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Optimal Genetic Algorithm-Pontryagin Minimum Principle Approach for Equivalent Fuel Consumption Minimization in Hybrid Electric Vehicle

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In present context, power management among various energy sources is the key requirement to achieve high efficiency in hybrid electric vehicle (HEV). The HEV usually utilizes the energy from fuel cell, battery and supercapcitor. Hence, the overall fuel consumption is required to be minimized by optimal strategy to identify optimal power distribution. The effectiveness of the strategy mainly depends on an accurate estimation of the equivalence factor to obtain the equivalent fuel consumption of the HEV. In the present work, estimation of equivalence factor using genetic algorithm (GA) tuned Pontryagin minimum principle (PMP) for optimal energy management is proposed. The proposed GA-PMP based method has the benefits of both GA and PMP. Simulation results show that the proposed GA-PMP approach outcomes more reduction in hydrogen consumption of fuel cell in comparison to PMP approach.

Keywords: Battery, energy management strategy, fuel cell, genetic algorithm, hybrid electric vehicle, Pontryagin minimum principle.

1. Introduction

Increased air pollutant emissions due to rapid development in the transport sector has grown up as a major threat to the public health in many countries worldwide [1]. Further, transport sector is the one of the primary consumers of crude oil and second largest emitter of greenhouse gases [2]. Protection of natural resource and environment are become the highly valued demand worldwide [3]. Therefore, HEVs are becoming key alternative to meet the enormous challenge of reducing the fossil fuel consumption and fighting against the climate change [4].

Nowadays, the HEVs are highly competitive against conventional internal combustion engine (ICE) vehicles; especially, when a relative extended range is required. In general, multiple energy sources employed with the HEV make them more efficient than ICE vehicles. Moreover, depending upon the type of energy source the HEVs can also be refueled quickly compared to EVs without requiring dedicated charging infrastructures and separate electrified tracks [5]. In addition, a comparable range with fuel economy and no carbon gas emission has made fuel cell electric vehicle (FCEV) as a popular alternative in the modern automobile industry. The advancements in fuel cell technologies and power electronics have empowered the significant development in FCEVs. The fuel cell (FC) has advantages as (i) high reliability, (ii) high efficiency, (iii) clean power generation and, (iv) low noise. In present time, there are many types of FCs are available and among them proton exchange membrane fuel cell (PEMFC) is the most suitable in automobile industry due to its low operating temperature and the quick start-up [6][7].

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However, FCEV must additionally use a secondary source of electrical energy in hybrid manner to cope with the fast-changing dynamic loads and its performance improvement. Use of high energy density battery or/and source of reversible power, such as super-capacitor (SC) as a secondary sources in FCEV overcome the issue of slow dynamic response of FCs and increase the overall efficiency of the vehicle [8]. The HEV consisting of multiple energy sources necessitates an energy management strategy (EMS) to manage the power flows among them, as well as to size the energy sources suitably in order to achieve overall maximum efficiency, minimal operative cost, optimal performances and enhanced lifetime [9].

The EMSs presented in literature can be broadly classified in two categories i.e. (i) rule-based strategy and (ii) optimization-based strategy with single objective or multiple objectives. Both types of strategies have their own advantages and disadvantages. Based on prior experiences, driving patterns and power train characteristics certain rules were designed to govern the HEV operating mode in rule-based strategy. There are some common rule-based energy management strategies include threshold [10], fuzzy logic [11][12] and filtration [13]. Although, such type of EMS are easy to implement and computationally efficient but due to lack of involvement of any optimization in this kind of EMS, the optimality of the solutions cannot be assured [14].

The optimization based EMSs for different designs of the HEV may further be classified in 'off-line' and 'real time optimization' strategies. An offline optimization strategies such as dynamic programming algorithm [15], genetic algorithm [16], particle swarm optimization [17] can obtain good control performance; however, they have high computational burden, which may be a major drawback for offline strategies. The real time optimization strategies reduce a global optimization problem into an instantaneous one and give a trade-off between optimal performance and calculation efficiency (Pareto Optimization) [18]. In continuation, equivalent consumption minimization strategy (ECMS) and model predictive control (MPC) are the two most popular real time optimization strategies among the researchers. In MPC [19] strategy the optimization problem was set over a prediction horizon, which again needs future driving cycle information. Further, the MPC problem with complex structure might affect the response time due to heavy computation. The proper determination of equivalence factor plays a key role in the ECMS, when the equivalence factor between different energy sources is properly tuned, the ECMS is considered as a practical approach for near-optimal solution [20].

Appropriate tuning of the equivalence factor is not easy due to its high sensitivity to drive cycle characteristics, battery state of charge (SOC) limits and the direction of electric current that are generally unpredictable. Equivalent to the costate of PMP, determination of the equivalent factor is proposed in [21]–[25] using PMP and gives promising results with less computational burden compared to other strategies. Further, optimal trajectory derived from PMP highly depend on initial value of costate. Therefore, the control based on PMP with improper selection of initial value of costate might not be a global optimal solution [23], [24]. Genetic algorithm tuned PMP approach for hybrid ICE and battery based vehicle for the proper selection of initial value of costate so that global solution of optimization problem of minimizing the fuel consumption can be achieved is presented in [26]. Proposed work focuses on the similar approach for the minimization of equivalent fuel consumption of hybrid Fuel Cell/ Battery/ Supercapacitor electric vehicle using genetic algorithm tuned PMP. The proposed method has the benefits of both GA and PMP.

2. Modelling of Components

In this section, dynamic modelling of the main components of the proposed HEV system such as fuel cell, battery and supercapacitor is presented.

(a) Fuel Cell System Modelling

Dynamic model of PEMFC is discussed in [27] and [28], which is used in this research work. The relationship between fuel cell stack voltage (V_{FC}) and Nernst voltage (V_N) can be represented as:

$$V_{FC} = n_o V_N - V_{Loss} \tag{1}$$

where, n_o: Number of fuel cell connected in series, V_{Loss}: Irreversible voltage losses.

Under the normal operating conditions, the output voltage of a fuel cell is determined by the irreversible voltage loss. The fuel cell output power is determined as follows,

$$P_{FC} = V_{FC} I_{FC} \tag{2}$$

where, I_{FC} is output current of the fuel cell.

(b) Battery Storage System Modelling

Lithium-ion battery model is used in this work. The battery model comprises a voltage source V_s and its internal series resistance R considering both are the function of battery SOC. The battery current can be given by [29]:

$$I_B = \frac{V_s(SOC) - \sqrt{V_s^2(SOC) - 4.R(SOC).P_B}}{2.R(SOC)} \tag{3}$$

Where P_B is the power supplied or absorbed by the battery. SOC of the battery which depends on the battery current can be given by:

$$SOC(t) = SOC_o - \frac{1}{Q_B}.\int_0^t I_B(\tau)d\tau \tag{4}$$

where, Q_B and i_B are the battery capacity and battery current, respectively.

The bidirectional dc–dc converter acts as a battery controller and stabilizes the dc link voltage during sudden load change.

(c) Supercapacitor Modelling

In comparison to a conventional capacitor, the supercapacitors can store and release more energy with high efficiency due to their high capacitance [30]. Here, Stern model, which combines both the Gouy–Chapman and Helmholtz models [31] is considered. The capacitance is expressed as,

$$C_S = [\frac{1}{C_{gc}} + \frac{1}{C_h}]^{-1} \tag{5}$$

where, C_h is Helmholtz and C_{gc} is Gouy–Chapman capacitance (in farads), respectively.

For a supercapacitor module of n_p cells in parallel and n_s cells in series, the total capacitance C_t is given by,

$$C_t = \frac{n_p}{n_s} C_S \tag{6}$$

The supercapacitor output voltage V_{SC} is expressed as

$$V_{SC} = \frac{q_t}{C_t} - R_{SC}\, i_{SC} \tag{7}$$

with

$$q_t = \int i_{SC}\, dt \tag{8}$$

where, q_t , R_{SC} and i_{SC} are total electric charge (in coulombs), supercapacitor module resistance and supercapacitor module current, respectively.

3. Equivalent Fuel Consumption Minimization Strategy

This section gives a brief about Equivalent Fuel Consumption Minimization control strategy for energy management of the hybrid (fuel cell, Battery and Supercapacitor) system as shown in Fig. 1. The brief specification of the different source parameters is given in Table I.

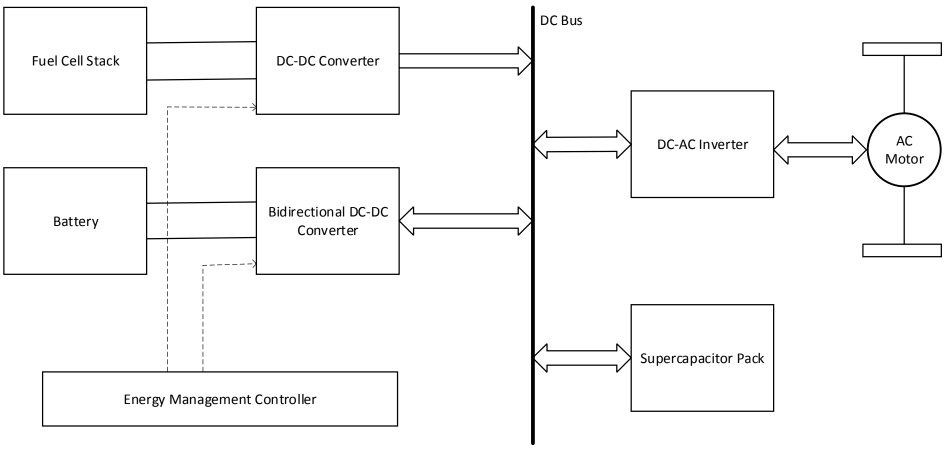

Fig. 1 Proposed Hybrid (FC-Battery-SC) System

Table I Energy Components of Hybrid System

Source	**Parameters**
Fuel Cell (PEMC)	12.5 kW (Peak), 30-60 V, Nominal power of 10 kW.
Battery System (Li-ion)	48 V, 40 Ah
Super-capacitor	291.6 V, 15.6 F (six cells in series of 48.6 v)

Power distribution among the various sources through ECMS is determined from the minimization of an instantaneous cost function, which consists of the fuel consumption of the fuel-cell system and the equivalent fuel consumption of the other energy sources.

In fuel cell/battery/SC based vehicles, both hydrogen and electrical energy can be used. Since, hydrogen consumption of fuel cell not only dependent on the energy supplied by the fuel cell to meet the load demand but it is also affected by maintaining the battery SOC to a desirable level by charging it, which in turn increases the hydrogen consumption or electrical energy supplied by battery to the load; however, it further decreases the hydrogen consumption. The electrical energy consumption of the battery may be transformed into equivalent hydrogen consumption as given in [32] to make the two comparable. The battery equivalent hydrogen consumption A_{batt} can be formulated as,

$$A_{batt} = \begin{cases} \dfrac{P_{batt}}{\eta_{dis}\eta_{chg\_avg}} \dfrac{A_{fc\_avg}}{P_{dc\_avg}}, P_{batt} \geq 0 & (9) \\ P_{batt}\eta_{chg}\eta_{dis\_avg} \dfrac{A_{fc\_avg}}{P_{dc\_avg}}, P_{batt} < 0 & (10) \end{cases}$$

where, P_{batt} is the battery power, A_{fc_avg} the average hydrogen consumption of fuel cell, P_{dc_avg} the average power of DC/DC converter, η_{dis} and η_{chg} are the battery discharging and charging efficiency, respectively. Further, η_{dis_avg} , η_{chg_avg} are the battery average discharging and charging efficiencies, respectively.

The main goal of ECMS is to achieve minimum fuel consumption by minimizing the fuel consumption of fuel cell and equivalent fuel consumption of battery. The objective function of ECMS, which express total hydrogen consumption 'C' of the hybrid system can be written as,

$$C = (A_{fc} + \alpha A_{batt})\Delta t \tag{11}$$

where, C is the total hydrogen consumption of the system, A_{fc} fuel cell hydrogen consumption, A_{batt} battery equivalent hydrogen consumption, Δt is the time interval of control and α is an equivalent hydrogen consumption coefficient of the battery system. Equivalent factor α can be expressed as,

$$\alpha = 1 - 2\,\mu \frac{(\,SOC - 0.5\,(SOC_{max} + SOC_{min}))}{SOC_{max} + SOC_{min}} \tag{12}$$

where, SOC is the current SOC value of the battery, SOC_{max} and SOC_{min} are the maximum and minimum SOC value of the battery, respectively. Here, μ is a value of balance coefficient, which has an impact on the change rate of battery's SOC. The value of μ is adjusted to 0.5 for better control of the battery in the present work.

The constrained condition of energy management in this work is expressed as (boundary conditions),

$$\begin{gathered} P_{fc\_\min} \leq P_{fc} \leq P_{fc\_\max} \\ P_{batt\_\min} \leq P_{batt} \leq P_{batt\_\max} \\ 0 \leq \alpha \leq 2 \end{gathered} \tag{13}$$

where, P_{fc_min}, P_{fc_max} , P_{batt_min} , P_{batt_max} are the minimum and maximum fuel cell and battery power, respectively.

Here, the SC power is not considered in the optimization problem as the dc bus voltage is controlled by the battery converters. As soon as the SC discharges, they are charged from the battery system; therefore, the load energy is effectively shared between fuel cell and the battery only.

The correct evaluation of the equivalence factor is of utmost importance. Hence, the proposed equivalent fuel consumption minimization strategy depends on various parameters, which may have co-relation. The proposed optimization technique has high searching ability for global optimum solution with high accuracy. In the presented work, a hybrid algorithm based on combination of the GA and the PMP is used for the accurate estimation of equivalence factor.

(a) Genetic Algorithm

Non-traditional optimization method GA mimics the principle of natural genetics and selection for an optimization procedure. Reproduction, mutation, and crossover used to create new and better population. The GA algorithm as shown in Fig. 2 is well proven optimization approach to find the global optimum solution. Further, local optimization techniques such as the PMP are quite efficient to find a local optimum very quickly.

Here, firstly the GA is used to find the optimized values of equivalence factor and thereafter the PMP approach is applied for further improvement in the parameters value. Since, the PMP technique generates a new space around the best point obtained from the GA, and search within this space about a better point. PMP propose an equivalence factor to convert battery power into equivalent fuel cell power, similar to costate of PMP. The initial value of costate variable is very important in the PMP algorithm, the optimal value obtained from GA is fitted into PMP for optimum fuel efficiency.

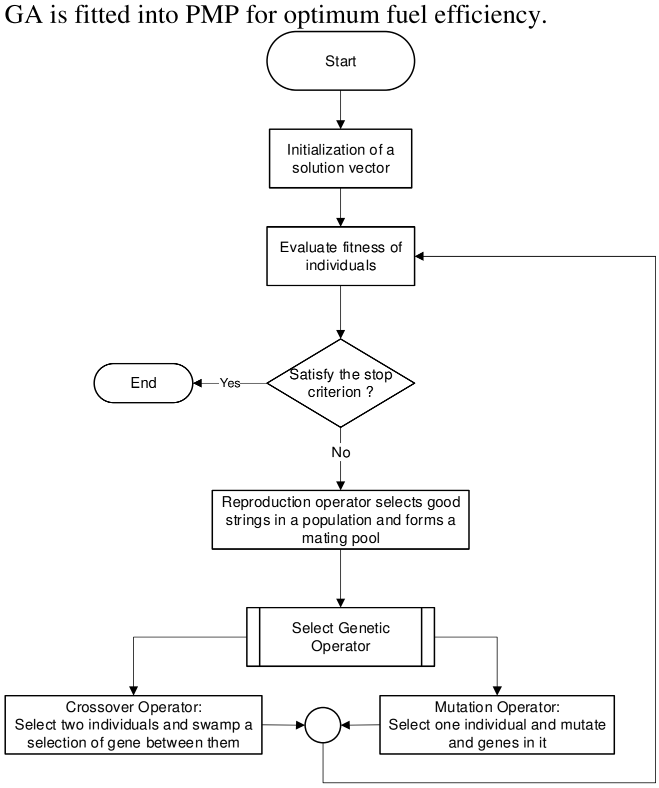

Fig. 2: Genetic Algorithm Flow Chart

(b) Pontryagin Minimum Principle Approach

The PMP turns the overall optimization problems into an instantaneous optimization problem and can be applied in a real time system with very less calculating burden. The Hamiltonian is formed first to optimize any problem using PMP, then state and costate equations are obtained. The main objective of the optimal control problem is to find an optimal power split trajectory such that it minimizes the overall fuel consumption in real time. As justified earlier that SC power is not considered in the optimization problem, so here fuel cell power and battery SOC are the control variable and sate variable of the optimal control problem, respectively.

The equation (3), which describe the dynamic of the state variable SOC can be simplified as follows [25]:

$$SOC(t) = f(S\dot{O}C(t), P_{batt}(t)) \tag{14}$$

Using a different function F the state equation (14) may be transformed as a constraint of optimal control problem, which is the first necessary condition:

$$S\dot{O}C(t) = F(SOC(t), P_{FC}(t)) \tag{15}$$

The performance measure J may be expressed as:

$$J\big(P_{FC}(t)\big) = \int_{t_0}^{t_f} \Big\{\dot{m}_{h_2}\big(P_{FC}(t)\big) + \lambda(t).\Big(F\big(SOC(t), P_{FC}(t)\big) - S\dot{O}C(t)\Big)\Big\}\, dt \tag{16}$$

Where $\dot{m}_{h_2}$ is the fuel consumption rate of fuel cell stack. Hamiltonian function H is defined as:

$$H\big(SOC(t), P_{FC}(t), \lambda(t)\big) = \dot{m}_{h_2}(P_{FC}(t)) + \lambda(t).F(SOC(t).P_{FC}(t)) \tag{17}$$

The costate equations which are the second necessary conditions and determines the optimal trajectory are given as:

$$\begin{aligned} \frac{\partial H}{\partial \lambda} &= S\dot{O}C \\ \frac{\partial H}{\partial SOC} &= -\acute{\lambda} \\ \frac{\partial H}{\partial P_{FC}} &= 0 \end{aligned} \tag{18}$$

The third necessary condition according to PMP, bound the optimal control trajectory $P_{FC}^{*}(t)$, optimal state trajectory $SOC^{*}(t)$, and optimal costate trajectory $\lambda^{*}(t)$ to minimize Hamiltonian such that:

$$H\big(SOC^{*}(t), P_{FC}^{*}(t), \lambda^{*}(t)\big) \leq H(SOC^{*}(t), P_{FC}(t), \lambda^{*}(t)) \tag{19}$$

4. Results and Discussions

This section presents the simulation results obtained for the control of hybrid standalone FC-Battery-SC system using proposed equivalent fuel consumption minimization strategy. The detail of the strategy has been explained in previous section. Fig. 3 shows the load power and power shared by fuel cell, battery and super-capacitor to meet the load demand.

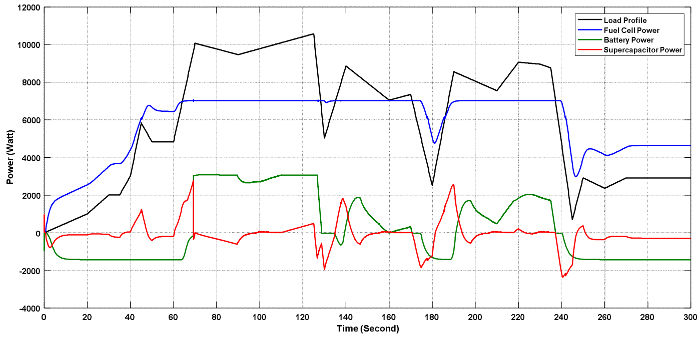

Fig. 3 Net load power demand and power shared by fuel cell, battery and super-capacitor

It can be seen from Fig. 3 that the maximum load power is supplied by fuel cell itself. Battery provides the power, when fuel cell alone is not able to meet the load demand. As discussed earlier, the super-capacitor power is not considered in the optimization problem as the dc bus voltage is controlled by the battery converters. Therefore, as soon as the super-capacitors discharge, they are charged with the same energy from the battery system. Hence, the total load energy is shared only between the fuel cell and the battery over a given load cycle. However, super-capacitors efficiently deliver power in sudden change in load demand.

Fig. 4 (a) and (b) shows the fuel cell voltage and current variation during the simulated load scenario. It is observed that the fuel cell output current increases according to fuel cell power output and the voltage drops simultaneously to some value due to the ohmic losses in the fuel cell stack. Fig. 4 (c) shows the battery SOC variation with time. Here, battery SOC is controlled through the penalty coefficient of the battery energy, so initially the fuel cell power is used to charge the battery and SOC increases. Afterwards, when fuel cell alone is not able to meet the load demand, battery share the load, it causes drop in the SOC. It remains constant, whereas the fuel cell alone is able to meet the load demand. Fig. 4 (d) shows the fuel consumption, the slope of the curve follows fuel cell power output.

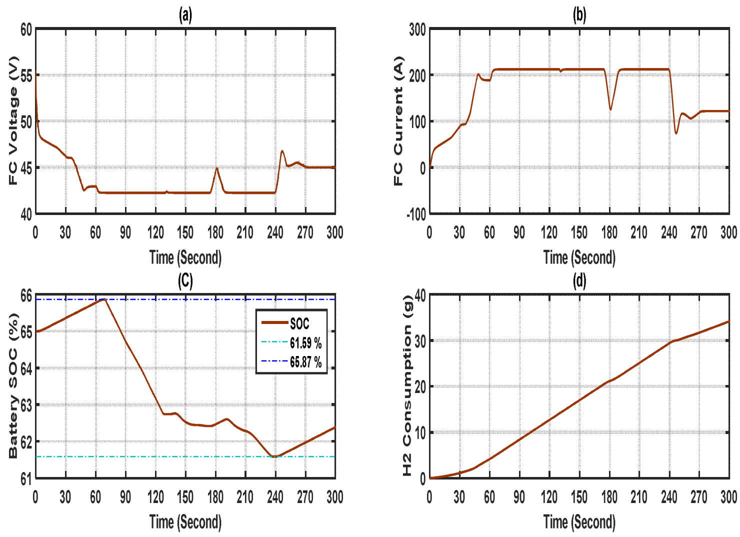

Fig. 4 (a) Fuel Cell Voltage (b) Fuel Cell Current

(c) Battery State of Charge (d) Fuel Consumption

The Supercapacitors voltage and current are shown in Fig. 5. The Super-capacitor current follows its power output, which in turn depends on the sudden change in load power. Charging/discharging of the supercapacitor causes to vary the voltage, which is further regulated to 270 volt by charging/discharging of the battery.

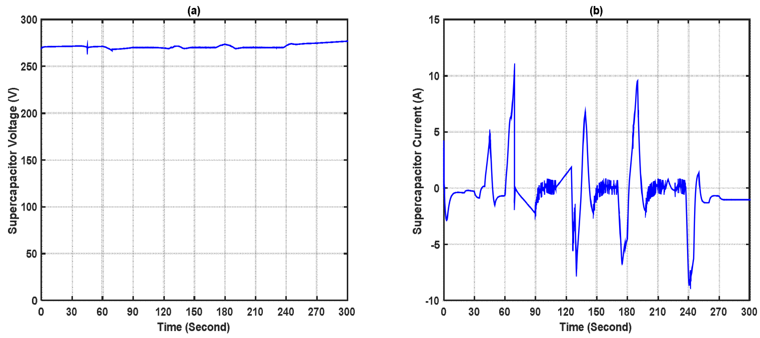

Fig. 5 Super-capacitor (a) Voltage (b) Current

Performance comparison between GA tuned PMP (GAPMP) and only PMP based equivalent fuel consumption minimization strategies in terms of fuel cell hydrogen consumption and battery SOC level are given in Fig. 6 and Fig. 7 respectively. Table II presents the overall hydrogen consumption and SOC range for proposed GAPMP and PMP strategy for the simulation period.

It can be revealed from the comparison that the proposed GAPMP based ECMS strategy reduce the hydrogen consumption approximately 9 % in compare to PMP based ECMS. Although this put slightly more burden on the battery which causes 1.7 % slightly lower SOC level in compare to PMP approach.

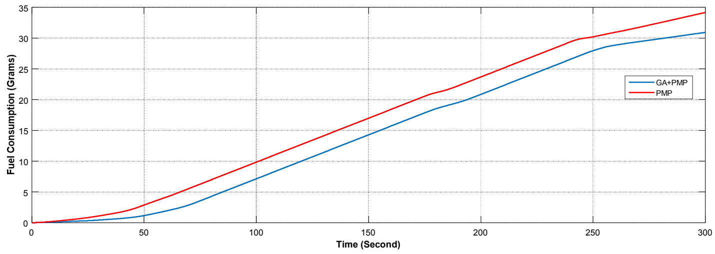

Figure 6: Hydrogen consumption comparison between GAPMP and PMP ECMS

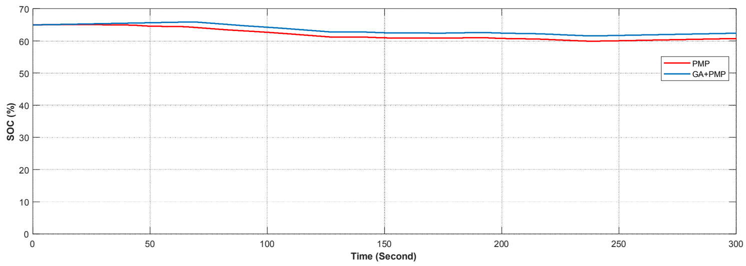

Fig. 7: Battery SOC Comparison for GAPMP and only PMP ECMS

Table II: Performance Comparison between GAPMP and PMP ECMS

	GA+PMP ECMS	PMP ECMS
H_2 Consumption (gm)	14.43	16.01
State of Charge (SOC)	59.98-65.17	61.59-65.87

5. Conclusion

This work presents a GA tuned PMP optimization technique for minimization of equivalent fuel consumption of fuel cell, battery and supercapacitor based hybrid electric vehicle. The developed energy management strategy splits the power demand between the fuel cell and battery in an optimal way. The main idea behind the hybrid GA-PMP algorithm is to combine the advantages of each algorithm in a way to avoid their disadvantages. The PMP as a local search technique generates a new space around the best point obtained from the GA and look for a better point within this space. The simulation results showed that the proposed technique causes lesser hydrogen consumption of fuel cell, which in turns reduce onboard hydrogen storage requirements and improve the performance of the HEV. The proposed GA-PMP based ECMS reduces the hydrogen consumption by approximately 9% at the cost of 1.7 % reduction in the battery SOC level in comparison to conventional PMP based ECMS.

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