties of the Game

6.1.3.1 Convexity

A game is convex if $v(S) + v(T) \geq v(S \cup T) + v(S \cap T)$ for all coalitions $S, T \subset N$. Assume that the coalition S is a strict subset of coalition T. In the three player trading game, convexity reduces into three conditions [14].

$$v(12) + v(13) \geq v(123) + v(1) \quad (38)$$
$$v(12) + v(23) \geq v(123) + v(2) \quad (39)$$
$$v(13) + v(23) \geq v(123) + v(3) \quad (40)$$

This can be checked as follows

$$v(12) + v(13) = 7.005 + 4.081 = 11.086 < v(123) + v(1) = 11.21 + 1.2575 = 12.4675 \text{ MW} \quad (41)$$

$$v(12) + v(23) = 7.005 + 5.672 = 12.677 < v(123) + v(2) = 11.21 + 3.471 = 14.681 \text{ MW} \quad (42)$$

$$v(13) + v(23) = 4.081 + 5.672 = 9.753 < v(123) + v(3) = 11.21 + 1.466 = 12.676 \text{ MW} \quad (43)$$

thus the game is not convex.

6.1.3.2 Additivity

The Shapley value allocation solution to the problem is oriented towards axiomatic approach rather than defining a solution concept. There is a possibility that Shapley value solution for losses allocation does not lie in the core as given in the previous section. Here the additive property is the strongest axiom among others and, is tested, as follows

$$\text{i.e.}\quad \begin{aligned} \nu(12) &= \nu(1)+\nu(2) \\ \nu(13) &= \nu(1)+\nu(3) \\ \nu(23) &= \nu(2)+\nu(3) \end{aligned} \tag{44}$$

In this case, for example

$$\nu(13) = 4.081 < \nu(1)+\nu(3) = 1.275 + 1.466 = 2.741\,\text{MW} \tag{45}$$

The allocation of losses using CGT may not satisfy the additive property, since loss, which is quadratic in nature, does not satisfy the principle of superposition

6.2. Case Study 2: IEEE 118Bus System

In this case study, a detailed explanation is presented about the suggested Proportional Nucleolus Method. The study is performed on the IEEE 118 bus system with 19 generating units, 118 load buses and 186 branches. The paper considers the IEEE 118 bus system that has been partitioned with specified four transactions. Each transaction may have multiple generator sources and multiple load centers. It is assumed that the total load at buying node is considered as it is in the transaction without partitioning. The table shown in below summarizes the transactional profiles.

TABLE 3

TRANSACTION DATA FOR IEEE-118 BUS TEST SYSTEM

Transaction Amount $t^{(m)}$ MW	Selling Nodes $\left(S_i^m, \alpha_i^{(m)} t_i^{(m)}\right)$ (Selling Node, transaction Amount)	$B_i^{(m)}$ Participation of Buying Nodes in the Transaction
574 MW	(10,63.00), (12,16.15), (25,44.00), (26,47.10), (31, 0.00), (46, 0.00), (49,28.56), (54, 6.24), (59,21.70), (61,22.40), (65,54.74), (66,47.04), (69,45.57), (80,57.24), (87, 0.00), (89,84.98), (100,35.28), (103, 0.00), (111, 0.00),	1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73 77 81 85 89 93 97 101105,109,113,117
1232 MW	(10,130.50), (12,20.40), (25,50.60), (26,87.92), (31, 4.90), (46, 5.70), (49,59.16), (54,14.40), (59, 0.00), (61, 0.00), (65,101.66), (6,121.52), (69,191.26), (80,147.87), (7,1.72), (89,176.03), (100,85.68), (103,17.20), (111,15.48),	2 6 10 14 18 22 26 30 34 38 42 46 50 54 58 62 66 70 74 78 82 86 90 94 98 102 106 110 114 118
1176 MW	(10,121.50), (12,22.95), (25,59.40), (26,84.78), (31, 0.00), (46, 7.60), (49,55.08), (54,12.96), (59,86.80), (61,89.60), (65,117.30), (6,105.84), (69,29.87), (80,128.79), (87, 1.08), (89,163.89), (100,68.04), (103,10.80), (111, 9.72),	3 7 11 15 19 23 27 31 35 39 43 47 51 55 59 63 67 71 75 79 83 87 91 95 99 103 107 111 115

1260 MW	(10,135.00), (12,25.50), (25,66.00), (26,94.20), (31, 2.10), (46, 5.70), (49,61.20), (54,14.40), (59,46.50), (61,48.00), (65,117.30), (6,117.60), (69,114.30), (80,143.10), (87, 1.20), (89,182.10), (100,63.00), (103,12.00), (111,10.80),	4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 96 100 104 108 112 116

6.2.1TRANSACTIONAL LOSSES OF 118-BUS SYSTEM

The characteristic values of four-player game, i.e. Coalition Characteristic Function for Transaction losses for IEEE-118 BUS TEST SYSTEM in MW are shown below

$\nu(1)$ =16.052 $\nu(2)$ =26.685 $\nu(3)$ =27.544 $\nu(4)$ =21.891

$\nu(12)$ =38.555 $\nu(13)$ =43.714 $\nu(14)$ =34.235 $\nu(23)$ =54.498

$\nu(24)$ =51.945 $\nu(34)$ =50.749 $\nu(123)$ =81.016 $\nu(124)$ =77.854

$\nu(134)$ =82.654 $\nu(234)$ =96.336 $\nu(1234)$ =136.863 (46)

By considering the coalition vectors shown in above , the loss allocation based on Proportional Nucleolus can be computed by solving a finite sequence of linear program formulation shown below

$$\min_{x_1,x_2,x_3,x4} \varepsilon \quad \text{(Where } \varepsilon = \max_S \frac{e(X:S)}{\nu(S)}\text{)}$$

Subject to

$$x_1 + x_2 + x_3 + x_4 = \nu(1234) = 136.863$$
$$x_1 \le \nu(1)(1-\varepsilon) = 16.052(1-\varepsilon)$$
$$x_2 \le \nu(2)(1-\varepsilon) = 26.685(1-\varepsilon)$$
$$x_3 \le \nu(3)(1-\varepsilon) = 27.544(1-\varepsilon)$$
$$x_4 \le \nu(4)(1-\varepsilon) = 21.891(1-\varepsilon)$$
$$x_1 + x_2 \le \nu(12)(1-\varepsilon) = 38.555(1-\varepsilon)$$
$$x_1 + x_3 \le \nu(13)(1-\varepsilon) = 43.7141(1-\varepsilon)$$
$$x_1 + x_4 \le \nu(14)(1-\varepsilon) = 34.235(1-\varepsilon)$$
$$x_2 + x_3 \le \nu(23)(1-\varepsilon) = 54.498(1-\varepsilon)$$
$$x_2 + x_4 \le \nu(24)(1-\varepsilon) = 51.945(1-\varepsilon)$$
$$x_3 + x_4 \le \nu(34)(1-\varepsilon) = 50.749(1-\varepsilon)$$
$$x_1 + x_2 + x_3 \le \nu(123)(1-\varepsilon) = 81.016(1-\varepsilon)$$
$$x_1 + x_2 + x_4 \le \nu(124)(1-\varepsilon) = 77.854(1-\varepsilon)$$
$$x_1 + x_3 + x_4 \le \nu(134)(1-\varepsilon) = 82.654(1-\varepsilon)$$
$$x_2 + x_3 + x_4 \le \nu(234)(1-\varepsilon) = 96.336(1-\varepsilon) \qquad (47)$$

The optimal solution for Proportional Nucleolus is obtained as

$$x_{1=} 19.7894MW, x_2 = 36.4166MW,$$
$$= x_3 = 42.2499MW, x_4 = 38.4071\text{MW} \qquad (48)$$

The core for multi transaction loss allocation for IEEE-118 bus system is evaluated as

$$\begin{aligned} 16.052 &\le x_1 \le 40.527 \\ 26.685 &\le x_2 \le 54.209 \\ 27.544 &\le x_3 \le 59.009 \\ 21.891 &\le x_4 \le 55.847 \end{aligned} \quad \text{MW} \tag{49}$$

where x_1, x_2, x_3, x_4 are the losses allocated to transactions 1,2,3,4 respectively. Similarly, results are obtained for all other methods also and are tabulated in Table 4

TABLE 4 : ALLOCATION RESULTS OF DIFFERENT METHODS FOR IEEE 118 BUS SYSTEM

Type of Method	x_1	x_2	x_3	$x4$	$x_1 + x_2 + x_3 + x_4$
ERTG	27.2248	37.8578	38.7168	33.0638	136.863
PRTG	23.8351	39.6236	40.8991	32.5052	136.863
ERNMCG	26.9653	40.6473	26.9653	42.2853	136.863
PRNMC	29.0233	38.8217	29.0233	39.9947	136.863
SCRB	25.3674	37.1609	39.5198	34.8149	136.863
ENSC	22.3448	36.0268	40.8268	37.6648	136.863
Shapley	24.5377	37.3917	39.9386	34.995	136.863
Nucleolus	27.2248	37.8578	38.7167	33.0637	136.863
W-Nu	27.2248	37.8578	38.7167	33.0637	136.863
Prop-Nu	19.7894	36.4166	42.2499	38.4071	136.863
Dis-Nu	25.3674	37.1609	39.5198	34.8149	136.863
AVG-DIS-NU	24.0437	38.2907	40.2723	34.2563	136.863
TLAMTF	23.513	36.14	37.9	39.31	136.863

It is observed that the game is not convex
For example

$$\begin{aligned} &v(12) + v(13) = 38.555 + 43.7141 = 82.2641 \\ &< v(123) + v(1) = 81.016 + 16.052 = 97.068 \end{aligned} \quad \text{MW} \tag{50}$$

The Shapley value lies in the core. But it could not satisfy the additivity property. It is observed that the system loss allocations in each method are marginally different from TLAMTF. No attempt has been made for choice of optimal in this paper.

The inherent properties of Proportional Nucleolus make its application suitable for allocation of losses using CGT.

7. CONCLUSION

Issue of 'loss allocation' in Multiple Transaction Electricity Market transactions has been taken up for study in this paper. Several contemporary solution methods for this problem have been discussed. Application of Game Theory techniques has been critically reviewed and a suitable approach of Cooperative Game Theory (CGT) has been proposed for considering the problem of loss allocation in the context of this study.

An attempt is made in this paper to identify an effective solution method for handling the problem of loss allocation in Multilateral Electricity Market Transactions and recommend the approach of Proportional Nucleolus of CGT. Superiority of the proposed approach has been demonstrated through a case study on IEEE 14 bus system and also on an IEEE 118 bus system. Results of the investigation presented in this paper are promising and exhibit potential for application in real-time electricity trading transactions of Multiple Transaction Electricity Market environment.

Acknowledgement

The authors acknowledge the support and encouragement extended to them by their respective organizations. However, the views expressed in this paper are of the authors in their individual capacity and not necessarily that of APTRANSCO of Andhra Pradesh.

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