Hamouda KHECHINI

Moncef GOSSA

J. Electrical Systems 3-3 (2007): 162-175

Regular paper

Fault Tolerant Robust Control Applied for Induction Motor (LMI approach)

JES Journal of Electrical Systems

This paper foregrounds fault tolerant robust control of uncertain dynamic linear systems in the state space representation. In fact, the industrial systems are more and more complex and the diagnosis process becomes indispensable to guarantee their surety of functioning and availability, that's why a fault tolerant control law is imperative to achieve the diagnosis. In this paper, we address the problem of state feedback H_2 / H_∞ mixed with regional pole placement for linear continuous uncertain system. Sufficient conditions for feasibility are derived for a general class of convex regions of the complex plan. The conditions are presented as a collection of linear matrix inequalities $(LMI's)$. The efficiency and performance of this approach are then tested taking into consideration the robust control of a three- phase induction motor drive with the fluctuation of its parameters during the functioning.

Keywords: induction motor, multi-objective control, H_2 / H_∞ guaranteed, pole assignment, tolerant control, state feedback, LMI approach.

1. Introduction

The aim of the fault tolerant control is to accommodate automatically the fault effects bearing the safeguarding of both the system stability and nominal performances; therefore, avoiding the immediate halt of the system and allowing its functioning within the degradation mode. The control design often involves tradeoffs among conflicting objectives. The controller is frequently required to satisfy simultaneously different performance and robustness objectives which are imposed on different channels of the locked loop plant. Some discussions about multi-objective control first appeared in [1]. So far there have not been any exact solution or methods using various approximations to find upper and lower bounds.

In particular, the mixed H_2 / H_∞ problem has received many attentions. State space feedback is well thought-out in [2]. Output feedback with some structural constraints is derived in [3]. Whereas a more general case is considered in [4] using convex infinite dimensional optimization. In this study, the first performance measure considered is H_2 / H_∞ with closed loop pole clustering constraints. In the state feedback case, a systematic LMI approach to mixed H_2 / H_∞ synthesis with pole placement in LMI regions is presented. This paper is organized as the following:
Section I introduces the robust performance (pole placement, H_2 or H_∞). Section II presents the LMI formulation of H_2, H_∞ and pole placement. Section III points up the implementation of mixed H_2 / H_∞ synthesis on a three- phase induction motor. As a final point, we come up by some remarks.

Higher School of Sciences & Techniques of Tunisia Taha HUSSEIN Avenue N° 5 Tunisia,
Email: hammoudaKhechini@yahoo.fr

2. Robust performance

The control structure is depicted by Figure 1. The plan $P(s)$ is given by LTI system:

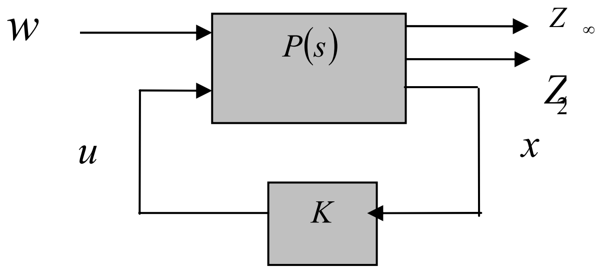

Figure 1: State feedback control

As:
$P(s)$: Transfer function of the plan,
w : Disturbance,
Z : Controlled output,
x : State vector
And u : Control vector.

In this paper, the performance measure considered is H_2 / H_∞ with closed-loop pole clustering constraints. The used plan is uncertain linear time, invariant $(LTI)_\infty$ system described as the following [7]:

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z_\infty = C_1 x + D_{11} w + D_{12} u \\ z_2 = C_2 x + D_{22} u \end{cases} \tag{1}$$

Denoting by $T_\infty(s)$ and $T_2(s)$ the closed loop transfer functions from w to Z_∞ and Z_2,

respectively, our goal is to design a state feedback law $u = kx$ such that:

1 Maintains the RMS gain (H_∞ norm) of T_∞ below some prescribed value $\gamma_0 \succ 0$.

2 Keeps the H_2 norm of T_2 ($LQG \cos t$) below some prescribed value $\nu_0 > 0$.

3 Minimizes an H_2 / H_∞ trade off criterion of the form

$$\alpha \|T_\infty\|_\infty^2 + \beta \|T_2\|_2^2 \tag{2}$$

(1) places the closed loop poles in a prescribed region D of the open left half plane.

2.1. Mixed H_2 / H_∞ performance

The norm H_2 of the transfer matrix between a perturbation w and a controlled output Z is obtained by [8]:

$$\|P_{wz}\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} trace\left[P_{wz}^*(jw).P_{wz}(jw)\right]dw \tag{3}$$

And the H_∞ norm is given by [5]:

$$\begin{aligned}\|P(s)\|_\infty &= \sup\sup\left(\sigma_i\left(P(jw)\right)\right)\\ &= \sup\sup\left(\lambda_i\left(P(jw).P(-jw)^T\right)\right)^{\frac{1}{2}}\end{aligned} \tag{4}$$

Where $\sigma_i(\,)$ denotes the i^{the} singular value and $\lambda_i(\,)$ is the i^{the} proper value. In its abstract "standard" formulation, the H_∞ control problem is one of disturbance rejection. Specifically, it consists of minimizing the closed loop gain from w to Z_∞ in the control loop of Figure 1. This can be interpreted as minimizing the effect of the worst case disturbance w on the output Z_2. The encountered concern is to determine the state feedback $K(s)$ such that $\gamma > 0$ and $\|F(P(s), K(s))\|_\infty < \gamma$, in order to settle on γ as small as possible.

2.2. Pole placement in LMI regions

The concept of LMI region [9] is useful to formulate pole placement objectives in LMI terms. LMI regions are convex subsets D of the complex plane characterized by

$$D = \left\{z \in C : L + Mz + M^T\overline{z} < 0\right\} \tag{5}$$

Where M and $L = L^T$ are fixed real matrices. The matrix-valued function:

$$f_D(z) := L + M.z + M^T\overline{z} \tag{6}$$

It is called the characteristic function of the region D. The class of LMI regions is fairly general since its closure is the set of convex regions symmetric with respect to the real axis. More practically, LMI regions include relevant regions such as sectors, disks, conics, strips, etc., as well as any intersection of the above. Another interesting region for control purposes is the set $S(\alpha, r, \theta)$ of complex number $x + jy$ such that:

$$x \prec -\alpha \prec 0, x^2 + y^2 \prec r^2, x \prec -\tan\theta^{|y|}$$

Strength of LMI regions is the availability of a "Lyapunov's theory" for such regions. Specifically, if $\left\{\lambda_{ij}\right\}_{1\le i,j\le m}$ and $\left\{\mu_{ij}\right\}_{1\le i,j\le m}$ denote the entries of the matrices L and M, a matrix A has all its eigenvalues in D if and only if there exists a positive definite matrix P such that [9]

$$\left[\lambda_{ij}P + \mu_{ij}PA^T\right]_{1\le i,j\le m} \prec 0 \tag{7}$$

Through the notation

$$\left[S_{ij}\right]_{1\le i,j\le m} := \begin{bmatrix} S_{11} & \cdots & S_{i1}\\ \vdots & \ddots & \vdots\\ S_{m1} & \cdots & S_{mm}\end{bmatrix} \tag{8}$$

Note that this condition is an LMI in *P* and that the classical Lyapunov's theory corresponds to the special case

$$f_D(z) = z + \overline{z} \tag{9}$$

3. LMI formulation

Given a state space realizations of the plan P in the form (1), the closed loop system is set in state space form by:

$$\begin{cases} \dot{x} = \left(A + B_2 K\right) x + B_1 w \\ z_\infty = \left(C_1 + D_{12} K\right) x + D_{11} w \\ z_2 = \left(C_2 + D_{22} K\right) u \end{cases} \tag{10}$$

The specifications and objectives in this work are H_2 and H_∞ performance with pole placement. Taken separately, our three design objectives have the following LMI formulation [5]:

1 H_∞ **performance:** the closed loop RMS gain from w to Z_∞ does not exceed γ if and only if there exists a symmetric matrix X_∞ such as :

$$\begin{pmatrix} (A + B_2 K) X_\infty + X_\infty (A + B_2 K)^T & B_1 & X_\infty (C_1 + D_{12} K)^T \\ B_1^T & -I & D_{11}^T \\ (C_1 + D_{12} K) X_\infty & D_{11} & -\gamma^2 I \end{pmatrix} < 0 \tag{11}$$

$$X_\infty > 0$$

2 H_2 **performance**: the closed loop H_2 norm of T_2 does not exceed ν if there exist two symmetric matrices X_2 and Q such that

$$\begin{pmatrix} (A + B_2 K) X_2 + X_2 (A + B_2 K)^T & B_1 \\ B_1^T & -I \end{pmatrix} < 0$$

$$\begin{pmatrix} Q & (C_2 + D_{22} K) X_2 \\ X_2 (C_2 + D_{22} K)^T & X_2 \end{pmatrix} > 0 \tag{12}$$

$$Trace(Q) < \nu^2$$

Pole placement: the closed loop poles lie in the LMI region

$$D = \left\{ z \in C : L + Mz + M^T \bar{z} < 0 \right\} \tag{13}$$

Where

$$L = L^T = \left[\lambda_{ij}\right]_{1 \le i,j \le m} \qquad M = \left[\mu_{ij}\right]_{1 \le i,j \le m} \tag{14}$$

If and only if there exists a symmetric matrix X_{pol} satisfying:

$$\left[\lambda_{ij} X_{pol} + \mu_{ij} (A + B_2 K) X_{pol} + \mu_{ij} X_{pol} (A + B_2 K)^T\right]_{1 \le i,j \le m} < 0 \tag{15}$$

$$X_{pol} > 0$$

These three sets of conditions are added up to no convex optimization problem with variables Q, K, X_∞, X_2 and X_{pol}. For tractability in the LMI framework, we seek a single Lyapunov matrix:

$$X := X_\infty = X_2 = X_{pol} \tag{16}$$

That enforces all the three objectives. With the change of variable $Y := KX$, this leads to the following suboptimal LMI formulation of our multi objective state feedback synthesis problem [2, 9, 10]:

Minimize $\alpha\gamma^2 + \beta trace(Q)$

Over Y, XQ and γ^2 satisfying

$$\begin{pmatrix} AX + XA^T + B_2Y + Y^TB_2^T & B_1 & XC_1^T + Y^TD_{12}^T \\ B_1^T & -I & D_{11}^T \\ C_1X + D_{12}Y & D_{11} & -\gamma^2 I \end{pmatrix} < 0 \tag{17}$$

$$\begin{pmatrix} Q & C_2X + D_{22}Y \\ XC_2^T + Y^TD_{22}^T & X \end{pmatrix} > 0 \tag{18}$$

$$\left[\lambda_{ij} + \mu_{ij}\left(AX + B_2Y\right) + \mu_{ij}\left(XA^T + Y^TB_2^T\right)\right]_{1\le i,j\le m} < 0 \tag{19}$$

$$Trace(Q) < \nu_0^2 \tag{20}$$

$$\gamma^2 < \gamma_0^2 \tag{21}$$

Denoting the optimal solution by (X^*, Y^*, Q^* , γ^*), the corresponding state feedback gain is given by

$$K^*Y^*\left(X^*\right)^{-1} \tag{22}$$

4. Illustrative example

1st case: Certain system

The use of this multi-objective control is illustrated on the three phase induction motor [6]. The state space description of this system is:

$$\begin{pmatrix} \dot{i}_{ds} \\ \dot{i}_{qs} \\ \dot{i}_{dr} \\ \dot{i}_{qr} \\ \dot{\omega}_r \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sigma T_s} & \omega_s + \frac{1-\sigma}{\sigma}\omega_r & \frac{M}{\sigma L_s T_r} & \frac{M\omega_r}{\sigma L_s} & 0 \\ -\omega_s - \frac{1-\sigma}{\sigma}\omega_r & -\frac{1}{\sigma L_s} & -\frac{M\omega_r}{\sigma L_s} & \frac{M\omega_r}{\sigma L_s T_r} & 0 \\ \frac{M}{\sigma L_r T_s} & -\frac{M\omega_r}{\sigma L_r} & -\frac{1}{\sigma L_r} & \omega_s + \frac{\omega_r}{\sigma} & 0 \\ \frac{M\omega_r}{\sigma L_r} & \frac{M}{\sigma L_r T_s} & -\omega_s + \frac{\omega_r}{\sigma} & -\frac{1}{\sigma T_r} & 0 \\ -(np)^2\frac{M}{J}i_{qr} & (np)^2\frac{M}{J}i_{dr} & 0 & 0 & -\frac{F}{J} \end{pmatrix} * \begin{pmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \\ \omega_r \end{pmatrix} +$$

$$\begin{pmatrix} \frac{1}{\sigma T_s} & 0 & 0 \\ 0 & \frac{1}{\sigma T_s} & 0 \\ -\frac{M}{\sigma L_s T_r} & 0 & 0 \\ 0 & -\frac{M}{\sigma L_s T_r} & 0 \\ 0 & 0 & \frac{1}{J} \end{pmatrix} * \begin{pmatrix} \nu_{ds} \\ \nu_{qs} \\ -npT_1 \end{pmatrix} \quad (23)$$

In this study, we highlight the three phase induction motor drive and describe its model in synchronous frame. The control has previously been realized through its disturbances. In fact, the system is unsteady in an opened loop. Many poles have some real parts which are positive.
The system parameters are:

$$A = 1.0e+003 * \begin{bmatrix} -0.1008 & 4.0148 & 0.0815 & 3.8386 & 0 \\ -4.0148 & -0.1008 & -3.8689 & 0.0815 & 0 \\ 0.0964 & -3.8340 & -0.0839 & 4.3218 & 0 \\ 3.8340 & 0.0964 & 3.6938 & -0.0839 & 0 \\ -0.3922 & 0 & 0 & 0 & -0.0002 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \end{bmatrix}^T; \quad B_2 = \begin{bmatrix} 84 & 0 & 0 \\ 0 & 84 & 0 \\ -80 & 0 & 0 \\ 0 & -80 & 0 \\ 0 & 0 & 90 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{11} = 0$$

$$D_{12} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$D_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Poles of the system on the opened loop are:

$P_1 = -0.2$

$P_2 = 5330.1$

$P_3 = 17$

$P_4 = -202.4$

$P_5 = -5514.2$

Next, we specify the LMI region for pole placement which is the disk with a center having an abscise =-10 and a radius = 1. Using state feedback control $u = kx$, we obtained the following results:

Poles of the system on the closed loop are:

$P_1 = -10.0481 + 0.9311i$

$P_2 = -10.0481 - 0.9311i$

$P_3 = -10.5171$

$P_4 = -9.9861$

$P_5 = -10.2313$

Guaranteed H_∞ performance: $1.00e - 001$

Guaranteed H_2 performance: $4.84e + 004$

$$K = \begin{bmatrix} -0.4666 & -47.9144 & -1.9935 & -594.2312 & -0.0000 \\ 47.9243 & 1.2057 & 46.1725 & -1.3407 & 0.0000 \\ 4.3581 & 0.0000 & 0.0000 & -0.0000 & -0.1120 \end{bmatrix}$$

The Lyapunov matrix is

$$X = 1.0e+008 * \begin{bmatrix} 1.0694 & -0.0000 & -1.2028 & -0.0008 & -0.0000 \\ -0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ -1.2028 & -0.0000 & 1.3528 & 0.0009 & 0.0000 \\ -0.0008 & 0.0000 & 0.0009 & 0.0000 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}$$

Where the eigenvalues are:

$$1.0e+008 * \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 2.4222 \end{bmatrix}$$

The simulation results are presented with the following figures:

- Figure 2: State variables with multi objectives control
- Figure 3: Corresponding control law
- Figure 4: Corresponding closed loop poles

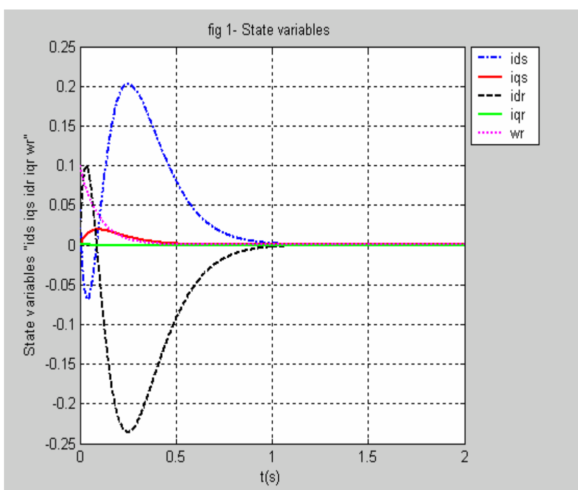

Figure 2: State variables with multi objectives control

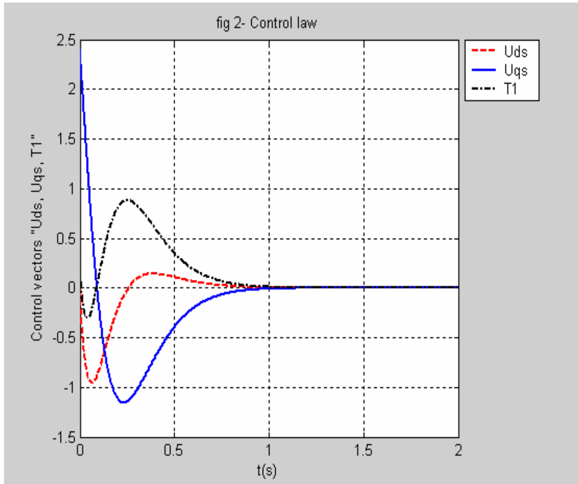

Figure 3: Corresponding control law

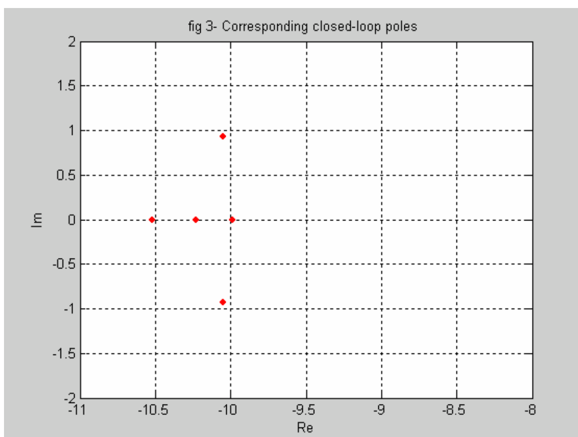

Figure 4: Corresponding closed loop poles

2nd case: Uncertain system

The use of tolerant robust control applied for three phases induction motor is validated by simulation on this system [7].We present the following uncertainty ranges for rotor resistor (R_r) and the inertial moment (J):

$$0.5\Omega \le R_r \le 1.5\Omega$$

$$0.010kg/m^2 \le J \le 0.015kg/m^2$$

The state space description of this system is given by **(**23**).**
We focus in this paper on the asynchronous motor and its model; hence, the control has already been realized through disturbances. It's worth saying that the system remains unsteady in an opened loop-many poles have positive real parts-
The system parameters are:

$$A = 1.0e+003 * \begin{bmatrix} -0.1008 & 4.0148 & a_{13} & 3.8386 & 0 \\ -4.0148 & -0.1008 & -3.8689 & a_{24} & 0 \\ 0.0964 & -3.8340 & a_{33} & 4.3218 & 0 \\ 3.8340 & 0.0964 & 3.6938 & a_{44} & 0 \\ a_{51} & 0 & 0 & 0 & a_{55} \end{bmatrix}$$

As the parameters a_{ij} are uncertain.

$40.18 \le a_{13} \le 120.58$

$40.18 \le a_{24} \le 120.58$

$-120.92 \le a_{33} \le -41.6$

$-120.92 \le a_{44} \le -41.6$

$-509.899 \le a_{51} \le -339.932$

$0.19994 \le a_{55} \le -0.13329$

$$B_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \end{bmatrix}^T ;\ B_2 = \begin{bmatrix} 84 & 0 & 0 \\ 0 & 84 & 0 \\ -80 & 0 & 0 \\ 0 & -80 & 0 \\ 0 & 0 & b_{53} \end{bmatrix}$$

Where $66.66 \le b_{53} \le 100$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{11} = 0$$

$$D_{12} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$D_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Poles of the system on the opened loop are:

$P_1 = -0.2$

$P_2 = 5330.1$

$P_3 = 17$

$P_4 = -202.4$

$P_5 = -5514.2$

Next, we specify LMI region for pole placement which is the intersection of the:

- Vertical $x < -0.1$
- Sector centered as the origin and with inner angle $3\frac{\pi}{4}$

Using state feedback control $u = kx$, we obtain the following results:

System poles on the closed loop are:

$$1.0e+006 * \begin{bmatrix} -0.0055 \\ -0.0053 \\ -0.0002 \\ -0.0000 \\ -3.9229 \end{bmatrix}$$

Guaranteed $H_{\inf}$ performance: $1.00e-001$

Guaranteed H_2 performance: $2.97e+001$

$$K = 1.0e+004 * \begin{bmatrix} -0.0010 & -0.0077 & -0.0009 & -0.0093 & -0.0000 \\ 0.0095 & 0.0041 & 0.0092 & 0.0176 & 0.0000 \\ 0.0004 & -0.0000 & -0.0000 & -0.0000 & -4.3587 \end{bmatrix}$$

The Lyapunov matrix is

$$X = 1.0e+004 * \begin{bmatrix} 0.0010 & -0.0001 & -0.0010 & -0.0000 & 0.0000 \\ -0.0001 & 0.0000 & 0.0001 & -0.0000 & 0.0000 \\ -0.0010 & 0.0001 & 0.0009 & 0.0000 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 2.6380 \end{bmatrix}$$

Where the **eigenvalues** are:

$$1.0e+004 * \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0020 \\ 2.6380 \end{bmatrix}$$

The existence of Lyapunov matrix X, symmetric definite positive has; thus, been proved. For this type of formulation we study the stabilization of the 4 points given by the uncertainty value of (R_r) and (J). The initial condition is $X_0 = \begin{bmatrix} 0.05 & 0 & 0 & 0 & 0.05 \end{bmatrix}^T$

The simulation results are exhibited with the following figures:

- Figure 5: State variables (Summit 1)
- Figure 6: State variables (Summit 2)
- Figure 7 State variables (Summit 3)
- Figure 8: State variables (Summit 4)

5. **Simulation Results**

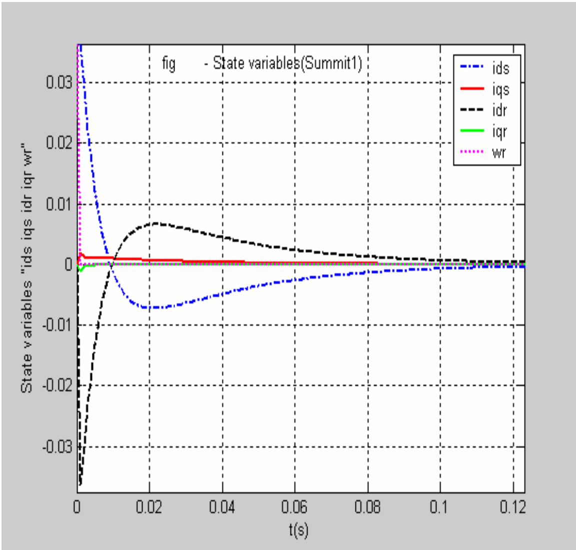

Figure 5: State variables (Summit 1)

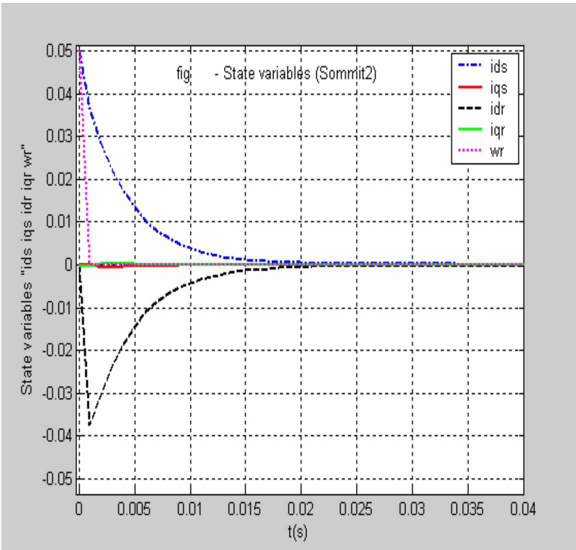

Figure 6: State variables (Summit2)

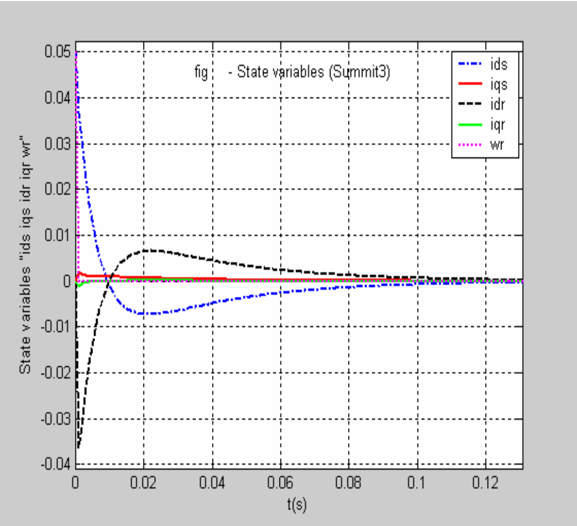

Figure 7: State variables (Summit 3)

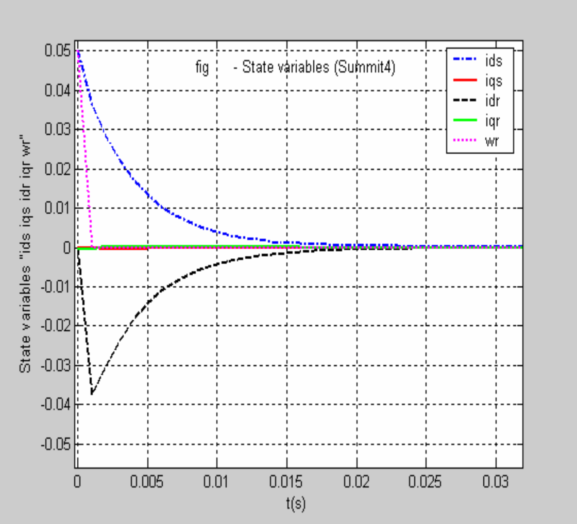

Figure 8: State variables (Summit4)

6. Conclusion

In this paper, we emphasize the fault tolerant robust control of induction motor right through the pole assignment with a combination H_2 / H_∞ constraints for the uncertain system. Moreover, we present in the state feedback case, a systematic LMI approach to mix H_2 / H_∞ synthesis with pole clustering in sector LMI region. Eventually, the numerical example for continuous time system has been exhibited showing the efficiency and the performance of the proposed method, Furthermore, a performance test of this control has been carried out with the presence of structural parameter fluctuations of the induction motor such that rotor and stator resistances, inertia moment and friction coefficient. The efficiency and robustness of this control algorithm are also verified through the simulation results which have been found in the MATLAB.

REFERENCES

[1] I. Kaminer, P. Khargonekar, M. Rotea, Mixed H_2/H_∞ control for discrete-time systems via convex optimization, Automatica, 29, pp.57-70, 1993.

[2] H. Mibar, A. Kochbati, M. Ksouri, Bernussou J., Pole assignment with mixed H_2 / H_∞ constraint: LMI Approach, CESA 1998, Hammamet, Tunisia, 1998.

[3] H. Noura, D. Sauter, C. Aubrun, "Fault-tolerant control via fault effect compensation", IMACS Multiconference CESA'96 in the Symposium "Control, Optimization and Supervision", Lille (France), Vol. 1, pp. 558-563, 9-12 July 1996.

[4] M. Chilali and P. Gahinet, *H_∞ design with pole placement constraint: LMI Approach*, proc conf. Dec. Control, pp 553-558, 1994.

[5] P. Khargonekar and M. Rotea, *mixed H_2 / H_∞ control: a convex optimization approach,* IEEE trans. Automat. Control, vol 39, pp 824-837, 1991.

[6] S. Boyd, C. Barrat, *Linear Controller Design: Limits of Performance*, Prentice –Hall, 1991.

[7] S. Boyd, L. El Ghaoui, E. Feron, V. Balaskrishnan, *Linear Matrix Inequalities in systems and control theory*, SIAM Publications, 1994.

[8] M. Gossa, N. Zaidi, T. Jebali, Commande robuste d'une machine à induction: Approche Quadratique, JTEA 2004, Hammamet, Tunisia, 2004..

[9] H. Rotstein, M. Sznair, An exact solution to general four-block discrete time mixed H_2/H_∞ problems via convex optimization, IEEE Trans. Control, 43, PP.1475-1480, 1998.

[10] M. Chilali, P. Gahinet, A. Nemirovski, A. Laub, *LMI control Toolbox for use MATLAB*, The math Works Inc., 1995