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Calculation of the leakage reactance in distribution transformers via numerical and analytical methods

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Different analytical and numerical methods for the calculation of the leakage reactance of the transformer are presented in this paper. Finite element analysis is used for the simulation of two-dimensional (2-D) and three-dimensional (3-D) finite model of the transformer. Results are also compared with the experimental and analytical methods. The obtained results from finite element analysis show that both 2-D and 3-D energy methods provide higher accuracy as compared to the conventional and inductance method. Energy method is also less time consuming as compared to other methods like flux element and image techniques. The relative error between experimental and finite element energy method is less than 1%. These methods can be helpful for the manufacturers of the transformers for minimizing the design time of the transformer and it can also increase the performance of the transformer.

Keywords: Energy method, Finite element analysis, Inductance method, Leakage reactance, Power transformer.

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1. Introduction

Transformers are one of the main components of the electrical system. Distribution transformers have the efficiency between 98 and 99 percent [1]. Leakage reactance plays a vital role in the designing of the transformer. Voltage drop and efficiency of the transformer on load are mainly affected by the leakage reactance [2]. Moreover, the stored energy in the leakage inductance leads to voltage spikes in the switches [3]. Therefore, accurate calculation of the reactance value is greatly important in the designing stage of the transformer. In addition, leakage reactance mainly depends on the mechanical dimensions and it is one of the main indicators to determine the mechanical condition and occurred faults in the transformer.

In literature, many different analytical and numerical techniques are available for the calculation of the leakage reactance [3-11]. When the height of the HV and LV windings are equal, most of the analytical methods give accurate results [6]. However, if the heights of windings are asymmetric, most of these techniques are non-reliable. Image method, inductance method, flux leakage elements and energy methods are most common techniques used to determine leakage reactance of the transformer. However, it is difficult to calculate or accurately test the self and mutual reactance [9].

Image technique is one of the methods that could be used for the calculation of the leakage reactance. Mutual and leakage reactance values are calculated by employing magnetic potential to the image of every turn of the winding [12,13]. Image technique is effective and the computation results highly depend on the current of the image conductor [12]. However,

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the main disadvantage of the imaging technique is that the reliability of this method decreases, when the difference between the axial length of the low voltage and high voltage windings increase.

Double Fourier series was used by Roth in 1926 for the calculation of the leakage reactance. This analytical method is useful for both uniform and non-uniform ampere-turn distributions of windings [9]. In this method, current density depends on the radial and axial position. However, calculation of the reactance become difficult in this method, when we also consider the winding curvature of the transformer.

In 1956, another method for calculation of the leakage reactance was presented by Rabin [14-15]. In this method, winding curvature was taken into account and calculation equation was improved [9, 14]. He also uses Fourier series for the leakage reactance calculation but he assumed that current density only depends on the axial position.

Estimation of the flux in the different parts of the transformer is one of the commonly used methods for the calculation of the leakage reactance [16-18].

In [7], practical approach for the analytical calculation of the leakage inductances of transformers with winding interturn short circuits was proposed. In [19], the calculation of the leakage inductance of core-type tape-wound transformers is considered in the context of a magnetic equivalent circuit-based analysis. Nevertheless, the implementation of these models are not simple. Furthermore, these both approach need detailed structural geometric data of the transformer windings.

The main objective of this study is to compare the simulation results with the analytical leakage reactance calculation methods. 2-D and 3-D finite element analyses are used in simulation studies, to obtain the leakage reactance of the transformer.

Results of the finite element and analytical methods are also compared and verified from experimental measurements.

2. Studied Transformer

A three phase, 1250 kVA, 50 Hz, 34.5/0.4 kV, two winding specially designed high efficient distribution transformer with Dyn11 connected windings is examined in this study. Transformer core was manufactured using M5 grain oriented silicon steel. Core induction was chosen as 1.53T in the design stage. Table-1 shows the main parameters of the transformer. Fig. 1 shows the active parts of the examined transformer.

Fig. 1. Active parts of the transformer

Table 1: Transformer parameters

Ratings	Capacity (kVA)	1250
	High Voltage (kV)	34.5
	Low Voltage (kV)	0.4
	HV Current (A)	12.08
	LV Current (A)	1804.37
	Frequency (Hz)	50
Core	Material	M5
	Flux Density	1.53
Windings	Material	Aluminum
	HV Turns	2390
	LV turns	16

Fig. 2 shows the 2-D geometric details of the examined transformer. Hysteresis, power loss and magnetization curve of the core material are given in Fig. 3, Fig. 4, and Fig. 5.

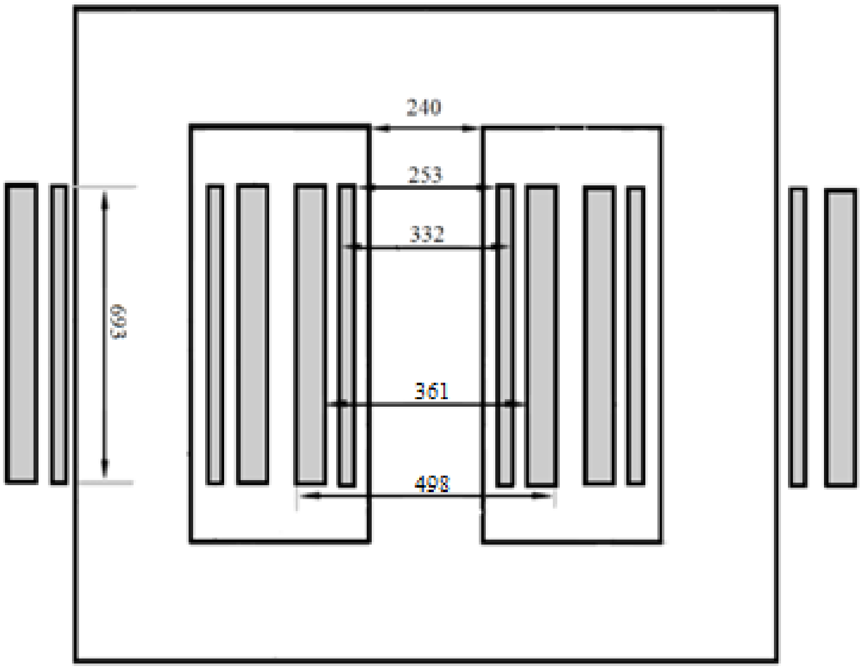

Fig. 2. 2-D geometry details of analyzed transformer (all dimensions are in mm)

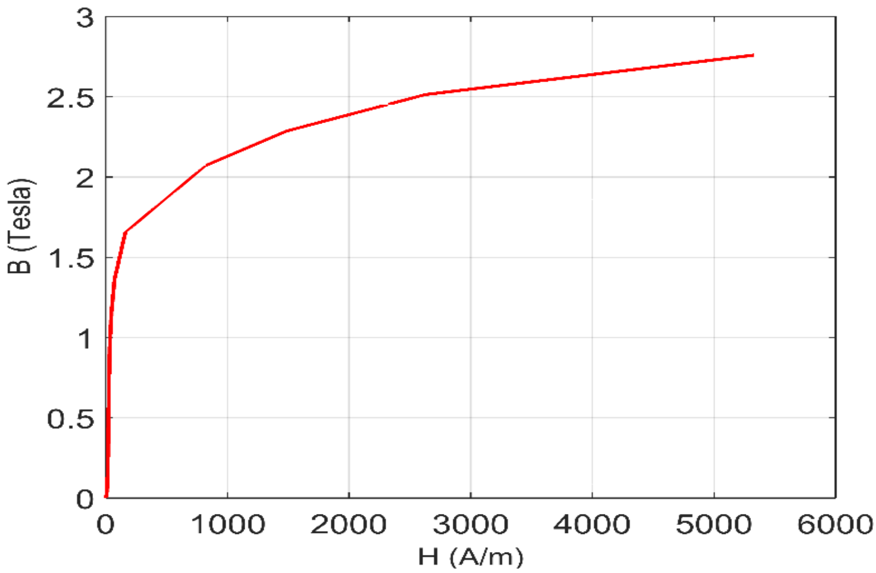

Fig. 3. B-H curve of the transformer

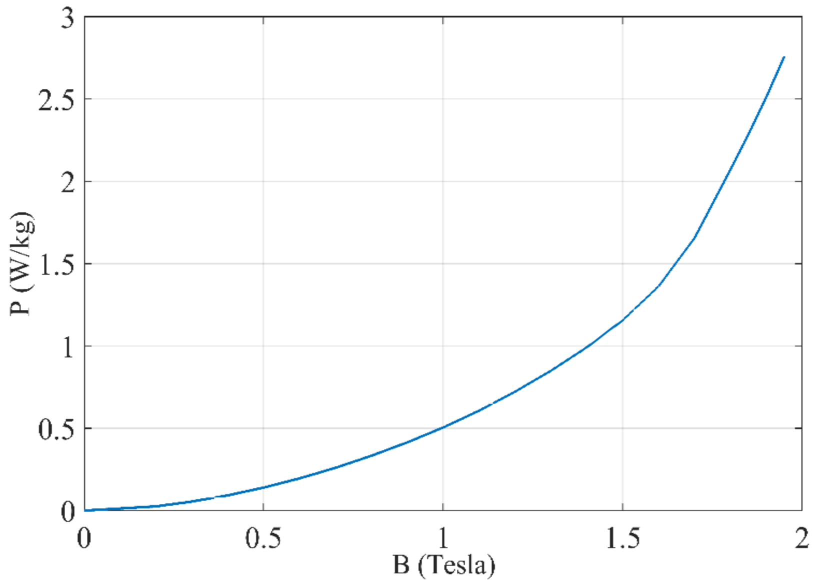

Fig. 4. B-P curve of the transformer

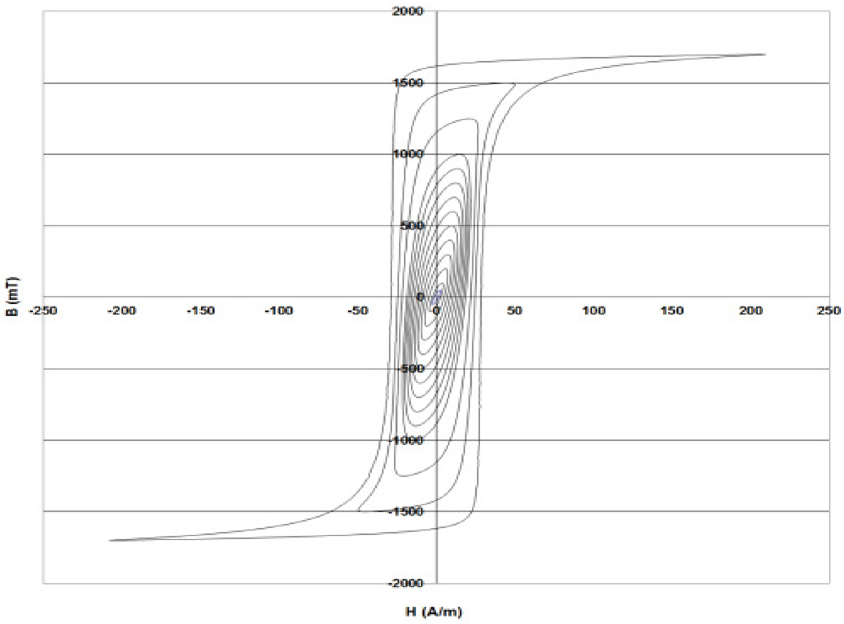

Fig. 5. Magnetization Curves of the core material

3. Leakage Reactance Calculation using Analytical Methods.

Power transformers are one of the most essential and expensive equipment in power systems [20]. Accurate calculation of the reactance value is greatly important in the designing stage of the transformer. Among a number of analytical methods in literature, two of most commonly used and accurate analytical expressions are presented in this work. In [9], leakage reactance is expressed as;

$$\%X=2.48\times10^{-5}\times f\times\frac{\text{Ampere Turn}}{H_{eq}\times\left(\frac{\text{volt}}{\text{turn}}\right)}\times\sum ATD \quad (1)$$

and

$$\sum ATD=\frac{1}{3}(C\times D_1)+(D\times D_2)+\frac{1}{3}(E\times D_3) \quad (2)$$

Where, C, D, D_1, D_2, D_3, E, and f are the radial depth of the LV winding, the radial depth of the gap between LV-HV windings, the mean diameter of the LV winding, the mean diameter of the gap between LV-HV windings, the mean diameter of the HV winding, the radial depth of HV winding, and frequency respectively.

In equation (1), H_{eq} is the equivalent height of the transformer and it can be obtained by dividing winding height H_w by the Rogowski factor K_R (<1.0).

$$H_{eq}=\frac{H_w}{K_R} \quad (3)$$

K_R can be calculated by using equation (4).

$$K_R=1-\frac{1-e^{-\frac{\pi\times H_W}{C+D+E}}}{\frac{\pi\times H_W}{C+D+E}} \quad (4)$$

Another analytical formula used to calculate the leakage reactance is given in equation (5) [17].

$$X\%=\frac{7.91\times f\times I_s\times\pi\times T^2\times D_{HL}}{V_s\times A_L}\times\left(D+\frac{C+E}{3}\right)\times 10^{-7} \quad (5)$$

where,

I_s= Rated secondary current per phase

T= Number of turns per phase in LV winding

D_{HL}=Mean diameter of LV and HV windings.

V_s= Rated secondary voltage per phase

A_L= Average length of LV and HV windings

Dimensions, given in equation (2) and equation (5) are also shown in Fig. 6.

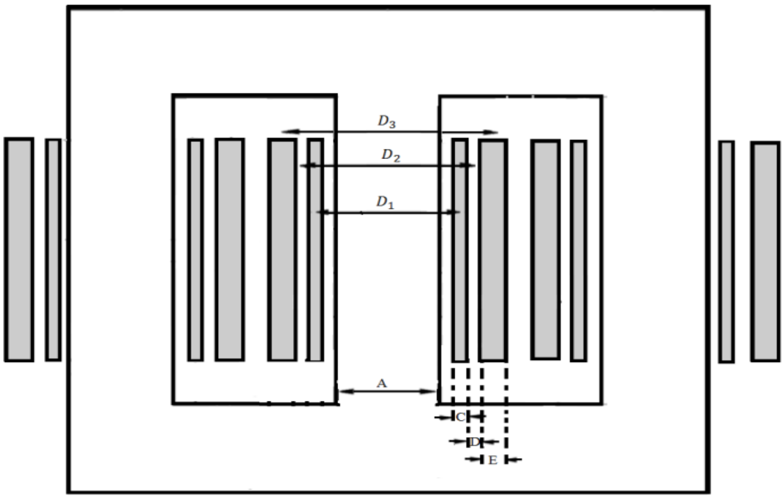

Fig. 6. Dimensions of the Transformer

4. FEM Based Reactance Computation

FEM is a numerical technique which can be used for the solutions of integral and differential equations such as electromagnetic, magnetostatic and thermal conductivity. In this study, total stored energy and inductance of the transformer is calculated by using ANSYS Maxwell software.

For the stored energy method, energy is converted into the inductance by using equation (6) and then leakage reactance is calculated from obtained inductance. For inductance method, inductance is directly obtained from FEM by using equation (10) and leakage reactance is calculated by using equation (15).

$$L=\frac{2W_m}{I^2} \quad (6)$$

Total magnetic field energy of the magnetic field in each part in a volume can be calculated as

$$W_m = \int_V \frac{1}{2}.B.H\ dV \quad (7)$$

and

$$H = \frac{B}{\mu} \quad (8)$$

Where B is magnetic flux density and H is magnetic field strength.

$$W_m = \frac{1}{2\mu}\int_V B^2\ dV \quad (9)$$

$$L = \frac{1}{I^2}\int_V B.H\ dV \quad (10)$$

The magnetic energy can also be calculated as the product of the magnetic vector potential and current density in volume.

$$W_m = \frac{1}{2}\int_V A.J\ dV \quad (11)$$

Inductance can be obtained by using equation (12).

$$L = \frac{1}{I^2}\int_V A.J\ dV \quad (12)$$

Where, A and J is magnetic vector field potential and current density vector, respectively. The stored magnetic energy in the window space in 2-D magnetic field can be calculated by using equation (13) and equation (14) [6, 21].

$$W_m = \frac{1}{2}.t \iint B.H.\ dxdy \quad (13)$$

$$W_m = \frac{1}{2}.t \iint J.A.\ dxdy \quad (14)$$

The leakage reactance of the transformer can be calculated as

$$X = \frac{2\times\pi\times f\times L}{Z_b} * 100 \quad (15)$$

Where; L is the inductance of the transformer and Z_b is base impedance.

$$Z_b = \frac{KV^2\ (\text{HV side})}{MVA} * 100 \quad (16)$$

Various magneto-static and transient analyses are performed for the determination of leakage reactance value and the flux distributions on the core of the transformer.

Fig. 7 shows the external excitation circuit of the transformer using FEM software. Fig. 8 shows the 3-D model of the studied transformer under mesh operation. The total number of the mesh generated in the 3-D model is 40369 elements. Fig. 9 shows the 2-D model of the studied transformer under mesh operation and total number of the mesh generated in the 2-D model is 1551 elements.

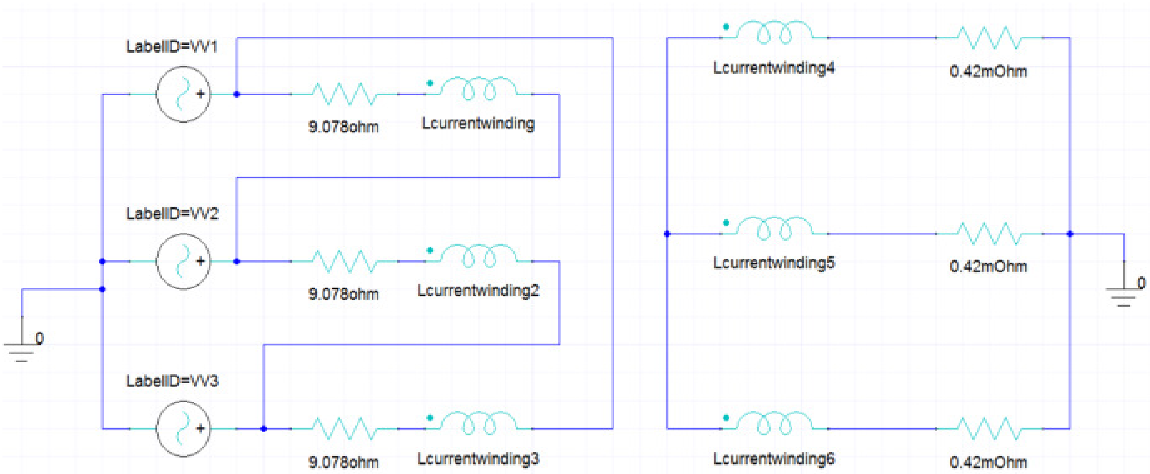

Fig. 7. External Excitation Circuit of Three-phase Transformer.

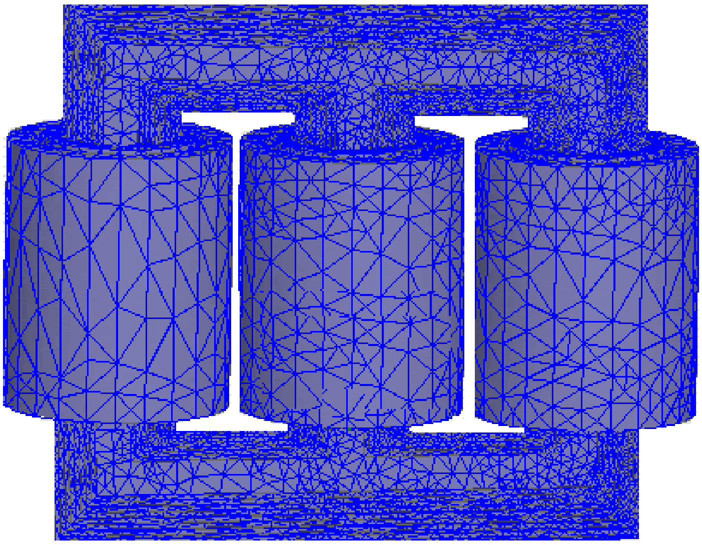

Fig. 8. 3-D. Mesh operation of Studied Transformer

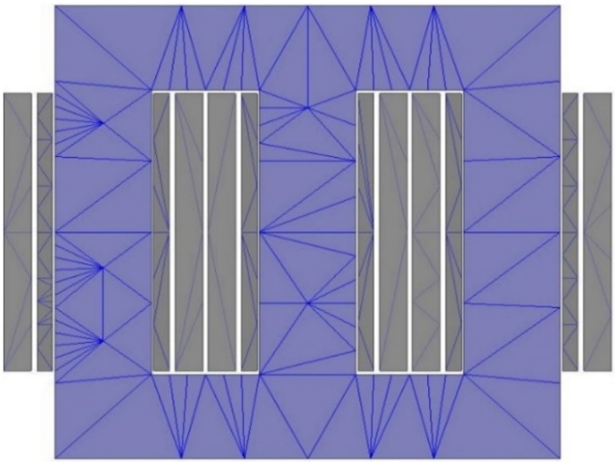

Fig. 9. 2-D. Mesh operation of Studied Transformer

Distribution of the magnetic flux density of the 3-D and 2-D transformer is shown in Fig. 10 and Fig. 11 respectively.

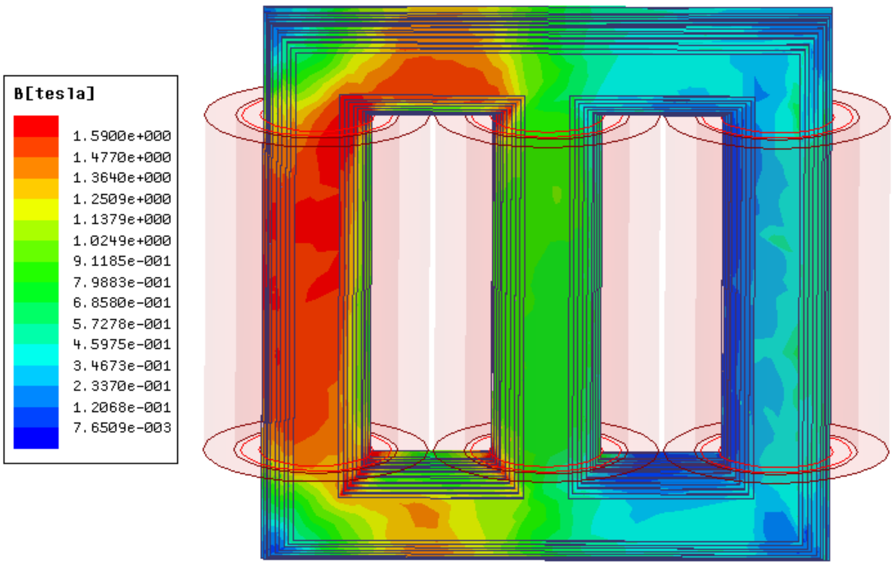

Fig. 10. 3-D. Magnetic flux density distribution

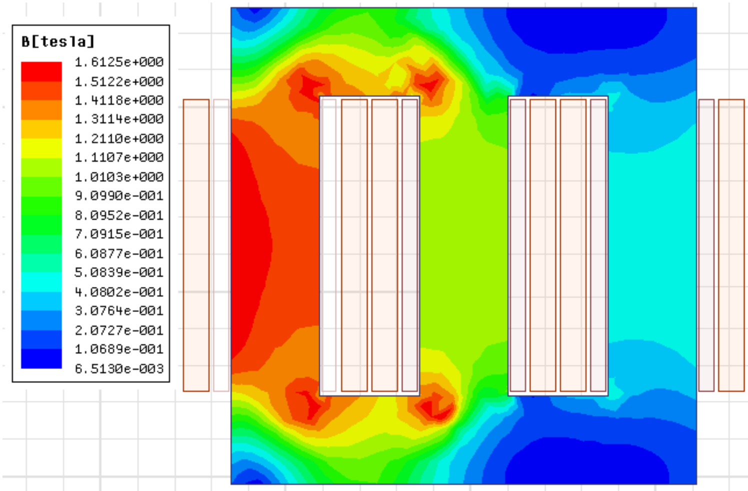

Fig. 11. 2-D. Magnetic flux density distribution

5. Comparison with the experimental results

Obtained simulation and analytical results are compared with experimental measurements. Experimental results of the leakage reactance of the transformer was obtained by short circuit test with rated winding current. In experimental stage, grid voltage was applied to the HV windings and LV windings were short-circuited. Test was performed according to the IEC 60076-1 standard. Measurement of the leakage reactance was performed quickly so that the temperature rise would not affect the results.

Obtained analytical, simulation and experimental results are given in table-2.

Table 2: Leakage reactance of power transformer

Approach	FEM Output	Leakage Reactance (%)	Error (%)
Experimental	-	6.15	-
Analytical method (Equation-1)	-	6.31	2.6
Analytical method (Equation-5)	-	6.53	6.18
Energy method (2-D)	41.14 J	6.20	0.82
Energy method (3-D)	41 J	6.16	0.16
Inductances Method (2-D)	188 mH	6.23	1.3
Inductances Method (3-D)	187 mH	6.19	0.65

Results show that the energy method is more accurate as compared to the other methods. The difference between the energy methods and experimental result is 0.82% and 0.16% for 2-D and 3-D method respectively. Inductance method is also efficient method but the difference between the inductance method and experimental result is high as compared to the energy method, error between inductance and experiment method is 1.3% and 0.65% for 2-D and 3-D method respectively.

3-D energy method is more accurate as compared to the 2-D method however, it is more time consuming and total simulation time of the 3-D method is more than 10 times as compared to the simulation time of the 2-D method.

6. Conclusion

This paper investigates the calculation accuracy of various leakage reactance methods of the transformer. Leakage reactance is calculated by using different analytical methods as well as inductance and energy method by using finite element analysis software. Results are also compared with the values obtained by experimentally.

Results prove that the energy method seems more accurate as compared to the analytical and inductance methods. 3-D energy method gives more accurate results as compared to the 2-D energy method. However, it consumes more time as compared to the 2-D energy method. Considering the importance and complexity of the leakage reactance in transformer designing stage, the use of these methods can provide advantages and simplicity to the transformer designers for the calculation of the leakage reactance.

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