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An Improved Differential Evolution Based Dynamic Economic Dispatch with Nonsmooth Fuel Cost Function

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Dynamic economic dispatch (DED) is one of the major operational decisions in electric power systems. DED problem is an optimization problem with an objective to determine the optimal combination of power outputs for all generating units over a certain period of time in order to minimize the total fuel cost while satisfying dynamic operational constraints and load demand in each interval. This paper presents an improved differential evolution (IDE) method to solve the DED problem of generating units considering valve-point effects. Heuristic crossover technique and gene swap operator are introduced in the proposed approach to improve the convergence characteristic of the differential evolution (DE) algorithm. To illustrate the effectiveness of the proposed approach, two test systems consisting of five and ten generating units have been considered. The results obtained through the proposed method are compared with those reported in the literature.

Keywords: Dynamic economic dispatch, ramp rate limits, nonsmooth fuel cost function, differential evolution

1. INTRODUCTION

Dynamic economic dispatch (DED) is an extension of the conventional economic dispatch problem used to determine the optimal generation schedule of on-line generators, so as to meet the predicted load demand over certain period of time at minimum operating cost under various system and operational constraints. Due to the ramp-rate constraints of a generator, the operational decision at hour t may affect the operational decision at a later hour. For a power system with binding ramp-rate limits, these limits must be properly modeled in production simulation. The DED is not only the most accurate formulation of the economic dispatch problem but also the most difficult dynamic optimization problem.

Most of the literature addresses DED problems with convex cost functions [1]–[3]. However, in reality, large steam turbines have steam admission valves, which contribute nonconvexity in the fuel cost function of the generating units [4]-[6]. Accurate modeling of DED problem will be improved when the valve point loadings in the generating units are taken into account. Furthermore, they may generate multiple local optimum points in the solution space. Previous efforts on solving DED problem have employed various mathematical programming methods and optimization techniques. Traditional methods like gradient projection method [1], Lagrangian relaxation [7], dynamic programming and so on, when used to solve DED problem, suffer from myopia for nonlinear, discontinuous search spaces, leading them to a less desirable performance and these methods often use approximations to limit complexity.

The stochastic search algorithms such as genetic algorithm (GA) [4],[8], evolutionary programming (EP) [5],[9],[10], simulated annealing (SA) [11], and particle swarm optimization (PSO) [6] may prove to be very effective in solving nonlinear ED problems

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without any restriction on the shape of the cost curves. They often provide a fast, reasonable nearly global optimal solution. The setting of control parameters of the SA algorithm is a difficult task and convergence speed is slow when applied to a real system. Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance. This degradation in efficiency is apparent in applications with highly epistatic objective functions. Moreover, the premature convergence of GA degrades its performance and reduces its search capability that leads to a higher probability toward obtaining a local optimum [12]. EP seems to be a good method to solve optimization problems, when applied to problems consisting of more number of local optima the solutions obtained from EP method is just near global optimum one. Also GA and EP take long simulation time in order to obtain solution for such problems. All these methods use probabilistic rules to update their candidates positions in the solution space. Sequential quadratic programming (SQP) method seems to be the best nonlinear programming method for constrained optimization problem but the objective function to be minimized is nonconvex , it ensures the local optimum solution.

Recently, SA [13], hybrid EP-SQP [14], DGPSO [15] and hybrid PSO-SQP [16] methods are proposed to solve dynamic economic dispatch problem with nonsmooth fuel cost functions. These hybrid methods utilize local searching property of SQP along with stochastic optimization techniques to determine the optimal solution of DED problem.

Differential Evolution developed by Storn and Price is one of the excellent evolutionary algorithms [17] . DE is a robust statistical method for cost function minimization, which does not make use of a single parameter vector but instead uses a population of equally important vectors. This paper develops an improved DE algorithm to determine the optimum generation schedule of the DED problem that takes into consideration of valve-point effects. In the proposed approach, the search capability of the DE algorithm is enhanced by introducing heuristic crossover operation and gene swap operator, which leads to a higher probability of getting global or near global optimal solutions. The proposed method is tested on five-unit and ten-unit sample test systems and the results are compared with a SA, hybrid EP-SQP, DGPSO and PSO-SQP methods. The effectiveness and potential of the proposed approach to solve DED problem is demonstrated.

2. FORMULATION OF DED PROBLEM

The classic DED problem minimizes the following incremental cost function associated to dispatchable units:

$$Min\ F = \sum_{t=1}^{T}\sum_{i=1}^{N} F_{it}(P_{it}) \quad (\$) \tag{1}$$

where F is the total generating cost over the whole dispatch period, T is the number of intervals in the scheduled horizon, N is the number of generating units, and $F_{it}(P_{it})$ is the fuel cost in terms of its real power output P_{it} at time t. Taking into account of the valve-point effects, the fuel cost function of i^{th} thermal generating unit is expressed as the sum of a quadratic and a sinusoidal function in the following form

$$F_{it}(P_{it}) = a_i P_{it}^2 + b_i P_{it} + c_i + \left|\ e_i \sin(f_i\ (P_{i\min} - P_{it}))\ \right| \quad (\$/\text{h}) \tag{2}$$

where a_i, b_i, and c_i are cost coefficients, e_i, f_i are constants from the valve point effect of the i^{th} generating unit, and P_i is the power output of the i^{th} unit in megawatts.

The minimization of the generation cost is subjected to the following equality and inequality constraints:

1) Real power balance constraint

$$\sum_{i=1}^{N} P_{it} - P_{Dt} - P_{Lt} = 0 \tag{3}$$

where $t = 1, 2, \ldots, T$. P_{Dt} is the total power demand at time t and P_{Lt} is the transmission power loss at time t in megawatts. P_{Lt} is calculated using the B-Matrix loss coefficients and the general form of the loss formula using B-coefficients is

$$P_{Lt} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{it} B_{ij} P_{jt} \tag{4}$$

2) Real power generation limit

$$P_{i\min} \leq P_{it} \leq P_{i\max} \tag{5}$$

where P_{imin} is the minimum limit, and P_{imax} is the maximum limit of real power of the i^{th} unit in megawatts.

3) Generating unit ramp rate limits

$$P_{it} - P_{i(t-1)} \leq UR_i, \qquad i = 1, 2, 3, \ldots\ldots\ldots, N \tag{6}$$

$$P_{i(t-1)} - P_{it} \leq DR_i \qquad i = 1, 2, 3, \ldots\ldots\ldots, N$$

where UR_i and DR_i are the ramp-up and ramp-down limits of i^{th} unit in megawatts. Thus the constraint of (6) due to the ramp rate constraints is modified as

$$\max(P_{i\min}, P_{i(t-1)} - DR_i) \leq P_{it} \leq \min(P_{i\max}, P_{i(t-1)} + UR_i) \tag{7}$$

such that

$$P_{it,\min} = \max(P_{i\min}, P_{i(t-1)} - DR_i) \quad \text{and} \tag{8}$$

$$P_{it,\max} = \min(P_{i\max}, P_{i(t-1)} + UR_i)$$

4) Constraint satisfaction technique

To satisfy the equality constraint of equation (3), a loading of any one unit is selected as the depending loading P_{Nt}. The power level of N^{th} generator is given by

$$P_{Nt} = P_{Dt} + P_{Lt} - \sum_{i=1}^{(N-1)} P_{it} \tag{9}$$

The transmission loss P_{Lt} is function of all the generators including that of dependent generator, and it is given by

$$P_{Lt} = \sum_{i=1}^{(N-1)} \sum_{j=1}^{(N-1)} P_{it} B_{ij} P_{jt} + 2P_{Nt} \left(\sum_{i=1}^{(N-1)} B_{Ni} P_{it} \right) + B_{NN} P_{Nt}^2 \tag{10}$$

Expanding and rearranging, equation (10) becomes

$$B_{NN} P_{Nt}^2 + \left(2 \sum_{i=1}^{(N-1)} B_{Ni} P_{it} - 1 \right) P_{Nt} + \left(P_{Dt} + \sum_{i=1}^{(N-1)} \sum_{j=1}^{(N-1)} P_{it} B_{ij} P_{jt} - \sum_{i=1}^{(N-1)} P_{it} \right) = 0 \tag{11}$$

The loading of dependent generator can be determined by solving (11) using standard algebraic method.

3. IMPROVED DIFFERENTIAL EVOLUTION ALGORITHM FOR DED PROBLEM

The detailed implementation of IDE to solve the dynamic economic dispatch problem, is as follows:

1) Initialization: DE uses NP D-dimensional parameter vectors

$$P_{k,G}; \quad k = 1,\ 2,\ 3,\ldots\ldots\ldots ,\ NP \tag{12}$$

in a generation *G*, with *NP* being constant over the entire optimization process. At the start of the procedure, i.e., generation $G = 1$, the population vectors have to be generated randomly within the limits. For *T* intervals in the generation scheduling horizon, there are *T* dispatches of generation by *N* generating units. An array of control variable vectors or positions of the each agent can be represented as

$$P_{k,G} = \left[\left(P_{11}\ P_{21}\ P_{31}\ \cdots\ P_{N1} \right)\ \cdots\ \left(P_{1T}\ P_{2T}\ P_{3T}\ \cdots\ P_{NT} \right) \right], \quad \text{for} \quad k = 1, 2, 3, \ldots\ldots\ldots, NP \tag{13}$$

Where P_{NT} is the generation power output of the N^{th} unit at T^{th} interval.

2) Heuristic Crossover operation: It is unique crossover operator because it uses values of the objective function in determining the direction of search and it produces only one offspring. The operator generates a single offspring X_3 from the randomly selected two parent vectors X_1 and X_2 in the population according to the rule

$$X_3 = \text{r}\ (X_2 - X_1) + X_2 \tag{14}$$

where *r* is a random number between 0 and 1 and the parent having higher fitness value is denoted by X_1 and lower X_2. The offspring produced due to crossover randomly replaces any one of the individuals in the population. A heuristic crossover operator with probability of 0.02 is implemented in this algorithm.

3) Mutation: For the following generation $G+1$, new vectors $V_{k,\ G+1}$ are generated according to the following mutation scheme

$$V_{k,G+1} = P_{k,G} + F\ .\ (P_{r1,G} - P_{r2,G}), \quad \text{for} \quad k = 1, 2, 3, \ldots\ldots\ldots, NP \tag{15}$$

The integers *r1* and *r2* are chosen randomly over [1, NP] and should be mutually different from the running index *k*. Under certain circumstances, the index *k* will be exchanged by an arbitrary random number r3 $\in$ [1, NP]. *F* is a scaling factor, which controls the amplification of the differential variation. The value of scaling factor is defined as follows:

$$F = 1 - \frac{\text{iter}}{\text{itermax}} \tag{16}$$

where *iter* and *itermax* are the number of current iteration and the maximum iteration, respectively. In DE, the mutation is solely derived from positional information of current population. This scheme provides for automatic self-adaptation and eliminates the need to adapt standard deviations of a probability density function.

4) Evaluation of Each Agent: Each individual in the population is evaluated using the fitness function of the problem to minimize the fuel cost function. The real power limit of the first generator and the unit ramp rate limits are constrained by adding them as a exact penalty term to the objective function to form a generalized fitness function f_k as given below.

$$f_k = \sum_{t=1}^{T}\sum_{i=1}^{N} F_{it}(P_{it}) + \sum_{t=1}^{T} \mu_1 \left| P_{1t} - P_{1t\lim} \right| + \sum_{t=2}^{T}\sum_{i=2}^{N} \mu_r \left| P_{it} - P_{r\lim} \right| \tag{17}$$

where μ_1 *and* μ_r are penalty parameters, and

$$P_{1t\lim} = \begin{cases} P_{1\min}, & \text{if } P_{1t} < P_{1\min} \\ P_{1\max}, & \text{if } P_{1t} > P_{1\max} \\ P_{1t}, & otherwise \end{cases} \tag{18}$$

$$P_{r\lim} = \begin{cases} P_{i(t-1)} - DR_i, & \text{if } P_{it} < P_{i(t-1)} - DR_i \\ P_{i(t-1)} + UR_i, & \text{if } P_{it} > P_{i(t-1)} + UR_i \\ P_{it}, & otherwise \end{cases} \tag{19}$$

The penalty terms associated with inequality constraints are added to the objective function. The penalty terms reflect the violation of the constraints and assign a high cost of the penalty function to candidate point far from the feasible region.

5) Estimation and Selection: The parent is replaced by its child if the fitness of the child is better than that of its parent. Explicitly, the parent is retained in the next generation if the fitness of the child is worse than that of its parent. DE selection scheme is based on local competition only. i.e., a child $V_{k,\,G+1}$ will compete against one population member $P_{k,\,G}$ and survivor will enter the new population. The number *NT* of children which may be produced to compete against $P_{k,\,G}$ should be chosen sufficiently high so that sufficient number of child will enter the new population. if $V_{k,\,G+1}$ is worse than that of its parent , the vector generation process defined by (15) & (19) is repeated up to *NT* times. If $V_{k,\,G+1}$ still worse than that of its parent, $P_{k,\,G+1}$ will be set to $P_{k,\,G}$. An insufficient number *NT* leads to

survival of too many old population vectors, which may induce stagnation. To prevent a vector $P_{k,\,G}$ from surviving indefinitely, DE employs the concept of aging. *NE* defines how many generations a population vector may survive before it has to be replaced due to excessive age. To this end $P_{k,\,G}$ in (13) is checked first for how many generations it has already lived. If $P_{k,\,G}$ has an age of less than NE generations it remains unaltered, otherwise $P_{i,\,G}$ is replaced by $P_{r3,\,G}$ with $r3 \neq k$ being a randomly chosen integer $r3 \in [1, NP]$. In short, if $P_{k,\,G}$ is too old it may not serve as a parent vector any more but will be replaced by a randomly chosen member of the current generation *G*.

6) Gene Swap operator: For a large scale optimization problems with difficult search spaces and lengthy chromosomes, the possibility of the DE algorithm to get trapped in local optima will be high. Maintaining diversity is especially important for dynamic optimization problems since the optimum of such a function changes over time and if the population is clustered in a tight region, the individuals may not be able to detect a change in the function landscape. In order to increase the diversity in the population of DE algorithm, a gene swap operator is introduced in the proposed algorithm. This operator randomly selects two genes in a chromosome and swaps their values. If the modified chromosome proved to have better fitness, it replaces original one in the new population. In the proposed algorithm, gene swap operator is applied with a probability of 0.05 that swaps the active power output of two units in the randomly selected individual.

7) Stopping Criterion: The procedure from 2-6 is repeated until the maximum number of iterations reached.

4. NUMERICAL SIMULATION RESULTS AND DISCUSSION

An improved DE algorithm for the DED problem described above has been applied to five-unit and ten-unit systems with nonsmooth fuel cost function to demonstrate the performance of the proposed method. The simulations were carried out on a PC with Pentium IV 2.8-GHZ processor. The software is developed using the MATLAB 6.5. An improved DE uses four control variables ie. population size *NP*, maximum number of generations *NG*, number of trials per iteration *NT*, number of generations a population vector may survive before it has to be replaced due to excessive age *NE*. The number of trials have been conducted with changes in the size of population, number of generations, and number of trials per iteration in order to obtain the best values to achieve the overall minimum cost of generation. The best solution obtained through the proposed method is compared to those reported in the recent literature.

Example-1: 5–unit system

The cost coefficients, generation limits, load demand in each interval and ramp-rate limits of five-unit sample system with valve-point loading are given in Appendix, which is taken from Ref. [13]. The scheduling time horizon is one day divided into 24 intervals. The transmission losses are calculated using B-coefficient loss formula. The results of the proposed method are compared with that of the simulated annealing (SA) method [13]. The IDE control parameters used in this example are $NP = 100$, $NG = 500$, $NT = 10$ and $NE = 5$. The optimal dispatch of real power for the given scheduling horizon using improved DE method is given in Table 1. The best total production cost obtained using proposed method is $45800, compared to $47356 of the SA method. The sum of total generating power in each interval satisfies the load demand plus transmission losses. The computation time taken by the algorithm is 3min, 17s.

Table 1 : Best scheduling of 5-unit system using improved DE method

Hour	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	Ploss (MW)
1	13.5391	99.3704	30.2998	127.1172	143.5164	3.8429
2	11.2817	98.5308	64.7365	124.7846	139.7971	4.1308
3	12.0242	105.7360	99.9374	125.0162	137.0990	4.8128
4	27.0177	106.5800	120.3798	125.2478	156.6716	5.8969
5	39.9826	99.3312	120.3491	124.9764	179.8703	6.5096
6	18.5381	111.6850	118.6123	137.7051	229.3824	7.9229
7	17.3986	94.4374	117.5525	174.9023	230.0848	8.3756
8	14.4462	99.2620	113.7126	211.2911	224.5312	9.2431
9	20.2907	99.2430	136.3110	210.9401	233.3671	10.1519
10	49.9993	101.3351	121.6219	210.4041	231.1839	10.5443
11	72.0943	103.7987	112.9096	209.4610	232.7864	11.0500
12	49.0762	95.2490	114.8818	210.0632	282.5364	11.8066
13	20.7067	97.9203	113.2666	211.2251	271.6483	10.7670
14	44.9018	101.9107	112.6850	211.5980	229.0945	10.1900
15	42.6234	106.1245	113.4645	210.1864	190.7303	9.1291
16	22.4686	98.0354	113.1596	210.1344	143.4480	7.2460
17	13.7916	100.0280	116.3379	194.7838	139.7523	6.6936
18	10.1286	97.0662	113.9712	210.3131	184.5040	7.9831
19	14.1660	99.0430	113.5148	203.6348	232.8794	9.2380
20	12.0689	98.6777	111.4379	209.7849	282.8782	10.8476
21	41.9365	86.6247	120.5802	195.4251	245.2676	9.8341
22	41.0368	76.6583	110.6187	166.3992	218.0152	7.7282
23	23.2548	92.1743	103.0519	130.8563	183.5350	5.8723
24	15.2391	70.2344	88.1839	130.7542	163.1208	4.5324

Example-2: 10–unit system

In this example, the DED problem of the 10-unit system is solved by the proposed method by neglecting transmission losses in order to compare the results of the improved DE method with hybrid methods such as Hybrid EP-SQP, Deterministically guided PSO and Hybrid PSO-SQP algorithms reported in literature [14], [15], & [16]. The load demand of the system was divided by 24 intervals. The system data for ten-unit sample system is taken from the Ref. [14] , as given in Appendix. Transmission losses have been ignored for the sake of comparison of results with those reported in literature. The following DE control parameters has been chosen for this example: $NP = 120$, $NG = 1500$, $NT = 10$, and $NE = 5$. The best results obtained through various hybrid methods and from the improved DE method are shown in Table 2. It clear from the table that the proposed method produces much better results compared to recently reported hybrid methods for solving DED problem. The optimum scheduling of generating units for 24 hours using proposed method is given in Table 3. The computation time of proposed method for ten-unit system is 14min, 15s. Figure 1 shows the convergence characteristics of the IDE for DED problem.

Table 2: Comparison of results for 10-unit system

Method	Total fuel cost (dollars/24h)	Difference (%) from improved DE
Improved DE	1026269	---
Hybrid EP-SQP[14]	1031746	0.5306
DGPSO[15]	1028835	0.1494
Hybrid PSO-SQP[16]	1027334	0.1037

Table 3: Best scheduling of 10-unit system using improved DE method

Hour	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P6 (MW)	P7 (MW)	P8 (MW)	P9 (MW)	P10 (MW)
1	226.653	135.010	232.146	60.155	73.031	57.000	129.995	47.006	20.005	55
2	226.843	135.030	305.610	60.137	73.000	57.540	129.813	47.027	20.001	55
3	303.957	150.391	312.253	60.046	122.743	57.000	129.603	47.000	20.007	55
4	303.351	229.524	331.327	60.035	172.743	57.039	129.975	47.006	20.001	55
5	302.830	309.507	337.410	60.001	122.762	95.439	129.996	47.006	20.049	55
6	379.834	389.487	338.729	60.188	73.016	135.264	129.467	47.009	20.005	55
7	379.967	459.959	302.114	75.387	73.006	159.999	129.523	47.022	20.021	55
8	379.841	396.569	339.865	124.731	122.964	159.983	129.991	47.000	20.057	55
9	456.523	397.284	339.842	174.728	172.960	130.739	129.917	47.006	20.000	55
10	456.582	459.983	338.274	211.290	222.960	130.908	129.994	47.003	20.005	55
11	456.444	459.996	339.832	255.159	222.712	159.850	129.983	47.010	20.014	55
12	465.802	459.793	339.511	299.877	242.993	159.996	129.974	47.007	20.047	55
13	457.002	459.997	300.751	257.028	222.503	122.856	129.844	47.018	20.001	55
14	380.226	396.844	305.844	207.145	222.744	159.786	129.399	47.006	20.006	55
15	303.669	390.425	339.992	157.603	172.744	159.820	129.618	47.129	20.001	55
16	302.696	310.449	286.680	107.634	172.215	122.332	129.987	47.001	20.006	55
17	379.823	230.498	260.451	61.008	172.642	123.557	129.999	47.002	20.020	55
18	380.774	310.259	300.307	60.420	172.515	151.611	129.990	47.125	20.000	55
19	380.458	390.118	300.653	71.348	221.933	159.925	129.428	47.137	20.000	55
20	456.353	459.884	339.993	121.252	223.660	159.003	129.987	77.127	49.742	55
21	380.012	459.916	335.361	76.326	237.966	122.307	129.986	107.125	20.002	55
22	303.007	382.899	261.195	60.745	222.631	72.543	129.901	119.997	20.083	55
23	229.435	304.436	181.198	61.010	172.717	58.245	129.955	119.994	20.011	55
24	293.992	224.469	101.208	60.285	122.725	57.102	129.988	119.228	20.002	55

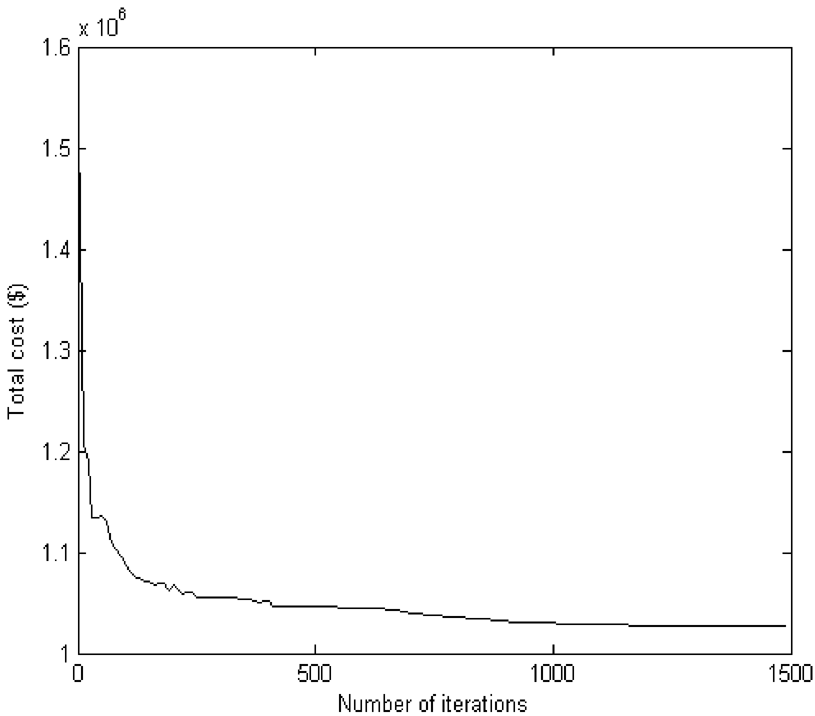

Fig. 1. Convergence characteristics of an improved DE method for 10-unit system.

5. CONCLUSION

An improved differential evolution based methodology has been developed for determination of optimal solution for DED problem with the generator constraints. The improved DE incorporates the heuristic crossover and gene swap operator to enhance its search capacity, which leads to a higher probability of getting the global or near global solution. The feasibility of the proposed method was demonstrated with five and ten-unit sample systems. The test results reveals that the optimal dispatch solution obtained through the improved DE lead to less operating cost than that found by other methods, which shows the capability of the algorithm to determine the global or near global solution for DED problem. The proposed approach outperforms SA, hybrid EP-SQP, DGPSO and PSO-SQP methods for DED problems in terms of quality of solution with better performance.

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APPENDIX

Table A: Data for the 5-unit system

Quantities	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
a ($/(MW)^2 h)	0.0080	0.0030	0.0012	0.0010	0.0015
b ($/MWh)	2.0	1.8	2.1	2.0	1.8
c ($/h)	25	60	100	120	40
e ($/h)	100	140	160	180	200
f (1/MW)	0.042	0.040	0.038	0.037	0.035
P_{min} (MW)	10	20	30	40	50
P_{max} (MW)	75	125	175	250	300
UR (MW/h)	30	30	40	50	50
DR (MW/h)	30	30	40	50	50

Transmission Loss Coefficient for 5-unit system

$$B = \begin{bmatrix} 0.000049 & 0.000014 & 0.000015 & 0.000015 & 0.000020 \\ 0.000014 & 0.000045 & 0.000016 & 0.000020 & 0.000018 \\ 0.000015 & 0.000016 & 0.000039 & 0.000010 & 0.000012 \\ 0.000015 & 0.000020 & 0.000010 & 0.000040 & 0.000014 \\ 0.000020 & 0.000018 & 0.000012 & 0.000014 & 0.000035 \end{bmatrix} \text{ per MW.}$$

Table B: Load demand for 24 hours (5-unit system)

Time (h)	Load (MW)	Time (h)	Load (MW)	Time (h)	Load (MW)	Time (h)	Load (MW)
1	410	7	626	13	704	19	654
2	435	8	654	14	690	20	704
3	475	9	690	15	654	21	680
4	530	10	704	16	580	22	605
5	558	11	720	17	558	23	527
6	608	12	740	18	608	24	463

Table C: Data for the 10-unit system

Quantities	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
a ($/(MW)^2 h)	0.00043	0.00063	0.00039	0.0007	0.00079
b ($/MWh)	21.60	21.05	20.81	23.90	21.62
c ($/h)	958.20	1313.6	604.97	471.60	480.29
e ($/h)	450	600	320	260	280
f (1/MW)	0.041	0.036	0.028	0.052	0.063
P_{min} (MW)	150	135	73	60	73
P_{max} (MW)	470	460	340	300	243
UR (MW/h)	80	80	80	50	50
DR (MW/h)	80	80	80	50	50

Quantities	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10
a ($/(MW)^2 h)	0.00056	0.00211	0.0048	0.10908	0.00951
b ($/MWh)	17.87	16.51	23.23	19.58	22.54
c ($/h)	601.75	502.70	639.40	455.60	692.40
e ($/h)	310	300	340	270	380
f (1/MW)	0.048	0.086	0.082	0.098	0.094
P_{min} (MW)	57	20	47	20	55
P_{max} (MW)	160	130	120	80	55
UR (MW/h)	50	30	30	30	30
DR (MW/h)	50	30	30	30	30

Table D: Load demand for 24 hours (10-unit system)

Time (h)	Load (MW)	Time (h)	Load (MW)	Time (h)	Load (MW)	Time (h)	Load (MW)
1	1036	7	1702	13	2072	19	1776
2	1110	8	1776	14	1924	20	2072
3	1258	9	1924	15	1776	21	1924
4	1406	10	2072	16	1554	22	1628
5	1480	11	2146	17	1480	23	1332
6	1628	12	2220	18	1628	24	1184