N. Ben Hadj
S. Tounsi
R. Neji
F. Sellami

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A Comparative Study of Permanent Magnet Motor Cost Minimization Dedicated to Electric Traction

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In this paper, we present a permanent magnet motor cost minimization dedicated to the electric traction based on a genetic algorithms (GAs) method. Our objective is to minimize this cost by taking account of certain constraints. The choice of a suitable coding is a critical element which depends largely on the genetic algorithm effectiveness. That is why we present a comparative study between two types of genetic algorithms i.e. a binary coded genetic algorithm (BCGA) and a real coded genetic algorithm (RCGA).

Keywords: Permanent magnet, radial flux, electric motor, optimization, GAs, electric traction

1. INTRODUCTION

The electric traction introduction into transport is accompanied by bodies optimization search into electric in order to minimizing the cost. This approach requires significant work on electric motors modeling especially the permanent magnet and radial flux synchronous motor. In road traction application (electric vehicle EV), the specific and voluminal power constraint leads originators to under dimensioning motor in order to reaching minimal cost [1].

In this paper, after presentation of the studied electric motor (EM) structure like his analytical modeling, we expose our optimization problem which is studied by two types of GAs the first one is with binary coding and the second is with real coding of which thereafter detailed comparative simulations.

2. PROBLEM FORMULATION

Any optimization problem requires a mathematical formulation, in particular our cost minimization problem. Basing on a studied motor analytical dimensioning model [2, 3], which starting from the schedule data conditions, the expert data, constants characterizing materials, and motor configuration, we deduces the geometrical and electromagnetic magnitudes motor (figure 1).

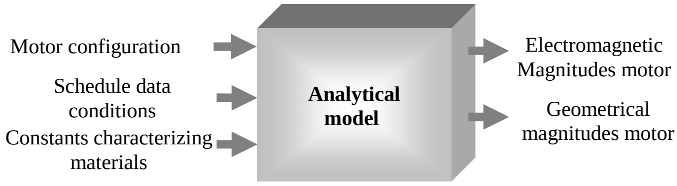

Figure 1: Analytical model

The EM cost deduced from its analytical dimensioning equations [2]:

$$Cost = C_{ma}M_m + C_c M_c + \left(M_{sy} + M_{st} + M_{toothi} + M_{ry}\right)\left(C_i + C_m\right) \qquad (1)$$

Laboratoire d'Electronique et des Technologies de l'Information LETI, Ecole Nationale d'Ingénieurs de Sfax B.P. W, 3038 Sfax – Tunisia.

where:

The magnets mass M_m is defined by:

$$M_m = pL_m W_m \left[\left(\frac{D_m - e}{2} \right)^2 - \left(\frac{D_m - e}{2} - H_m \right)^2 \right] M_{va} \tag{2}$$

The copper mass M_c is defined by the equation:

$$M_c = \frac{3 I_n N_{sph} L_{sp}}{\sqrt{2}\delta} M_{vc} \tag{3}$$

The stator teeth mass M_{st} is defined by the equation:

$$M_{st} = \left[\left(\frac{D_m + e}{2} + Hd \right)^2 - \left(\frac{D_m + e}{2} \right)^2 \right] L_m M_{vt} \tag{4}$$

The stator yoke mass M_{sy} is defined by the equation:

$$M_{sy} = \pi \left[\left(\frac{D_m + e}{2} + H_d + H_{cs} \right)^2 - \left(\frac{D_m + e}{2} + H_d \right)^2 \right] L_m M_{vt} \tag{5}$$

The rotor yoke mass M_{ry} is defined by the equation:

$$M_{ry} = \pi \left[\left(\frac{D_m - e}{2} - Ha \right)^2 - \left(\frac{D_m - e}{2} - Ha - H_{cr} \right)^2 \right] L_m M_{vt} \tag{6}$$

The inserted teeth mass M_{toothi} is defined by the equation:

$$M_{toothi} = \frac{M_{ds} A_{toothi}}{A_{tooth}} \tag{7}$$

where:

C_{ma} : Cost of one kilogram of magnets

C_c : Cost of one kilogram of copper

C_i : Cost of one kilogram of iron

C_m : Manufacture cost per kilogram of iron

P : Number of poles pairs

A_{tooth} : The principal angular tooth width

A_{toothi} : The angular inserted tooth width

L_m : The average motor length

D_m : The average motor diameter

E : The air-gap thickness

H_a : The magnet height

H_d : The principal tooth height

H_{cs} : The motor rotor yoke height

L_{sp} : Inductance per phase

I_n : Motor rated current

N_{sph} : Number of spires per phase

M_{va} : Volumic Mass of magnets

M_{vc} : Volumic Mass of copper

H_{vt} : Volumic Mass of metal sheets

After mathematical cost formulation of electric traction motor, we note that this cost depends primarily on the average motor length 'L_m', average motor diameter 'D_m', wheel radius '$rroue$', air-gap flux density 'be', air-gap thickness 'e', and the reducer ratio 'rd'. Consequently, we notethat this cost can be expressed differently by the following equation:

$$Cost = F\left(D_m, L_m, be, e, rroue, rd\right) \tag{8}$$

The two-dimensional cost evolution according to these parameters is illustrated by figures 2, 3, 4. This evolution is not linear, which validates the application of stochastic methods like genetic algorithm in order to find the optimal six parameters.

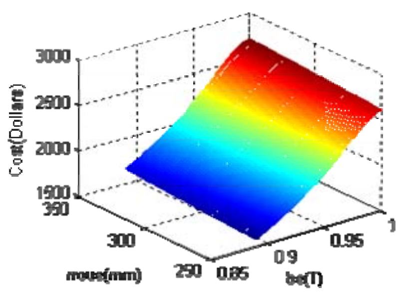

Figure 2: Motor cost as a function of '$rroue$' and 'be'

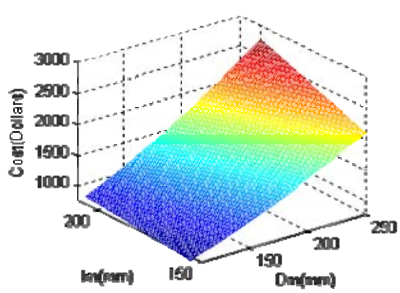

Figure 3: Motor cost as a function of 'L_m' and 'D_m'

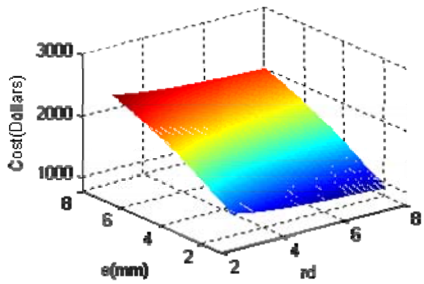

Figure 4: Motor cost as a function of ‘ e ’ and ‘ rd ’.

3. OPTIMIZATION PROBLEM

Electromagnetic optimization problems, generally involve several parameters, which can be continuous or discrete and are often bounded. Moreover, the objective functions that arise in electromagnetic optimization problems are often nonlinear, stiff, multi-extremal and non-differentiable. GAs are robust, stochastic-based methods which can handle the common features of electromagnetic optimization problems that are not readily handled by other traditional optimization techniques.

The optimization problem consists in determining optimal parameters values L_{mopt}, D_{mopt}, $rroue_{opt}$, be_{opt}, e_{opt}, rd_{opt} which correspond to minimal motor cost ‘$cost_{opt}$’. The beach of each parameter variation must respect the following constraint $X_{i\min} \leq X_i \leq X_{i\max}$, where $X_i \in (D_m, L_m, be, e, rroue, rd)$.

The values of the lower limit $X_{i\min}$ and the upper limit $X_{i\max}$ are established following technological, physical and expert considerations, for example:

- The wheel radius is delimited by the space reserved in electric vehicle.

- The air-gap flux density variation beaches are defined starting from the iron B-H curve to avoid saturation problem.

-The current density is an expert data.

The optimization problem consists on optimizing the motor cost by keeping efficiency higher than 0.95% and respecting the constraints illustrated by Table 1:

Table 1: Optimization limits parameters.

Lower limit	Variables	Upper limit
100	D_m (mm)	250
150	L_m(mm)	200
0.89	*be*(T)	1.05
2	*rd*	8
2	*e*(mm)	8
0.25	*rroue*(m)	0.35

3.1 Optimization with GAs method

GAs are stochastic optimization techniques founded on natural selection and genetics concepts [4]. It starts with a set of solutions called population. Solutions from a population are used to form a new population. This is motivated by the hope that the new population will be better than the old one.

Solutions that will form new solutions are selected according to their fitness: the more suitable they are, the more chances they have to reproduce. This is repeated until some condition (for example, number of generations or improvement of the best solution) is satisfied.

Figure 5 illustrate the applied procedure of optimization. The first step is the characterization of the individuals that will form the population. The individuals are composed by the six parameters of the motor cost.

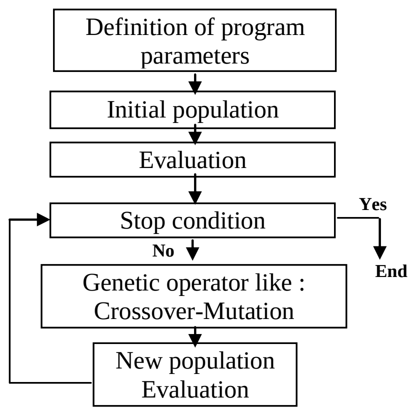

Figure 5: Optimization procedure.

The initial values assigned to the population are random values in the allowable range, as shown in Table 1. Each individual of this population is evaluated using the fitness function. The convergence criterion is based on a maximum allowed number of generations.

If convergence is not attained, genetic operators (selection, crossover and mutation) are applied. The selection procedure is responsible for forming the pairs that will be submitted to the other genetic operators. Selection is a mechanism related to individual fitness. Crossover and mutation are mechanisms used to change the genetic materials of the individuals. They are the main tools for the success of the optimization process and must be implemented in order to allow an effective exploration of the search space. The new individuals created by the genetic operators described above will be evaluated and the iterative process will be repeated until one of convergence criteria is reached [5].

The principal difficulty encountered with *GAs* is the coding problem in a specific form to genetics, several possibilities exist in [5] especially the traditional binary coding and real coding.

3.2 A binary coded genetic algorithm

Initially, to solve the optimization problem, we use a BCGA which our variables are coded in binary with discretization of research space. Thus a coding on k bits implies a discretization of intervals in $g_{\max} = 2^k - 1$ discrete values. We consider a finished space of research:

$$\mathrm{X}_{\mathrm{imin}} \leq \mathrm{X}_{\mathrm{i}} \leq \mathrm{X}_{\mathrm{imax}} \qquad \forall i \in [1; n\,\mathrm{var}] \tag{9}$$

where $n\,\mathrm{var}$ represent the number of variables.

To each real variable X_i we associates therefore a long whole g_i :

$$0 \leq g_i \leq g_{\max} \qquad \forall i \in [1; n\,\mathrm{var}] \tag{10}$$

where:

$$g_i = \sum_{j=0}^{6} b_j 2^j \tag{11}$$

Coding and decoding formulae are then following:

$$g_i = \frac{X_i - Xx_{i\min}}{X_{i\max} - X_{i\min}} g_{\max} \tag{12}$$

$$X_i = X_{i\min} + \left(X_{i\max} - X_{i\min}\right) \frac{g_i}{g_{\max}}$$

(13)

The various procedure of this algorithm is in [3].

3.3 A real coded genetic algorithm

In second phase, we used a RCGA, where each individual is then one digit with actual values in the research space [5].This coding consists simply in the concatenation of variables *Xi* of an individual *X* defined by:

$$X = X_1 X_2 \ldots . N_{n\,\mathrm{var}} \tag{14}$$

The first stage of the algorithm is the generation of initial population, which consists in creating randomly genes according to uniform distribution. We consider the case where the population is given by:

$$pop^n = \begin{bmatrix} D_m^{n,1} & L_m^{n,1} & e^{n,1} & be^{n,1} & rroue^{n,1} & rd^{n,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ D_m^{n,Np} & L_m^{n,Np} & e^{n,Np} & be^{n,Np} & rroue^{n,Np} & rd^{n,Np} \end{bmatrix} \tag{15}$$

where each line represents an individual (a point in the optimization space), *n* is the generation and *Np* is the population size.

Then, after generating initial population, each individual is evaluated according to equation 1 which is given by the following structure:

$$\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{Np} \end{bmatrix} \begin{matrix} individu_1 \\ individu_2 \\ \vdots \\ individu_{Np} \end{matrix} \tag{16}$$

After evaluation, we apply selection operator which determines and chooses population members who survive and who reproduce. The “Tournament” method was used as selection procedure which increases the chances for poor quality individuals to take part in the population improvement.

Then, we applies arithmetic crossing operator which carries out a simple linear combination between two parents:

$$\begin{cases} X = X_1 X_2 ... X_{n\,\mathrm{var}} \\ Y = Y_1 Y_2 ... Y_{n\,\mathrm{var}} \end{cases} \tag{17}$$

by generating a random variable, $\alpha \in (0,1)$ [6]-[7], we obtains two children defined by:

$$\begin{cases} X' = \alpha X + (1-\alpha) Y \\ Y' = (1-\alpha) X + \alpha Y \end{cases} \tag{18}$$

After crossing application, we apply uniform mutation operator where we take variable $X_i \in X$ [6, 7]. This last, will be changed according to certain probability into random number in a uniform distribution on the interval $[X_{i\min}, X_{i\max}]$.

The new individual is:

$$Y_i = X_{i\min} + r(X_{i\max} - X_{i\min}) \tag{19}$$

where r is a random variable in the interval [0,1].

We carried out some simulations in order to validate our GAS: BCGA and RCGA. We used the Rastrigin function as test function. The results obtained with RCGA were practically the same obtained using BCGA.

4. RESULTS

GAs is programmed in MATLAB 7.0 on PC Pentium 4, 2.0 GHz and 128 MB of RAM using characteristics tabulated in table 2.

Table 2: GAs characteristics.

Characteristics	BCGA	RCGA
Selection	Roulette wheel	Tournament
Crossover	Simple- point	Arithmetic
Mutation	No uniform	Uniform
Number of population	350 individuals	350 individuals
Number of generation	800 iterations	800 iterations
Crossover probability	0.85	0.85
Mutation probability	0.01	0.01

The optimization procedure was executed several times. In the great majority of the cases, the algorithm found practically the same best individual. This demonstrates the convergence of the applied methodology. For a stochastic optimization method, the final solution can only be considered optimal by repetition of the results [8].

Figure 6 shows a comparison between application of BCGA and RCGA for minimal cost motor determination.

We notice that the obtained results by the application of RCGA method are definitely better compared to those found by the BCGA. We obtain with RCGA a cost of 929.3$ compared to BCGA, i.e. a cost of 930.4$.

The $\cos t_{opt}$ was obtained after 20.25 minutes for the 800 generations. However the improvement was faster and more effective concerning RCGA, where the optimum was obtained after only 5.12 minutes for the 800 generations.

The GAs methodology used in this work allows obtaining results with good precision. Real coding has advantages related to the convergence time (few generations) and simplicity to assemble the individuals (it is not necessary to code them in binary representation).

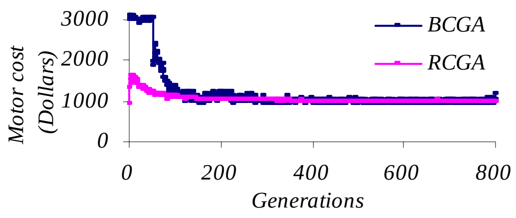

Figure 6: EM cost according to the number of generation.

The simulation results are tabulated by table 3.

Table 3 : GAs results

Optimised variables	Optimised variables value with RCGA	Optimised variables Value with BCGA
D_{mopt}(mm)	101.13	100
L_{mopt}(mm)	150.53	196.82
be_{opt}(T)	0.891	1
e_{opt}(mm)	2.041	7.994
$rroue_{opt}$ (mm)	251.80	258.21
rd_{opt}	7.95	7.15
$Cost_{opt}$ ($)	**929.3**	**930.4**

5. CONCLUSION

GAs have a strong potential of practical application. The choice of coding individuals remains one of the problems of GAs, it is very difficult to find a good coding adapted to the structure of the problem. Results found by BCGA are acceptable but the computing time is rather significant, on the other hand RCGA give satisfactory results with a reasonable computing time, for this reason we may find it beneficial to use the real representation of individual when the problem parameters have great values and require a high degree of accuracy.

The use of RCGA in electric traction field can constitute an interesting alternative seen its effectiveness for the problems resolution to several variables and constraints, and it treats a population of solutions.

Finally, the EV designed around a reduced cost of permanent magnets EM presents an interesting solution in the world of electric vehicles.

REFERENCES

[1] R. Trigui, F. Harel, B. Jeanneret, F. Badin, S. Dérou : ‘Optimization globale de la commande d’un moteur synchrone à rotor bobiné. Effets sur la consommation simulée de véhicules électriques et hybrides’. Colloque National Génie Electrique Vie et qualité; GEVIQ’2000. Marseille, 21-22 Mars 2000.

[2] S. Tounsi, R. Neji, N. Ben hadj, F. Sellami : ‘Global optimization of electric vehicle design parameters’. The 21st worldwide battery, hybrid and fuel cell electric vehicle symposium and exhibition; EVS 21 Monaco 2-6 April 2005; CD FFP 312.

[3] S. Tounsi, R. Neji et F. Sellami : ‘Optimization and Design for a Radial Flux Permanent Magnet Motor for Electric Vehicle’. Journal of Electrical Systems,volume 1,issue 4,2005.

[4] J. H. Holland : ‘Adaptation in Natural and Artificial Systems: An introductory analysis with applications to biology, control, and artificial intelligence’, MIT Press, Cambridge, 1992.

[5] S. L. Avila, W. P. Carpes Jr. and J. A. Vasconcelos : ‘Optimization of an offset reflector antenna using genetic algorithms’, IEEE – Transactions on Magnetics, v.40, n. 2, pp. 1256-1259, Mar. 2004.

[6] Z. Michalewitcz, Genetic Algorithms + Data Structures=Evolution Programs’, Springer Verlag.1992.

[7] Y. Fefermann: ‘Optimization directe de systèmes hétérogènes en Génie Electrique’, PHD thesis of INP Toulouse. France, December 2002.

[8] D.E. Goldberg: ‘Genetic Algorithms in Search, Optimization a Machine Learning’. Addison Wesley Longman, 2000.