

In this paper, a novel robust adaptive sliding-mode control scheme is presented high precision position control for a permanent magnet synchronous linear motor (PMSLM) servo system under the influence of unknown parameters, nonlinear friction, cogging forces, and external distance. The proposed strategy effectively combines the design techniques of adaptive control and sliding-mode control. In contrast to existing control approach, such as proportional-integral-derivative (PID) control and adaptive robust control (ARC), the developed robust adaptive sliding-mode control not only has better tracking performance, but also provides stronger robustness. Moreover, in the ARC, the selection of robust control gain must meet certain conditions, and an observer is needed to estimate the unmeasurable states in the LuGre friction model, which makes the controller more complicated to realize. The system's stability is proved via Lyapunov stable theory. Finally, the effectiveness of the presented algorithm is demonstrated by numerical simulation for different situations.

**Keywords:** Permanent magnet synchronous linear motor, adaptive sliding-mode control, unknown parameters, LuGre friction.

Article history: Received 21 February 2014, Received in revised form 24 September 2014, Accepted 3 October 2014

## 1. Introduction

Permanent magnet synchronous linear motor(PMSLM) direct drive systems have been widely employed in modern mechanical systems requiring linear motor at high speed and high accuracy, such as machine tools<sup>[1]</sup>, maglev vehicles<sup>[2]</sup>, semiconductor and microelectronics manufacturing equipment<sup>[3]</sup>, automatic inspection machines<sup>[4]</sup>. That is because that PMSLM have several benefits including high thrust density achievable, low thermal losses, higher acceleration/deceleration capability. Moreover, PMSLM based direct drive systems is the lack of mechanical reduction and transmission devices as in gear boxes, chains, and screws coupling, which will greatly reduce the effects of the typical nonlinearities such as friction and backlash. However, the lack of indirect coupling mechanisms makes the influence of load disturbances and various uncertain electromagnetic phenomenons (e.g. cogging force, friction, etc.) much more significant than in the case of conventional rotary machines.

Numerous methods have been investigated to compensate the unknown system parameters, load disturbance, cogging friction, and friction<sup>[5-8]</sup>. However, most of those methods consider only the effect of viscous friction. In fact, friction is a very complicated phenomenon relying on the physical properties of the contact surface, such as relative velocity, material property, and lubrication condition. Therefore, a new LuGre friction model based nonlinear dynamic compensation approaches are developed<sup>[9-12]</sup>. Though above motioned approaches can achieve precision position tracking and effective compensation of the nonlinear dynamic uncertainties, the structure of the designed

\* Corresponding author: tcs111@163.com

Chuansheng Tang, Yuehong Dai, Yong Xiao (School of Mechatronics Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China)

controllers are too complicated to realize in practice. A nonlinear observer is needed to estimate the unmeasured state in the LuGre model. The selection of the robust control gain requires meeting certain conditions in adaptive robust control as seen in [10].

To resolve those problems, we develop an adaptive sliding control scheme to improve system performance. As we know that sliding-mode control has strong robustness for the parametric uncertainties and external disturbances, which has been widely used to achieve motor control [13-16]. But this scheme has inherent chattering phenomenon. The reason for this is a sign function is used in the traditional sliding-mode control. Combine sliding-mode theory and fuzzy neural network technology, Lin [14] has proposed intelligent complementary sliding-mode control for PMSLM, but it is very difficult to realize. In this paper, sigmoid function is introduced to substitute the sign function. Moreover, adaptive approach is employed to estimate the important uncertain parameters in the system to further improve its performance.

## 2. Controller design for PMSLM

In this section, it is given the dynamic model of the PMSLM drive system and at the same time the controller is designed in detail. Then, The stability of the proposed control scheme is verified via Lyapunov stable theory.

### 2.1. Dynamic model of PMSM

The mechanical motion equation of surface-mounted PMSLM can be expressed as follows:

$$M_n \ddot{x} + F_f + F_r + F_l = F_e, \tag{1}$$

where  $x$  denotes the mover position.  $M_n$  denotes the mover mass.  $F_e$ ,  $F_l$ ,  $F_r$  and  $F_f$  denote the electromagnetic thrust, external disturbance, cogging force and nonlinear dynamic friction, respectively. Following [17], we assume that friction can be modeled by the dynamic LuGre model as

$$F_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x}, \quad \dot{z} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z \tag{2}$$

where  $z$  denotes the friction state that physically stands for the average deflection of the bristles between the two contact surfaces. The friction force parameters  $\sigma_0$ ,  $\sigma_1$  and  $\sigma_2$  can be physically explained as the stiffness of bristles, damping coefficient, and viscous coefficient, respectively. The function  $g(\dot{x})$ , which characterizes the Stribeck effect can be described as

$$g(\dot{x}) = F_c + (F_s - F_c) e^{-|\dot{x}/v_s|}, \tag{3}$$

where  $v_s$  denotes the Stribeck velocity.  $F_c$ , and  $F_s$  denote the levels of the normalized Coulomb friction and stiction force respectively.

The cogging force  $F_r$  is represented by a single dominant spatial frequency  $\Omega$  sinusoidal function with phase shift  $\phi$  [8]

$$F_r = C \sin(\Omega x + \phi) = \bar{C}_1 \cos(\Omega x) + \bar{C}_2 \cos(\Omega x + \phi), \tag{4}$$

Substitute (2) into (1), we obtain that

$$M_n \ddot{x} = F_e - \sigma \dot{x} + \Delta, \tag{5}$$

where  $\Delta = \sigma_1 \frac{|\dot{x}|}{g(\dot{x})} z - \sigma_0 z - F_r - F_l$  denotes the uncertainty including matched and unmatched disturbances. In this paper, we assume that the uncertainty  $\Delta$  and the uncertain parameters  $m$  and  $\sigma = \sigma_1 + \sigma_2$  are bounded, that is, they meet the following assumptions.

Assumption 1: The upper bound of the uncertain parameters  $m$  and  $\sigma$  are defined as:

$$m \in \Sigma_1 \xrightarrow{\text{def}} \{m : 0 \leq m \leq m_{\max}\}, \quad \sigma \in \Sigma_2 \xrightarrow{\text{def}} \{\sigma : 0 \leq \sigma \leq \sigma_{\max}\} \tag{6}$$

Assumption 2: The upper bound of the uncertainty  $m$  meets:

$$|\Delta| \leq d. \tag{7}$$

## 2.2. Controller design

The control objective is to design a control law to force the mover position  $x$  to track its desired position value  $x_{1d}$ . The difficulties in designing a high-performance controller for the aforementioned PMSLM servo system are to overcome the unknown parameters and uncertain disturbances. The following will give the detail designing process of the proposed adaptive sliding controller.

In order to design the controller, the model of the system (5) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ m\dot{x}_2 = F_e - \sigma x_2 + \Delta \end{cases} \tag{8}$$

Denote the position tracking error as

$$e_1 = x_1 - x_{1d}. \tag{9}$$

Select the nonlinear sliding mode surface as

$$\begin{cases} s = \dot{e}_1 + k_p e_1 + k_I \chi = x_2 - q \\ q = \dot{x}_{1d} - k_p e_1 - k_I \chi \end{cases} \tag{10}$$

where  $\chi = \int_0^t e_1(\tau) d\tau$ ,  $k_p > 0$  and  $k_I > 0$  are the integral term, proportional gain and integral gain, respectively.

**Theorem.** Consider the uncertain dynamic system (8). If the controller is designed as

$$\begin{cases} F_e = f_a + f_s \\ f_a = \hat{m}\dot{q} + \hat{\sigma}x_2 \\ f_s = -ks - \varepsilon \tanh(s) \end{cases} \tag{11}$$

Then, the closed-loop system's stability is achieved with the following adaptive law:

$$\begin{cases} \dot{\hat{m}} = -\gamma_1 \dot{q}s \\ \dot{\hat{\sigma}} = -\gamma_2 x_2 s \end{cases} \tag{12}$$

where  $f_a$  and  $f_s$  are the adaptive term and robust sliding-mode term, respectively.  $\hat{m}$  and  $\hat{\sigma}$  are the estimation value of  $m$  and  $\sigma$ .  $\gamma_1 > 0$  and  $\gamma_2 > 0$  are the adaptive gains.  $k > 0$  and  $\varepsilon \geq d$  are the sliding-mode gains.

It should be noted that, if  $\varepsilon$  is too small, the reaching time will be too long; on the other hand, too large a  $\varepsilon$  will cause severe chattering and robust sliding-mode term proportional gain and integral gain, respectively. In this paper, adaptive term  $u_a$  is added to compensate some uncertain term, which can avoid the  $\varepsilon$  selected to be large. Moreover, Tangent function is introduced to substitute the sign function to further reduce the chattering.

**Proof.** If the candidate Lyapunov function is defined as  $V = \frac{1}{2}ms^2 + \frac{1}{2\gamma_1}\tilde{m}^2 + \frac{1}{2\gamma_2}\tilde{\sigma}^2$ , and  $\tilde{m} = \hat{m} - m$  and  $\tilde{\sigma} = \hat{\sigma} - \sigma$  are the estimation errors. Then the time derivative of  $V$  along the trajectory of (8) is

$$\begin{aligned} \dot{V} &= sm\dot{s} + \frac{1}{\gamma_1}\tilde{m}\dot{\hat{m}} + \frac{1}{\gamma_2}\tilde{\sigma}\dot{\hat{\sigma}} \\ &= s(m\dot{x}_2 - m\dot{q}) + \frac{1}{\gamma_1}\tilde{m}\dot{\hat{m}} + \frac{1}{\gamma_2}\tilde{\sigma}\dot{\hat{\sigma}} \\ &= s(F_e - \sigma x_2 + \Delta - m\dot{q}) + \frac{1}{\gamma_1}\tilde{m}\dot{\hat{m}} + \frac{1}{\gamma_2}\tilde{\sigma}\dot{\hat{\sigma}} \\ &= s(\hat{m}\dot{q} + \hat{\sigma}x_2 - ks - \varepsilon \tanh(s) - \sigma x_2 + \Delta - m\dot{q}) + \frac{1}{\gamma_1}\tilde{m}\dot{\hat{m}} + \frac{1}{\gamma_2}\tilde{\sigma}\dot{\hat{\sigma}} \\ &= -ks^2 - \varepsilon|s| + \Delta s + \tilde{m}(s\dot{q} + \frac{1}{\gamma_1}\dot{\hat{m}}) + \tilde{\sigma}(sx_2 + \frac{1}{\gamma_2}\dot{\hat{\sigma}}) \end{aligned}$$

Note the assumption (7) and the adaptive laws (12), we can get

$$\dot{V} = -ks^2 - \varepsilon|s| - \Delta s \leq -ks^2 - (\varepsilon - \Delta)|s| \leq -ks^2 \leq 0$$

So, the closed-loop system is stable under the presented control law and parameter adaptive update law.

In order to avoid  $\hat{m}$  and  $\hat{\sigma}$  which are too big, we make the control input  $F_e$  too big, we rewrite the adaptive update law (12) by using the discontinuous projection mapping proposed in [18] as

$$\begin{cases} \dot{\hat{m}} = Proj_{\hat{m}}(-\gamma_1 \dot{q}s) \\ \dot{\hat{\sigma}} = Proj_{\hat{\sigma}}(-\gamma_2 x_2 s) \end{cases} \quad (13)$$

Where

$$Proj_{\beta}(\square) = \begin{cases} 0, & \text{if } \beta = \beta_{\max} \text{ and } \square > 0 \\ 0, & \text{if } \beta = \beta_{\min} \text{ and } \square < 0 \\ \square & \text{otherwise} \end{cases} \quad (14)$$

### 3. Simulation results

In this section, to verify the feasibility of the suggested method and compare it with other schemes, numeric simulations of different methods, including PID control, sign function based adaptive sliding-mode control (ASMC) and modified sigmoid function based adaptive sliding-mode control (MASMC), are conducted on a PMSLM drive system described in Section 2.1. The simulation is carried out using the Matlab package. In the simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size of 0.001. The main parameters of PMSLM are described in Table 1 as seen in reference [18]. The model parameters of the friction and ripple forces used in this paper are given as

$$F_c = 10N, F_s = 20N, v_s = 0.1m/s, \sigma_0 = 12Nm/rad, \sigma_1 = 0.1Nm/rad, \sigma_2 = 13.2Nm/rad$$

Table 1. Parameters of PMLSM

Stator resistance $R_s$	2.6 $\Omega$
d-axis inductance $L_d$	30.4e-3 H
q-axis inductance $L_q$	30.4e-3 H
Magnet flux linkage $\psi_f$	0.1728 Wb
Number of pole pairs P	2
mover mass $m$	7.3 kg

The spatial cogging frequency <sup>[5]</sup> is assumed to be  $\Omega = 314 \text{ rad / s}$ , and  $C = 8.5 \text{ N}$ ,  $\phi = 0.005 \pi$ . The desired position tracking signal in the simulations is chosen as

$$x_{1d} = 0.25 \sin(2\pi \frac{t}{T_s} - \frac{\pi}{2}) + 0.25 \text{ m},$$

where  $T_s = 4$  is the sample time and the corresponding speed is

$$x_2 = v_d = 0.5\pi \frac{1}{T_s} \cos(2\pi \frac{t}{T_s} - \frac{\pi}{2}) \text{ m / s},$$

The adaptive updating rates and control gains of the presented controller are selected as follows:

$$k_p = 8.5, k_l = 0.01, \varepsilon = 1000, k = 500, \gamma_1 = \gamma_2 = 500$$

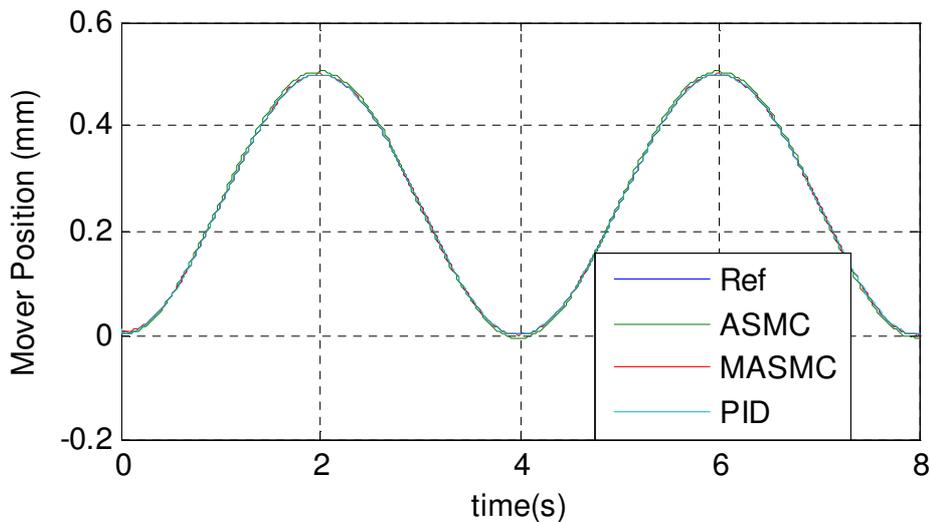


Fig. 1 The position tracking response curve

All the control gains and update rates in the proposed presented control system are chosen to achieve the best transient control performance in the simulations considering the requirement of stability. If the control gains and updating rates are chosen too small, the convergence of the tracking error is slow; on the other hand, too large control gains and updating rates is chosen, unstable tracking response may be resulted due to the saturation of actuator. Simulation results are shown in Fig. 1~7.

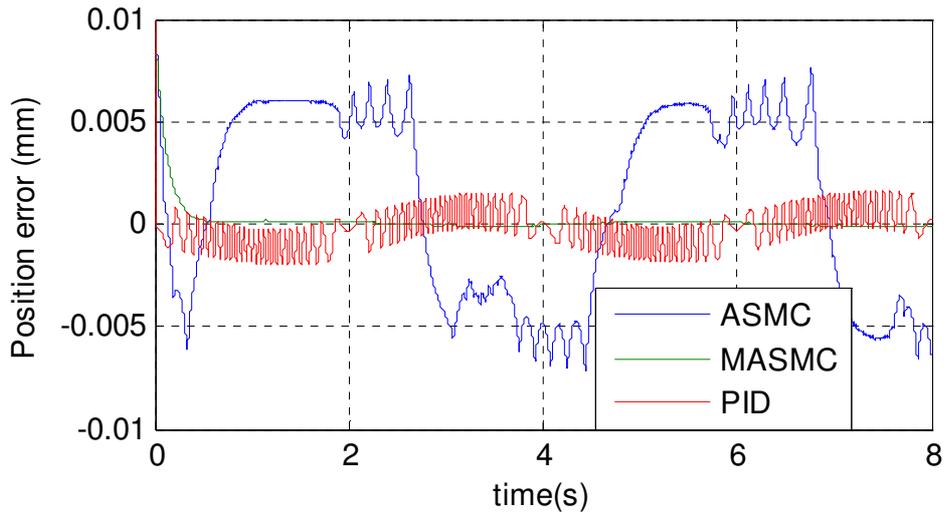


Fig. 2 The position tracking error curve

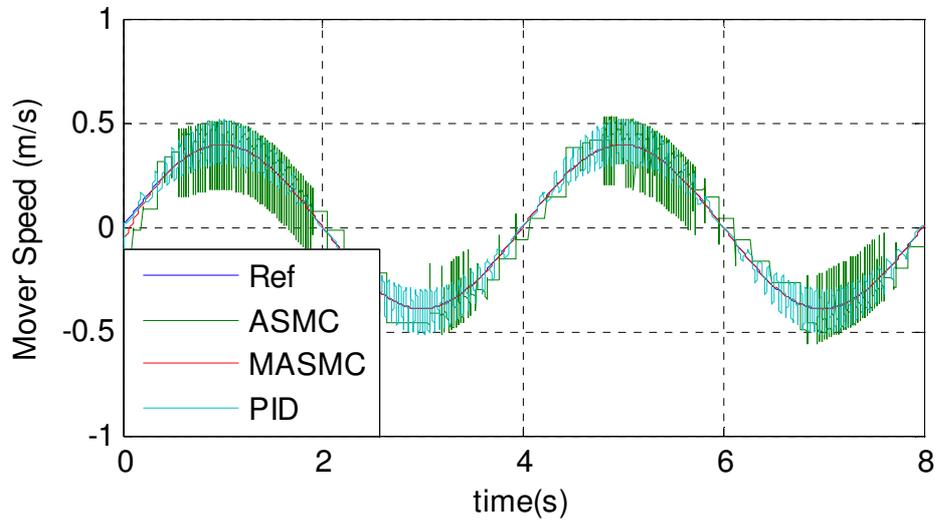


Fig. 3 The speed response curve

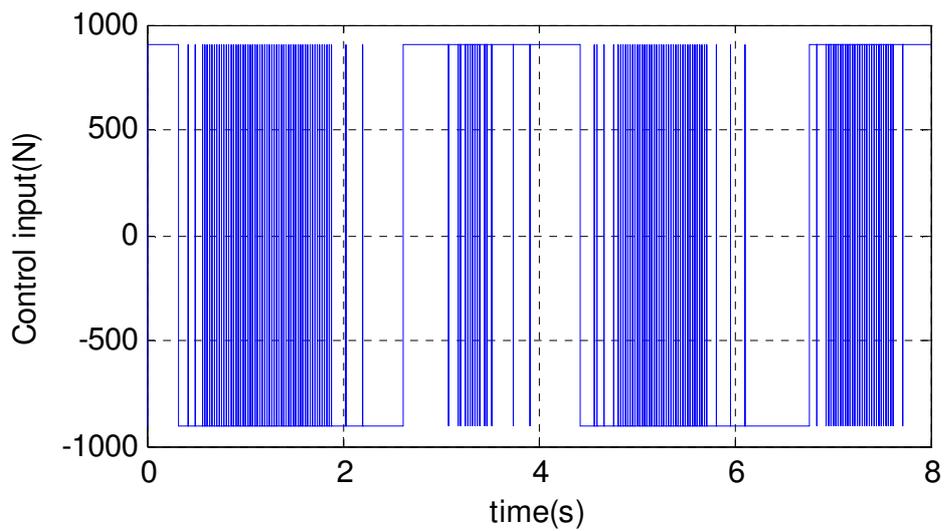


Fig. 4 The control input of the adaptive sliding-mode control with sign function

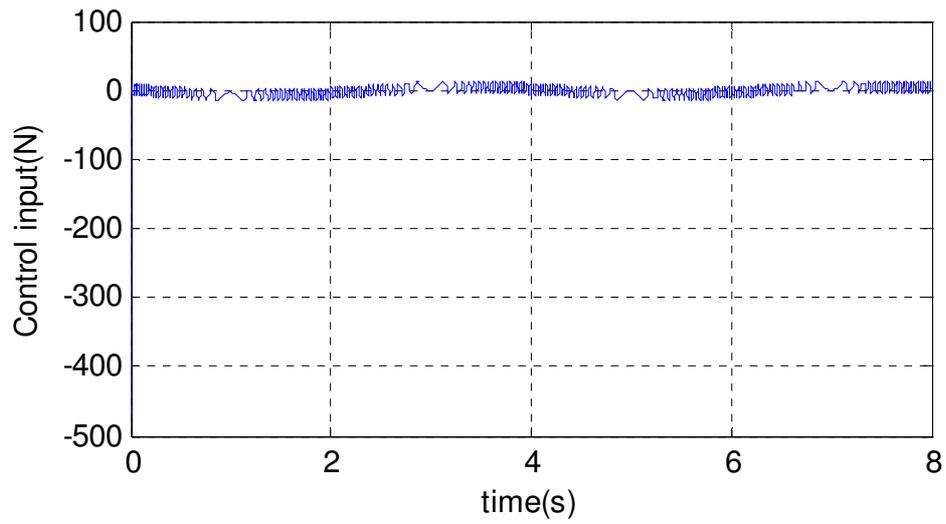


Fig. 5 The control input of the adaptive sliding-mode control with sigmoid function

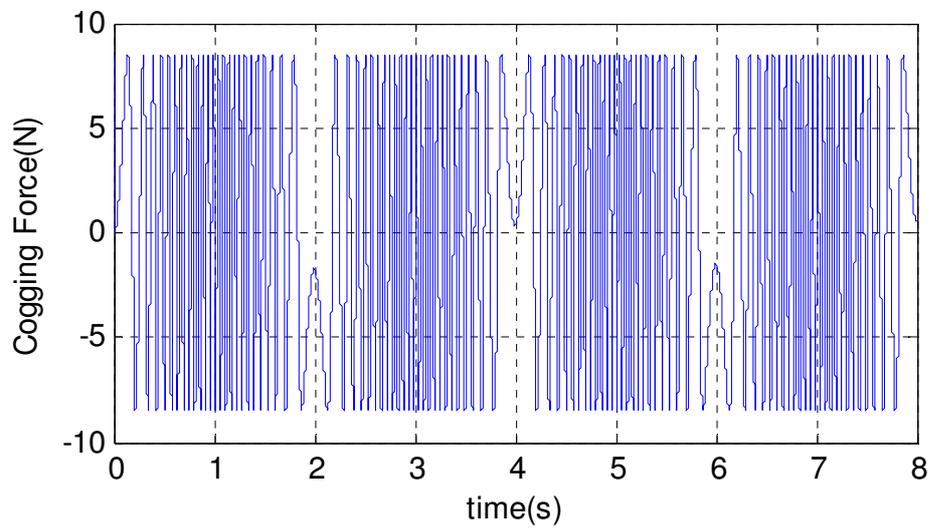


Fig. 6 The cogging force curve

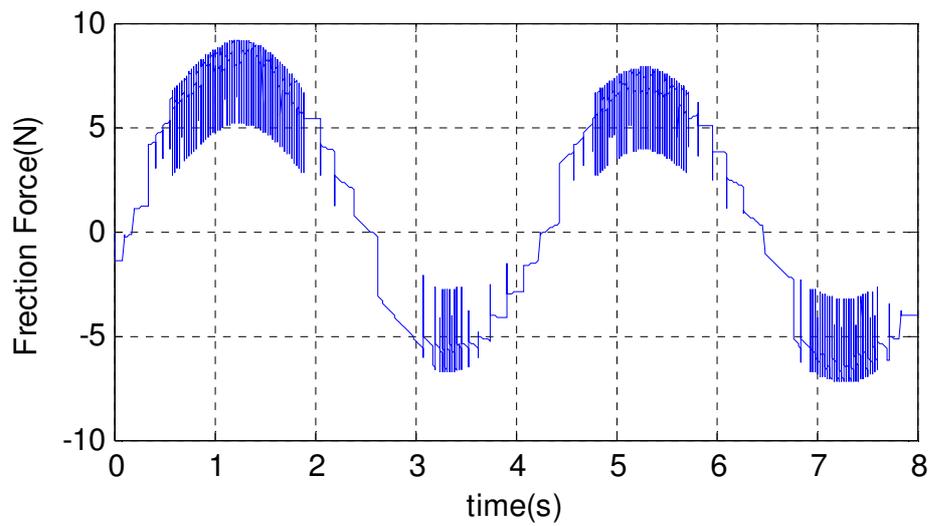


Fig. 7 The friction force curve

It can be seen from Fig. 1-3 that the proposed method has smaller position and speed tracking error than those of PID control and ASMC. And the proposed method can achieve the desired reference speed and position as soon as possible, which has better transient and steady performance than other two methods.

Fig. 4 and Fig. 5 show that the proposed method can effectively reduce the chattering phenomenon caused by traditional sliding-mode control.

Fig. 6 and Fig. 7 show the cogging force and friction, respectively. We can see that the cogging force and friction present a complex nonlinear behavior.

#### 4. Conclusion

In this paper, a novel nonlinear position control scheme is proposed for PMSLM servo system in the presence of system uncertainties. This method combines sliding-mode theory and adaptive approach to achieve the high precision position control of PMSLM. Based on Lyapunov theory, the stability of the presented closed control system is verified. Simulation results illustrate that the proposed controller exhibits better response performance and smaller tracking error than traditional methods. Future research should investigate the implementation of the proposed control scheme using an experimental setup.

#### Acknowledgment

This work was supported by the National Science and Technology Major Project of the Ministry of Science and Technology of China (Project No. 2009ZX04001).

#### References

- [1] X. Li, R. Du, B. Denkena and J. Imiela, Tool breakage monitoring using motor current signals for machine tools with linear motors, *IEEE Trans. Ind. Electron.*, 52 (5), 1403-1408, 2005.
- [2] J. F. Hoburg, Modeling maglev passenger compartment static magnetic fields from linear Halbach permanent-magnet arrays, *IEEE Trans. Magn.*, 40 (1), 59-64, 2004.
- [3] C. Hu, B. Yao and Q. Wang, Coordinated adaptive robust contouring control of an industrial biaxial precision gantry with cogging force compensations, *IEEE Trans. Ind. Electron.*, 57 (5), 728-735, 2010.
- [4] B. Yao, C. Hu and Q. Wang, An orthogonal global task coordinate frame for contouring control of biaxial systems, *IEEE/ASME Trans. Mechatronics.*, 17 (4), 622-634, 2012.
- [5] K. K. Tan, S. N. Huang and T. H. Lee, Robust adaptive numerical compensation for friction and force ripple in permanent magnet linear motors, *IEEE Trans. Magn.*, 38 (1), 221-228, 2002.
- [6] Y. Hong and B. Yao, A globally stable high-performance adaptive robust control algorithm with input saturation for precision motion control of linear motor drive systems, *IEEE Trans. Mechatronics*, 12 (2), 198-207, 2007.
- [7] F. J. Lin, L. T. Teng and H. Chu, A robust recurrent wavelet neural network controller with improved particle swarm optimization for linear synchronous motor drive, *IEEE Trans. Power. Electron.*, 23 (6), 3067-3077, 2008.
- [8] S. L. Chen, K. K. Tan, S. Huang and C. S. Teo, Modeling and compensation of ripples and friction in permanent-magnet linear motor using a hysteresis relay, *IEEE Trans. Mechatronics*, 15 (4), 586-594, 2010.
- [9] C.-I. Huang and L.-C. Fu, Adaptive approach to motion controller of linear induction motor with friction compensation, *IEEE/ASME Trans. Mechatronics.*, 12 (4), 481-490, 2007.
- [10] L. Lu, B. Yao, Q. Wang and Z. Chen, Adaptive robust control of linear motor with dynamic friction compensation using modified LuGre model, *Automatica.*, 45, 2890-2896, 2009.
- [11] L. Freidovich, A. Robertsson, A. Shiriaev and R. Johansson, LuGre-model-based friction compensation, *IEEE Trans. Control Syst. Techn.*, 18 (1), 194-200, 2010.
- [12] H. Xiang, W. Tan, X. Li and C. Zhang, Adaptive friction compensation based on LuGre model, *Journal of Mechanical Engineering*, 48 (17), 70-74, 2012.

- [13] F. Cupertino, D. Naso, E. Mininno and B. Turchiano, Sliding-mode control with double boundary layer for robust compensation of payload mass and friction in linear motors, *IEEE Trans. Ind. Appl.*, 45 (5), 1688-1696, 2009.
- [14] F. J. Lin, P. H. Chou, C. S. Chen and Y. S. Lin, DSP-Based cross-coupled synchronous control for dual linear motors via intelligent complementary sliding model control, *IEEE Trans. Ind. Electron.*, 59 (2), 1061-1073, 2012.
- [15] M. L. Corradini, G. Ippoliti, S. Longhi and G. Orlando, A quasi-sliding mode approach for robust control and speed estimation of PM synchronous motors, *IEEE Trans. Ind. Electron.*, 59 (2), 1096-1104, 2012.
- [16] N. T.-T. Vu, D.-Y. Yu, H. H. Choi and J.-W. Jung, T-S fuzzy-model-based sliding-mode control for surface-mounted permanent-magnet synchronous motors considering uncertainties, *IEEE Trans. Ind. Electron.*, 60 (10), 4281-4291, 2013.
- [17] C. C. D. Wit, H. Olsson, K. Astrom and P. Lischinsky, A new model for control of systems with friction, *IEEE Trans. Autom. Control*, 40 (3), 419-425, 1995.
- [18] C. Tang and Y. Dai, Adaptive robust control of permanent magnet synchronous linear, *Modular Machine Tool & Automatic Manufacturing Technique*, 11, 64-67, 2013.