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A Complete Solution of Harmonics Elimination Problem in a Multi-Level Inverter with Unequal DC Sources

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In this paper, the problem of eliminating harmonics in a multi-level inverter with unequal DC sources is considered. The main objective is to improve the method of solving the equations for proper switching angles which reduce the total harmonic distortion (THD) in the output voltage. The basic concept of this reduction is to eliminate specific harmonics, which are generally the lowest orders, with an appropriate choice of switching angles. This paper employs Homotopy algorithm to solve the transcendental equations for finding the switching angles. This method solves the nonlinear transcendental equations with a much simpler formulation and without complex analytical calculations for any number of voltage levels. Also, several informative simulation results verify the validity and effectiveness of the proposed algorithm.

Keywords: Multi-Level Voltage-Source Inverter (VSI), Harmonic Elimination, Transcendental Equations, Unequal DC-Sources, Homotopy Algorithm

1. INTRODUCTION

In recent years, multi-level inverters are widely used as static power converter for high-power applications such as FACTS devices, HVDC light transmission, AC drives, and active filters [1-6]. One of the significant advantages of multi-level configuration is the harmonic reduction in the output waveform without increasing switching frequency or decreasing the inverter power output. The output voltage waveform of a multi-level inverter is composed of a number of levels of voltages, typically obtained from capacitor voltage sources. The so-called multi-level starts from three levels and as the number of levels increases, the output total harmonic distortion (THD) decreases. The number of achievable voltage levels, however, is limited by voltage unbalance problems, voltage clamping requirement, circuit layout, and packaging constraints. Therefore, an important key in designing an effective and efficient multi-level inverter is to ensure that THD in the output voltage waveform is small enough [5].

The well-known multi-level topologies are: 1. cascaded H-bridge multi-level inverter, 2. diode-clamped multi-level inverter, and 3. flying capacitor multi-level inverter [7].

The Multi-level inverter using cascaded H-bridges with separated DC sources, hereafter called a cascade multi-level inverter, appears to be superior to other multi-level inverters in terms of its structure that is not only simple and modular but also requires the least number of components. This modular structure makes it easily extensible for higher number of output voltage levels without undue increase in power circuit complexity. In addition, extra clamping diodes or voltage balancing capacitors are not necessary [2].

It is generally accepted that the performance of an inverter, with any switching strategies, can be related to the harmonic contents of its output voltage. Power electronics researchers

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have always studied many novel control techniques to reduce harmonics in such waveforms [5]. Up-to-date, there are many techniques, which are applied to inverter topologies. In multi-level topology, there are several well-known modulation techniques as follows [4-5]:

- Selected Harmonics Elimination or Optimized Harmonic Stepped-Waveform (OHSW) technique;
- Space Vector PWM (SVPWM) technique;
- Carrier-Based PWM (CBPWM) technique.

This paper focuses on the selected harmonics elimination technique applied to a cascaded H-bridge multi-level inverter with unequal DC sources. By employing this technique along with the multi-level topology, the low THD output waveform without any filtering circuit is possible. Switching devices, in addition, turn on and off once per cycle, that can overcome the switching loss problem, as well as EMI [4-5]. For a cascaded H-bridge multi-level inverter with unequal DC sources, a novel method based on mathematical Resultant theory for all solutions of switching angles is proposed in [1-2]. However this method has a number of problems. One of the problems is finding a set of proper initial values for the numerical iteration that will lead to a valid solution. This method converts the transcendental equations into an equivalent set of polynomial equations. These equations are polynomials of 22nd degree, and are very difficult and time consuming to compute. Also, for any change in the number of voltage levels or input DC voltages, new polynomials are required. In this paper, the derived equations are solved by a mathematical algorithm called Homotopy [8], with a much simpler formulation. This method can be used for any number of voltage levels without complex analytical calculations. The paper is organized as follows. In Section 2, the cascaded H-bridge multi-level inverter is reviewed briefly and the problem is formulated in a generalized form for some selected harmonics (the lowest orders) to be eliminated while maintaining the fundamental component at its desired value. In Section 3, Homotopy algorithm is introduced and its model construction and method of solving Homotopy mapping function is explained. A seven-level inverter with unequal Dc sources is formulated and solved according to the mentioned algorithm and the simulation results are presented in section 4.

The results are summarized in section 4, concluding the validity and effectiveness of the proposed algorithm.

2. CASCADED H-BRIDGE MULTI-LEVEL INVERTER

Fig. 1 shows the single-phase structure of a cascaded H-bridge multi-level inverter with unequal DC sources [9]. It consists of S H-bridge inverter cells. Each inverter cell generates an output voltage V_{ok} (k=1, 2, …, S) which can take three different values (levels), $-U_k$, 0, and $+U_k$ by connecting the DC source $U_k=V_kV_{dc}$ to the AC output side according to the states of the four switching devices. The output phase voltage of the multi-level inverter is then the sum of individual H-bridge cell's output, i.e.:

$$V_{an} = V_{o1} + V_{o2} + \cdots + V_{o(S-1)} + V_{oS} \qquad (1)$$

Fig. 2 illustrates the output voltage waveform of the individual H-bridge cells as well as the output phase voltage waveform of the whole multi-level inverter for the lowest possible switching frequency scheme. With S bridge inverters cascaded, the number of output voltage levels of the multi-level inverter will be $2S$+1.

As indicated in Fig. 2, the output voltage is a stepped waveform in which $\theta_1, \theta_2, \ldots, \theta_s$ are switching angles and should be determined in order that some specific harmonics are eliminated or THD is minimized.

The Fourier series expansion of the stepped output voltage waveform of the multi-level inverter with unequal DC sources is:

$$v_{an}(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \left[\frac{4V_{dc}}{n\pi} \sum_{k=1}^{S} \left[V_k * \cos\left(n\theta_k\right) \right] \right] \sin\left(n\omega t\right) \tag{2}$$

Where S is the number of H-bridge cells, n is odd harmonic order, $U_k=V_kV_{dc}$ is voltage of the kth DC source (if all DC sources have the same voltage V_{dc}, then $V_1=V_2=\ldots=V_S=1$).

The aim here is to calculate the switching angles $0^\circ < \theta_1 < \theta_2 < \ldots < \theta_s < 90^\circ$ so as to eliminate (S-1) certain lower frequency harmonics from the output voltage and make the fundamental component equal to the desired value V_f. This implies, mathematically, that S equations derived from equation (2) must be solved for θ_1 to θ_S .Considering that only the odd order harmonics are present in the waveform and taking into account the fact that in a three-wire three-phase system, the triplen harmonics need not to be eliminated from each phase as they are automatically cancelled from the line to line voltage, the following set of equations are derived and must be solved for switching angles θ_1 to θ_S :

$$\begin{aligned}
&V_1 \cos(\theta_1) + V_2 \cos(\theta_2) + \cdots\cdots + V_S \cos(\theta_S) = S \cdot m \\
&V_1 \cos(5\theta_1) + V_2 \cos(5\theta_2) + \cdots\cdots + V_S \cos(5\theta_S) = 0 \\
&V_1 \cos(7\theta_1) + V_2 \cos(7\theta_2) + \cdots\cdots + V_S \cos(7\theta_S) = 0 \\
&\vdots \\
&V_1 \cos(h\theta_1) + V_2 \cos(h\theta_2) + \cdots\cdots + V_S \cos(h\theta_S) = 0
\end{aligned} \tag{3}$$

Where $m = \dfrac{V_f}{4SV_{dc}/\pi}$ and h =3S-k is the highest harmonic order which is to be eliminated, with k=1 or 2 for even and odd values of S respectively.

m is known as modulation index, which follows the fact that each H-bridge cell has a DC source with nominal value of V_{dc}, therefore, the output voltage of the multi-level inverter at its maximum value will be a square wave of SV_{dc} amplitude with the fundamental component amplitude of $V_{fM} = \dfrac{4SV_{dc}}{\pi}$.

Equations (3) can be shown in the following vector form:

$$\mathbf{F}(\theta) = \left[f(\theta), f(5\theta), f(7\theta), \cdots, f(h\theta)\right]^T = \left[Sm,\ 0,\ \cdots,\ 0\right]^T \tag{4}$$

where

$$\theta = \left[\theta_1, \theta_2, \cdots, \theta_S\right]^T \quad \text{and} \quad f(\theta) = V_1 \cos\theta_1 + V_2 \cos\theta_2 + \cdots + V_S \cos\theta_S \tag{5}$$

The constraint to the vector θ is

$$D: 0^\circ < \theta_1 < \theta_2 < \ldots < \theta_s < 90^\circ \quad (6)$$

Unfortunately, equation (4) is nonlinear as well as transcendental in nature. One approach to solve equation (4) is to use an iterative procedure such as Newton–Raphson (NR) [4-6]. Another approach is to use mathematical Resultant theory [1-2]. This methodology is based on the mathematical theory of resultants of polynomials which is a systematic procedure for finding the roots of systems of polynomial equations [1-2]. However these methods have a number of problems. One of the problems is finding a set of proper initial values for the numerical iteration which leads to a valid solution. Normally the iterative methods convert the transcendental equations into an equivalent set of polynomial equations. These equations are polynomials of high orders and are very difficult and time consuming to compute. Also, for any change in the number of voltage levels or input DC voltages, new polynomials are required.

In this paper, an alternative mathematical method is extended to find all solutions to (4). This methodology is based on the mathematical Homotopy algorithm [3 and 8].

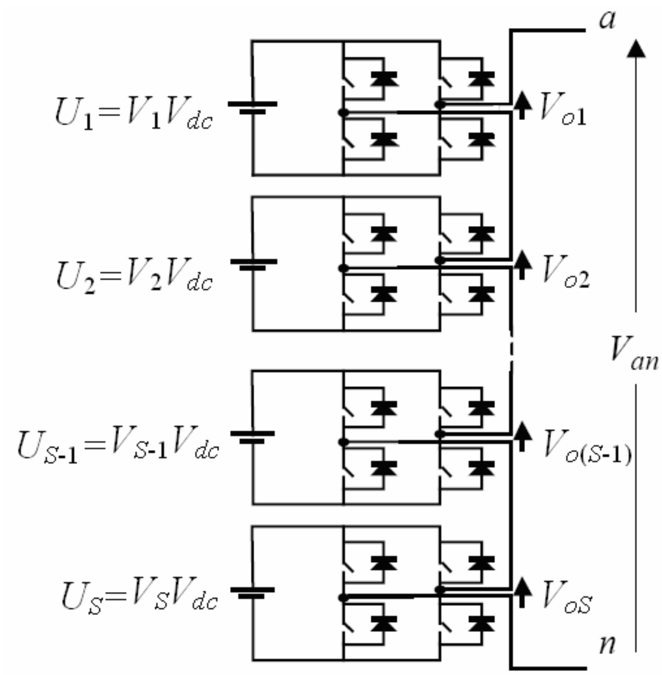

Fig. 1. Single-phase structure of a cascaded H-Bridge multi-level inverter with *unequal* DC sources.

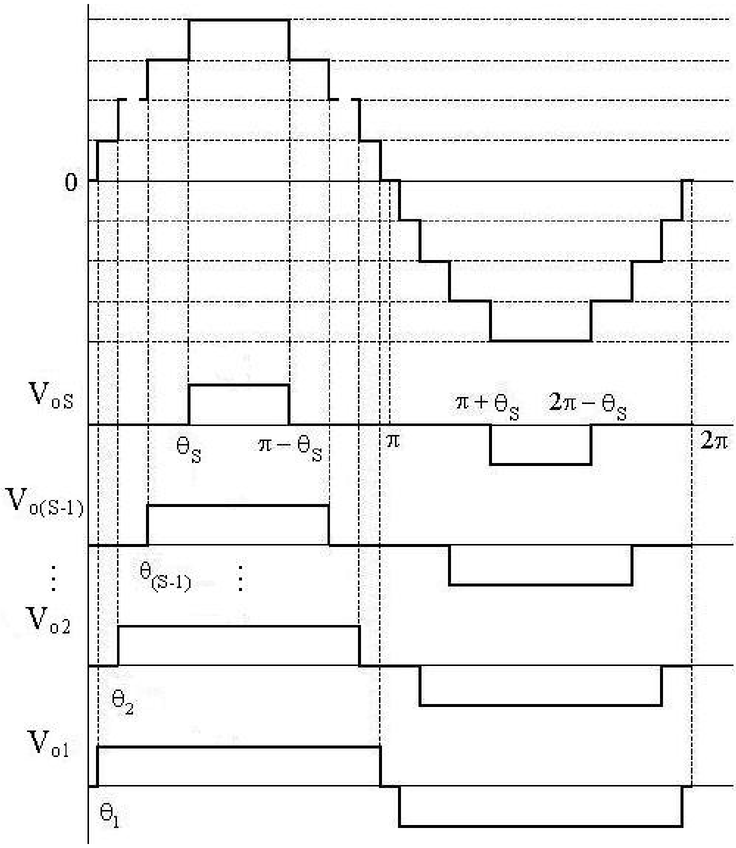

Fig. 2. Waveforms showing the cascaded multi-level inverter output phase voltage and each H-bridge output voltage.

3. SOLVING EQUATIONS

3.1. Construction of Homotopy Algorithm Model

Equation (4) can be rewritten as

$$\mathbf{F}(\theta) = \left[f(\theta) - S.m,\ f(5\theta),\ \ldots,\ f(h\theta)\right]^T = \mathbf{0} \tag{7}$$

Two steps are involved with the Homotopy algorithm: 1) to introduce a Homotopy parameter t into the equation (7), and 2) to construct a set of mapping **H**. When t is a fixed value (e.g. t =1), **H** is the mirror **F**; when t is another fixed value (e.g. t=0), **H** is the mirror **G**, where **G** is an Easily Solved Equation (ESE). Therefore, we can construct a Homotopy equation, based on which the original problem becomes finding a solution to the Homotopy equation with a fixed t (such as t =1).

The Homotopy equation is defined as

$$H(\theta, t) = 0,\ t \in [0, 1],\ \theta \subset D \tag{8}$$

where

$$H(\theta, 0) = G(\theta) = 0, \quad H(\theta, 1) = F(\theta) = 0, \quad \forall\theta \subset D \tag{9}$$

In the above equation, the solution $\theta(0)$ of $\mathbf{H}(\theta, 0) = \mathbf{G}(\theta) = 0$ is known, and the equation $\mathbf{H}(\theta, 1) = \mathbf{F}(\theta) = 0$ is the original problem equation. In other words, construct a set of mapping

$$H : D \times [0, 1] \subset R^{n+1} \to R^n \tag{10}$$

to substitute for the single mirror **F**, so the original problem changes into solving $\theta = \theta(t)$ of Homotopy equation (8).

whether $\theta : [0, 1] \subset R^n$ is continuous or not, it is determined by *Homotopy* parameter ***t***, i.e., $\theta = \theta(t)$ is a curve in the *n* dimensions space R^n, one end of which is fixed at $\theta(0)$ and the other end is $\theta = \theta(1)$, which is the solution of original problem equations $H(\theta, 1) = F(\theta) = 0$.

3.2. Method of Solving Homotopy Mapping Function

Homotopy equations satisfying (8) are many. We can define

$$\mathbf{H}(\theta, t) = t\mathbf{F}(\theta) + (1-t)\mathbf{G}(\theta) = 0 \ \forall t \in [0, 1],\ \theta \subset \mathbf{D} \tag{11}$$

where $\mathbf{G}(\theta) = 0$ is an ESE and its solution $\theta(0)$ is known.

Taking different ESE $\mathbf{G}(\theta) = 0$, we can obtain different *Homotopy* mapping. In this paper, the ESE can be selected as $\mathbf{G}(\theta) = \mathbf{F}(\theta) - \mathbf{F}(\theta(0))$, from which the *Homotopy* equation can be derived:

$$\mathbf{H}(\theta, t) = \mathbf{F}(\theta) + (t-1)\,\mathbf{F}(\theta(0)) = 0 \ \forall t \in [0, 1],\ \theta \subset \mathbf{D} \tag{12}$$

To solve Equation (12), various methods can be employed, such as Numeric Extended Method, Arc-Length Method and Differential Equation Method. Due to the limited space, this paper only introduces Differential Equation Method.

Considering θ as a function of the Homotopy parameter t, the derivative of equation (12) with respect to *t* will be:

$$\frac{\partial \mathbf{H}(\theta, t)}{\partial \theta}\theta'(t) + \frac{\partial \mathbf{H}(\theta, t)}{\partial t} = \frac{\partial \mathbf{F}(\theta)}{\partial \theta}\theta'(t) + \mathbf{F}(\theta(0)) = 0 \tag{13}$$

from which

$$\theta'(t) = -[\mathbf{J}(\theta(t))]^{-1}\mathbf{F}(\theta(0)),\, t \in [0, 1] \tag{14}$$

where $\mathbf{J}(\theta(t)) = \dfrac{\partial \mathbf{H}(\theta, t)}{\partial \theta} = \dfrac{\partial \mathbf{F}(\theta)}{\partial \theta}$.

Equation (14) is an Ordinary Differential Equation (ODE). The initial value is known. The equation can be effectively solved by 4th order Runge-Kutta Method

4. SIMULATION RESULTS

To verify the proposed Homotopy algorithm, a simulation model for a three-phase 7-level cascaded H-bridge inverter is implemented. 5th and 7th harmonics are selected to be eliminated from the output voltage and the fundamental component is specified by the modulation index *m*. DC source voltages are selected to be $U_1=V_1V_{dc}$=63.00 V, $U_2=V_2V_{dc}$=51.00 V, and $U_3=V_3V_{dc}$=60.60 V. The results for phase *a* are plotted in Fig. 3 which shows the switching angles $\{\theta_1, \theta_2, \theta_3\}$ versus *m*. Comparing Fig. 3 with the simulation and experimental results of [2] confirms validity of the proposed algorithm.

A three-phase induction motor model with the following parameters is attached to the multi-level inverter :

- Rated Power = 1/3 hp
- Rated Current = 1.5 A
- Rated Speed = 1425 rpm
- Rated Voltage = 208 V line to line rms at 50 Hz

With the switching angles corresponding to *m* = 0.52, i.e. $\theta_1 = 40.0978^\circ$, $\theta_2 = 54.3146^\circ$, $\theta_3 = 75.6119^\circ$ (taken from Fig. 3), the simulation results of the 50 Hz three-phase output voltages, both phase and line to line voltages, are presented in Fig. 4. Normalized FFT of the phase *a* voltage and line to line voltage between phases *a* and *b* are shown in Fig. 5 and Fig. 6. Note that 5th and 7th harmonics are zero in phase and line-to-line voltages. Also, it is noted that although phase voltage contains triplen harmonics such as 3rd and 9th, these harmonics do not appear in line to line voltage. THD for the phase voltage and line to line voltage can be computed from the FFT given in Fig. 5 and Fig. 6 which are found to be 46.36%, and 11.50%, respectively.

Fig. 7 shows the three-phase motor currents resulting from applying the voltages of Fig. 4 to the motor. The normalized FFT of phase *a* current waveform is shown in Fig. 8. The harmonic content of the current is significantly reduced compared to that of the voltage because of filtering by the motor's inductance. THD of the current waveform of phase *a*, computed using the FFT data of Fig. 8, is found to be 0.76%.

In another set of simulations, the modulation index *m* is considered to be equal to 0.70 and the frequency is set to 50 Hz. The switching angles are taken from Fig. 3 with *m* = 0.70 ($\theta_1 = 17.4122^\circ$, $\theta_2 = 41.9400^\circ$, $\theta_3 = 62.5332^\circ$). The resulting three-phase voltages are simulated, and both the phase and line to line voltages are shown in Fig. 9. Normalized FFT of the phase *a* voltage and line to line voltage between phases *a* and *b* are shown in

Fig. 10 and Fig. 11 respectively. Similar results as those of the previous case can be deduced again, with a considerable reduction in the phase voltage harmonics. The THD of phase *a* voltage and line to line voltage between phases *a* and *b* are computed using the information given in Figures 10 and 11 and found to be 18.36%, and 10.53%, respectively.

To obtain an accurate result, the harmonic components up to the 200th have been considered in calculating the voltage THD. The phase and line to line voltage THD of the 7-level inverter, as a function of the modulation index *m*, are shown in Fig. 12. It is seen that the phase voltage THD increases dramatically, when *m* decreases. The line to line voltage THD, however, increases slightly with decreasing *m*. Also, for a given *m*, the line to line voltage THD is much less than the phase voltage THD, due to cancellation of the triplen harmonics in the line to line voltage. For example, at *m* = 0.63, THD of the output phase voltage is 32.97%, whereas, that of the line to line voltage is 9.42%.

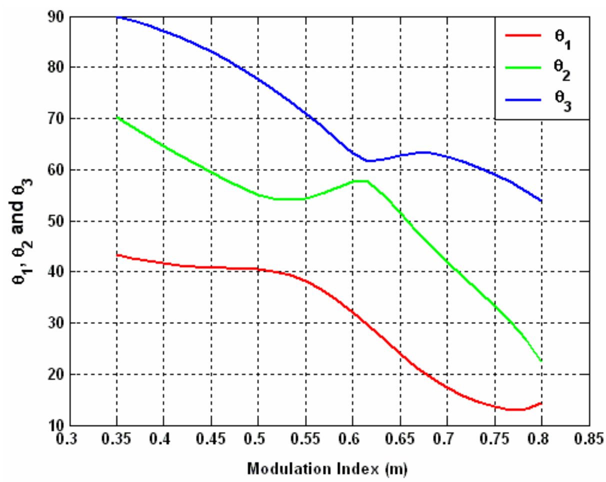

Fig. 3. Switching angles versus *m*.

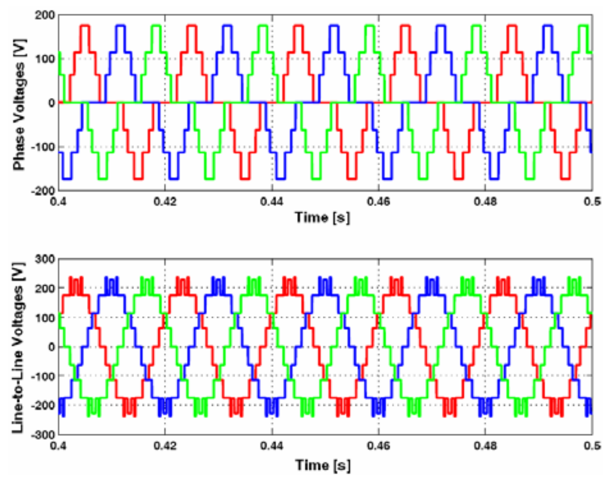

Fig. 4. Phase and line to line voltage waveforms for *m*=0.52.

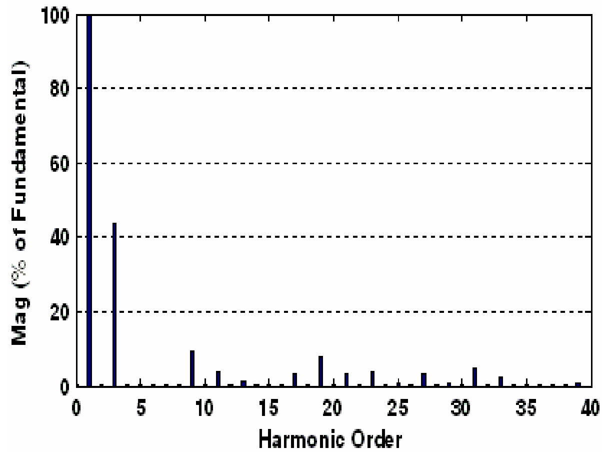

Fig. 5. Normalized FFT of the phase *a* voltage waveform shown in Fig. 5.

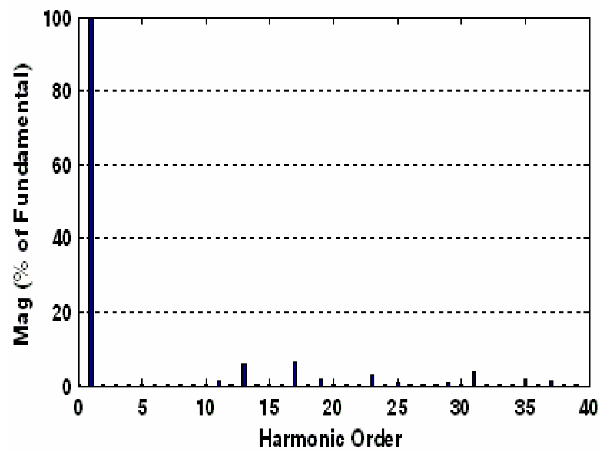

Fig. 6. Normalized FFT of the line to line voltage between phases *a* and *b* shown in Fig. 5.

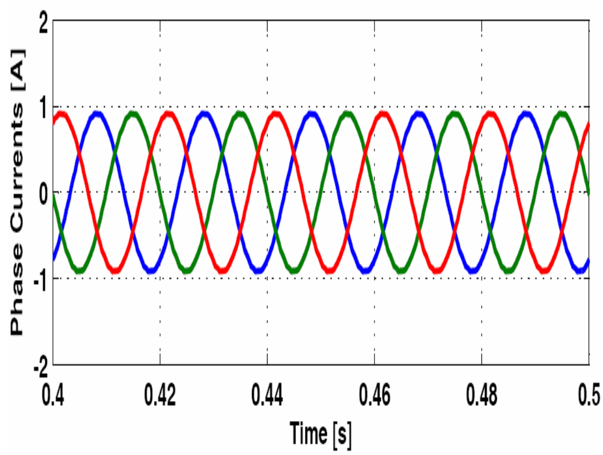

Fig. 7. Output current waveforms for *m*=0.52.

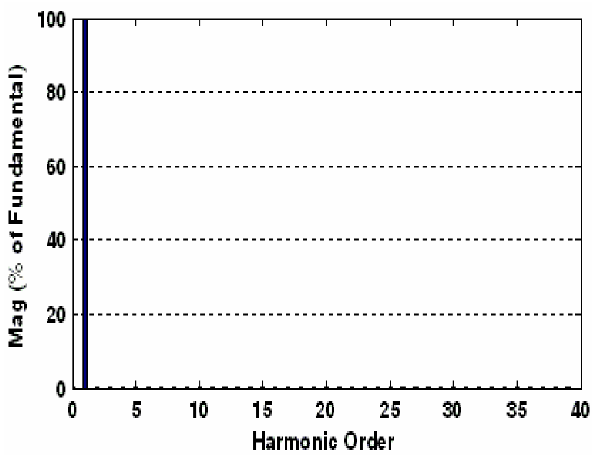

Fig. 8. Normalized FFT of the phase *a* current waveform shown in Fig. 7.

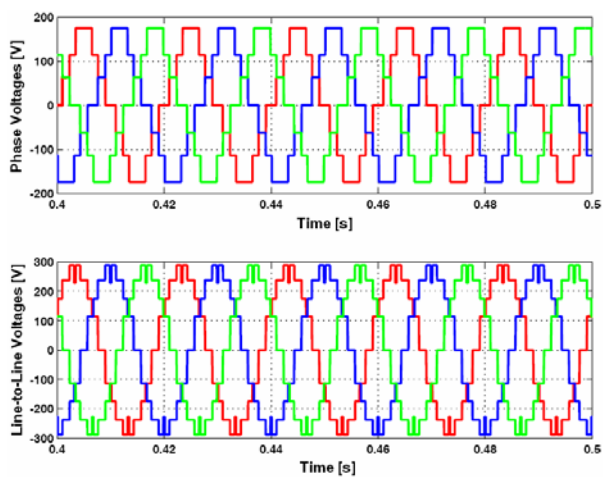

Fig. 9. Phase and line to line voltage waveforms for m=0.70

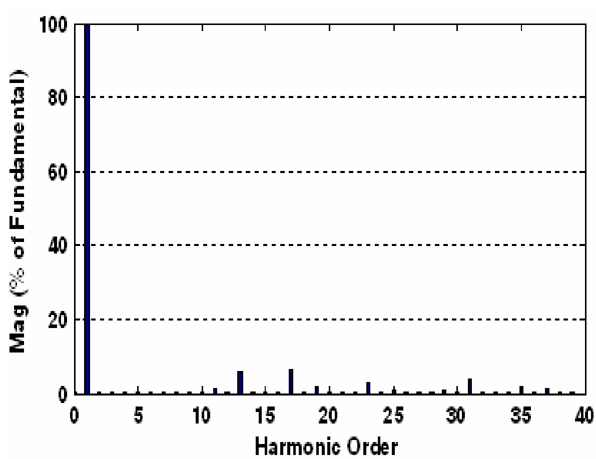

Fig. 10. Normalized FFT of the phase *a* voltage waveform shown in Fig. 9.

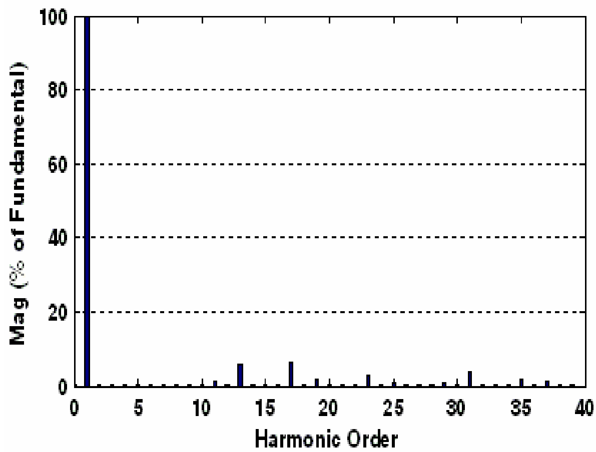

Fig. 10. Normalized FFT of the phase *a* voltage waveform shown in Fig. 9.

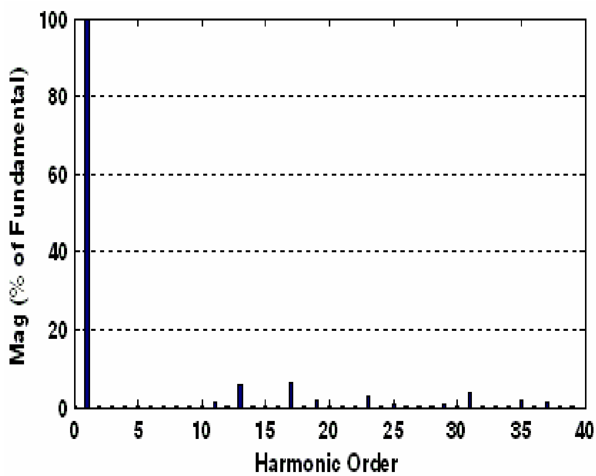

Fig. 11. Normalized FFT of the line to line voltage between phases *a* and *b* shown in Fig. 9.

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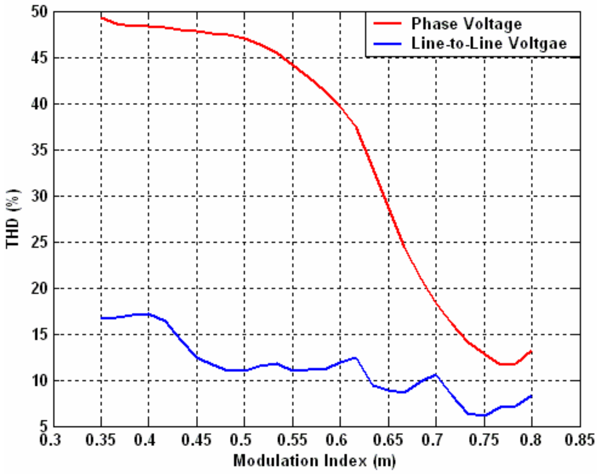

Fig. 12. The phase and line to line output voltage THD of a 7-level inverter as a function of *m*

5. CONCLUSION

This paper employs Homotopy algorithm to solve the nonlinear transcendent equations which are formed to find switching angles of the devices in a cascaded H-bridge multi-level inverter with unequal DC sources, in order to eliminate some selected harmonics from the output voltage. The proposed algorithm is very effective, efficient and reliable in finding solutions to high-order nonlinear equations. This algorithm solves the nonlinear transcendent equations with a much simpler formulation. Also, it can be used for any number of voltage levels without complex analytical calculations. Computer simulations based on a seven-level cascaded H-bridge inverter have been provided for the verification of validity of the proposed algorithm.

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