Ahmed Elsheikh[1], Yahya Helmy[1], Yasmine Abouelseoud[1,*], Ahmed Elsherif[1]

J. Electrical Systems 10-1 (2014): 63-77

Regular paper

Journal of Electrical Systems

Optimal Power Flow and Reactive Compensation Using a Particle Swarm Optimization Algorithm

This paper addresses two important optimization problems in power systems planning; namely, the optimal power flow (OPF) problem and the optimal capacitor placement and sizing (OCPS) problem. The aim of studying the OPF problem is to determine the optimal setting of the generators' active powers and voltages such that the total power generation cost is minimized, taking into account the system security constraints. Particle Swarm Optimization (PSO) has been selected as an optimization tool. The contribution of this paper in this respect lies in the formulation of the objective function in such a way that that yielded results superior to those formerly reported in literature. As for the OCPS problem, its goal is to determine the optimal locations and sizes of capacitors such that the total cost of the power losses and the capacitors is minimized, while the voltage profile is improved. The loss sensitivity factors are used to select the candidate buses for capacitor placement and then the discrete PSO algorithm is used to determine the optimal setting of the capacitors. This enabled improving the results reported earlier in literature, where the capacitor sizes were treated as continuous variables and then rounded.

Keywords: Optimal power flow (OPF), electric power system security, particle swarm optimization (PSO), capacitor placement, loss sensitivity factors.

1. Introduction

The OPF problem objective is to meet the required demand at minimum production cost, satisfying units' and system's operating constraints, by adjusting the power system control variables. The power system must be capable of withstanding the loss of some or several transmission lines, transformers or generators, guaranteeing its security; such events are often termed probable or credible contingencies [1,2].

In its most general formulation, the OPF problem is a nonlinear, non-convex, large-scale, static optimization problem with both continuous and discrete control variables. Even in the absence of non-convex unit operating cost functions, unit prohibited operating zones, and discrete control variables, the OPF problem is non-convex due to the existence of the nonlinear (AC) power flow equality constraints. The presence of discrete control variables, such as switchable shunt devices, transformer tap positions, and phase shifters, further complicates the problem solution. Several techniques have been used for solving the OPF problem, for instance: Sequential Linear Programming, Newton Raphson, Sequential Quadratic Programming and the Interior point method. Classical methods suffer from three main problems. Firstly, they rely on convexity to find the global optimum. However, due to the nonlinear and non-convex nature of the OPF problem, the methods based on these assumptions are not guaranteed to find the global optimum. Secondly, all these methods involve complex calculations. These calculations take a long time. Finally, all these methods cannot be applied with discrete variables, which are transformer taps and shunt capacitors. Therefore, many other powerful deterministic, probabilistic and stochastic

Manuscript received September 27, 2013; revised January 21, 2014; accepted January 29, 2014.
* Corresponding author: Yasmine Abouelseoud of Engineering mathematics department, Faculty of Engineering Alexandria university, Alexandria, Egypt, E-mail address: yasmine.abouelseoud@gmail.com
[1] Faculty of Engineering, University of Alexandria, Alexandria, Egypt

Copyright © JES 2014 on-line : journal/esrgroups.org/jes

techniques are used for solving these high dimensional optimization problems. The meta-heuristic optimization techniques such as Genetic algorithms (GA), Particle Swarm Optimization (PSO), Artificial Bees Colony (ABC), Fuzzy Logic (FL), Simulated Annealing (SA) and Ant Colony Optimization (ACO) are examples of these techniques [1, 3]. Pandya and Hoshi in [6] presented a comprehensive survey of various optimization methods used to solve OPF problems and suggested a methodology for selecting suitable methods for solving the different variations of the basic OPF problem.

The use of capacitors in power systems has many well-known benefits that include improvement of the system power factor, improvement of the system voltage profile, decreasing the flow through cables and transformers and reduction of losses due to the compensation of the reactive component of power flow. These benefits depend greatly on how capacitors are placed in the system. The general capacitor placement problem is how to optimally determine the locations to install capacitors and the sizes of capacitors to be installed in the busses of distribution systems. Many researchers focused on optimal capacitor placement problem in balanced distribution feeders [4, 5]. Because capacitor sizes and locations for allocation are discrete variables, this makes the capacitor placement problem have a combinatorial nature. The problem is a zero-one decision making problem with discrete steps of standard capacitor bank sizes. In some cases, each step of the capacitor bank size has a different installation cost. Such zero-one decisions and discrete steps make the capacitor placement problem be a nonlinear and non-differentiable, mixed integer programming problem [7].

N. Rugthaicharoencheep and S. Auchariyamet in [7] introduced an overview of capacitor placement problem in distribution systems. The objective functions and constraints of the problem were listed and the methodologies for solving the problem were summarized. They stated that the objective function might be minimizing the total energy losses in a day or minimizing the cost of energy losses or minimizing the sum of energy loss cost and the investment cost required for capacitors allocation or minimizing the total cost due to energy losses, peak losses and the investment cost of capacitors. The optimum solution should satisfy the power flow equations, bus voltage limits (to maintain the voltage profile within the upper and lower limits), and the limitation of the KVAR to be installed at each bus (because of limited space for placement). They reviewed the solution techniques that had been applied to the capacitor placement problem such as fuzzy logic, tabu search, simulated annealing, genetic algorithms, particle swarm optimization and ant colony optimization.

In this paper, the OPF is firstly solved for a given interconnected electric power system to determine the optimal setting of the generators' active powers and voltages such that the total generating cost is minimum, taking into account the system security constraints. Then, the loss sensitivity factors are used to determine the candidate buses for capacitor placement. Finally, the discrete PSO algorithm is used to determine the optimal sizes of capacitors such that the cost of power losses and the investment in capacitor placement is minimal.

2. Related work

2.1. Solution of the OPF Problem

The optimal power flow (OPF) problem was first introduced by Carpentier in 1962. He introduced a generalized, continuous nonlinear programming formulation of the economic dispatch problem including voltage and other operating constraints [8].

Newton Raphson method is widely used to solve power flow problem. Dommel and Tinney developed in [9] a practical method based on power flow solution by Newton's method for solving the OPF problem with control variables such as voltage magnitudes on generator buses (PV buses), transformer tap ratios and real generated powers (P_G) available for economic dispatch to minimize costs or losses. They used the method of steepest descent. The basic idea is to move from one feasible solution point in the direction of steepest descent (negative gradient) to a new feasible solution point with a lower value for the objective function. By repeating these moves in the direction of the negative gradient, the minimum will be reached. Alsac and Stott in [10] extended Dommel-Tinney's approach by incorporating the steady state security constraints into the OPF solution based on an exact mathematical formulation of the problem. This enables reactive power and voltage constraints to be considered in the outage cases. The inclusion of steady state security constraints makes the OPF solution a more powerful and practical tool for system operation and design.

Sumpavakup et al. in [11] used the ABC algorithm, which is a new population based meta-heuristic approach inspired by intelligent foraging behavior of honeybee swarm. The advantage of ABC algorithm is that it does not require external parameters such as cross over rate and mutation rate as in case of genetic algorithm and differential evolution and it is hard to determine these parameters in prior. The other advantage is the global search ability in the algorithm. The proposed method was tested on the IEEE-14 and the IEEE-30 bus systems. The results obtained by the ABC algorithm were compared with other swarm intelligence techniques such as GA and PSO. The results obtained showed that the ABC algorithm could converge towards better solutions slightly faster than GA and PSO methods.

Madhu Gargin [12] presented a solution technique for the OPF problem of large transmission systems via a simple GA. The main objective was to minimize the fuel cost and keep the power outputs of generators; bus voltages, shunt capacitors/reactors and transformer tap setting within their secure limits. The OPF problem with reactive power compensation was also investigated in his work. The compensating device investigated was a shunt capacitor. Shunt capacitors were installed to provide capacitive reactive compensation and power factor correction. Shunt capacitors were used to improve the quality of the electrical supply and to ensure the efficient operation of the power system. Also, the flat voltage profile on the system can significantly reduce line losses. Shunt capacitors are relatively inexpensive and can be easily installed anywhere on the network between any one of the buses and the ground. The proposed approach was tested on the IEEE 30-bus system.

M. A. Abido in [13] proposed a novel PSO based approach to solve the OPF problem. The problem was formulated as an optimization problem with mild constraints. Different objective functions were considered to minimize the fuel cost, to improve the voltage profile, and to enhance power system voltage stability. The dependent variables were the slack bus power, load bus voltages, generator reactive power outputs and transmission line loadings. The independent or control variables were generator voltages, generator real power outputs except at the slack bus, transformer tap settings and shunt VAR compensations. The control variables were self constrained. The hard inequalities of the dependent variables were incorporated in the objective function as quadratic penalty terms. The search was terminated if one of the following criteria was satisfied. The number of iterations reached the maximum allowable number or the last change in the best solution was less than a pre-specified number. An annealing procedure was incorporated in order to make a uniform search in the initial stages and very local search in the later stages. A decrement function for decreasing the inertial weight given as w(t)=α w(t-1), where α was a decrement constant smaller than but close to 1, was proposed in that study. A feasibility check procedure for the particles positions was imposed after the position updating to prevent the particles from flying outside the feasible search space. The proposed approach was tested on the IEEE-30 bus system.

Pablo Oñate et al., in [1] and [2], presented a novel approach to solve the OPF problem with embedded security constraints represented by a mixture of continuous and discrete control variables. The objective function was to minimize the total operating cost taking into account both operating security constraints and system capacity requirements. The total cost included the pre-contingency cost plus the cost of each contingency. The cost functions included two terms: one related to the generating costs, and a second one associated with a consumer benefit. The PSO algorithm with reconstruction operators was used as the optimization tool. The reconstruction operators allowed that all particles representing possible solutions are only within the feasible region reducing the computational time and improving the quality of the achieved solution. Any particle was composed of continuous and discrete control variables. The continuous ones included the generators' active power output, and the discrete variables included the transformers tap setting and the VAR-injection values of the switched shunt capacitors. The proposed approach was tested on two systems: a 39-bus system (New England power system) and a 26-bus system. They represented the valve-point loading effects with a sinusoid component and added it to the quadratic active power generation cost function.

2.2. Solution of the OCPS Problem

Chung and Shaoyun in [14] presented a recursive linear programming based approach for minimizing line losses and finding the optimal capacitor allocation in a distribution system. The approach included two steps. First, the optimal capacitor placement was formulated as an LP problem and treated capacitor sizes as continuous variables. Second, an enumeration-based method was used to solve for the sizes of capacitors by taking practical aspects of capacitors. The load effect and operational constraints were considered. This approach did not require any matrix inversion, thus saved computational time and memory space. The economic justification of capacitor installation was also discussed. They presented a computer program that could be used as a tool to assist power system planning engineers in selecting the location and size of the capacitors in order to optimize the system losses. The approach was tested on two systems; LEKR0004 which is a 6-bus system, and YCHR0007 which is a 14-bus system. From the results of these two examples, it was shown that there would be a significant change in the system loss after capacitor installation.

K. Prakash and M. Sydulu in [14] developed a novel approach that determines the optimal location and size of capacitors on radial distribution systems to improve the voltage profile and reduce the active power loss. Capacitor placement and sizing were solved for using loss sensitivity factors and particle swarm optimization, respectively. The concept of loss sensitivity factors was considered as the contribution of this work in the area of distribution systems. Loss sensitivity factors determined the candidate nodes for the placement of capacitors. The estimation of these candidate nodes basically helps in reduction of the search space for the optimization procedure. These factors were determined using a base case load flow. Particle swarm optimization was applied and found to be very effective in radial distribution systems. PSO was used for estimation of the required level of shunt capacitive compensation to improve the voltage profile of the system. The proposed method was tested on 10, 15, 34, 69 and 85 bus distribution systems. The main advantage of that proposed method was that it systematically decided the locations and size of capacitors to realize the optimum reduction in active power loss and significant improvement in voltage profile. The method placed capacitors at a fewer number of locations with optimum sizes and offered much saving in initial investment and regular maintenance. The disadvantage of that proposed algorithm was that the capacitor sizes were considered as continuous variables, then the capacitor sizes were rounded off to the nearest available capacitor value.

R. Srinivasa Rao in [16] presented a new method which applied an artificial bee colony algorithm (ABC) for capacitor placement in distribution systems with an objective of improving the voltage profile and reduction of power loss. To demonstrate the validity of the proposed algorithm, computer simulations were carried out on a 69-bus system and the results were compared to the other approaches available in the literature. The proposed method was superior to the other methods in terms of the quality of solution and the computational efficiency. The objective of the capacitor placement problem in the distribution system was to minimize the annual cost, subjected to certain operating constraints and load pattern. For simplicity, the operating and maintenance cost of the capacitors placed in the distribution system was not taken into consideration. The three-phase system was considered as balanced and loads were assumed to be time invariant. The considered cost included the cost of power loss and the capacitor placement. The voltage constraint was added to the objective function as a penalty term. If the voltage constraint is not violated, then the penalty term was multiplied by zero, and if the constraint is violated, a significant penalty is imposed to cause the objective function to move away from the infeasible solution. The bus reactive compensation power was limited to be less than or equal to the total reactive power of the system. The loss sensitivity factors were used to determine the candidate buses for compensation. Top three nodes were selected as candidate locations to reduce the search space and then the amount to be injected in the selected nodes was optimized by the ABC algorithm**.**

3. OPF problem formulation

In this section, the optimal power flow mathematical formulation is presented. The objective function is to minimize of fuel cost (generating cost).

$$\sum_{i=1}^{N_G}(a_i P_{Gi}^2 + b_i P_{Gi} + c_i)\ \$/hr \qquad (1)$$

The valve point loading effect can be added to the quadratic curve of the fuel cost as a sinusoid component

$$\sum_{i=1}^{N_G}\left(a_i P_{Gi}^2 + b_i P_{Gi} + c_i + \left|d_i \sin\left(e_i (P_{Gi\,min} - P_{Gi})\right)\right|\right)\ \$/hr \quad (2)$$

where a_i, b_i, c_i, d_i and e_i are generator cost curve coefficient. N_G is the total number of generators. P_{Gi} is the active power of the ith generator.

The control variables include the active powers (P_G) of all generators except one generator called the slack generator and the voltages (V_G) of all generators including slack generator voltage. These variables are continuous control variables. The discrete control variables include the VAR-injection values of the switched shunt capacitors (Q_C) and the transformers' tap settings (Tap).

The constraints include the "Power balance" constraints given as

$$\sum_{i=1}^{N_G} P_{G_i} = \sum_{j=1}^{N_{load}} P_{load_i} + P_{loss}\ (3)$$

$$\sum_{i=1}^{N_G} Q_{G_i} = \sum_{j=1}^{N_{load}} Q_{load_i} + Q_{loss} \quad (4)$$

where N_{load} is the total number of loads, P_{loss} and Q_{loss} are the total active and reactive power losses respectively, and Q_{G_i} is the reactive power of the i[th] generator.

The remaining constraints deal with imposing limits on generators active and reactive powers, bus bar voltages, line flow, shunt capacitors and transformers' taps as given in equations (5)-(10), respectively.

$$P_{Gmin} < P_{Gi} < P_{Gmax} \quad (5)$$
$$Q_{Gmin} < Q_{Gi} < Q_{Gmax} \quad (6)$$
$$V_{min} < V_i < V_{max} \quad (7)$$
$$Flow_i < Flow_{i,max} \quad (8)$$
$$Q_{C_{min}} < Q_C < Q_{C_{max}} \quad (9)$$
$$Tap_{min} < Tap_i < Tap_{max} \quad (10)$$

4. OCAS Problem Formulation

The mathematical formulation of the optimal capacitor allocation and sizing is presented in what follows. The objective function is to minimize the total annual cost due to capacitor placement and power losses with constraints that include limits on voltage and size of installed capacitors

$$F = K^p P_{loss} + \sum_{j=1}^{J} K_j^c Q_j^c \quad (11)$$

where F is the total annual cost function, P_{loss} is the total power losses, K^p is the annual cost per unit of power losses ($/KW), K_j^c is the capacitor annual cost ($/KVAR), Q_j^c is the shunt capacitor size placed at bus j and J is the number of candidate buses for capacitor placement.

The control variables are the shunt capacitors (Q_C), which are discrete variables. The constraints on shunt capacitors, bus bar voltages, and line flow are given in equations (12)-(14), respectively.

$$Q_C^{max} \leq Q_{total} \quad (12)$$

(where Q_C^{max} is the largest capacitor size and Q_{total} is the total reactive load)

$$V_{min} < V_i < V_{max} \quad (13)$$
$$Flow_k < Flow_{k,max} \quad (14)$$

where $Flow_k$ is the power flow in kth-line and $Flow_{k,max}$ is the maximum allowable power flow.

5. The Proposed Algorithm

In this section, the proposed algorithm is presented. It proceeds in two phases. In the first phase, the OPF problem is solved for a given interconnected electric power system to determine the optimal setting of the generators' active powers and voltages such that the total power generation cost is minimized, taking into account the system security constraints. In the second phase, the loss sensitivity factors are used to determine the candidate buses for optimal capacitor placement. Finally, the discrete PSO algorithm is used to determine the optimal sizes of capacitors such that the cost of power losses and the investment of capacitor placement is kept minimal. In what follows, the steps of the proposed algorithm are provided.

5.1 The First Phase

In this phase, the OPF is solved using a PSO-based technique.

Step 1: In this step, all the input data are specified and fed to the algorithm. These data include:

a) The number of buses.
b) The load demand (active (watt) and reactive (VAR) power) at each bus.
c) Bus voltage limits (V_{min}, V_{max}).
d) Transmission lines' impedances (resistance and reactance).
e) Transmission lines' capacity (Maximum allowable power flow).
f) Generator data (location; i.e. connected to which bus, P_{Gmin}, P_{Gmax}, Q_{Gmin}, Q_{Gmax}, cost coefficients).
g) Particle swarm optimization algorithm parameters (number of particles NIND, initial velocity vector, maximum number of iterations, acceleration constants C_1 and C_2, initial inertial weight).

Step 2: Determine the slack generator which has the maximum ratings (i.e. the one with the greatest $P_{G\,max}$)

Step 3: Randomly initialize the positions of particles between the lower and the upper limits of the corresponding control variable. The position vector (or encoded solution vector) of the i^{th} particle is given as:

X_i = [P_G (all generators except slack), V_G (for the slack generator), V_G (for other generators)] (15)

Step 4: Initialize the velocity of particles. The initial velocity for the active power is selected randomly in the range of $0.25*(P_{Gmax} - P_{Gmin})*[-1, 1]$ and the initial velocity for the voltage is selected randomly in the range of $0.5*(V_{Gmax} - V_{Gmin}) * [-1, 1]$.

Step 5: Perform the power flow (or load flow) analysis using Gauss Seidel or Newton Raphson algorithm for each particle to get the following

a) The active power of the slack generator (P_{slack}).
b) The voltage at each bus (V_{bus}).
c) Transmission line flows to determine the overloaded lines.

Step 6: Calculate the cost for each particle.

Step 7: Calculate the fitness function (FF) for each particle.

$$\begin{aligned} FF = {} & \text{power generation cost} + (\text{penalty factor}) * \left(P_{slack} - P_{slack_{max}}\right)^2 \\ & + \sum_{i=1}^{N_{load}} (\text{penalty factor}) * (V_i - V_{max})^2 \\ & + \sum_{i=1}^{N_{load}} (\text{penalty factor}) * (V_{min} - V_i)^2 \\ & + \sum_{i=1}^{N_{line}} (\text{penalty factor}) * \left(Flow_i - Flow_{i,max}\right)^2 \end{aligned} \quad (16)$$

where the penalty factor is selected as follows:

$$\text{penalty factor} = \begin{cases} 0, & \text{if the constraints are not violated} \\ (0.5 * \text{cost} * \text{iteration}^2), & \text{if the constraints are violated} \end{cases} \quad (17)$$

Step 8: Compare particles' fitness values and determine the personal (individual) best ($pbest_i$) for each particle and global best (gbest) for all the particles, where $pbest_i$ is the position of the i^{th} particle that gives a minimum FF among all its previously calculated FFs and gbest is the position of the particle that gives the so-far calculated minimum FF through all particles.

Step 9: Update the particles' velocities according to the following equation

$$v_i^{t+1} = w^t * v_i^t + C_1 * r_1(pbest_i - X_i^t) + C_2 * r_2 * (gbest - X_i^t) \qquad (18)$$

where v_i^{t+1} is the velocity of the i^{th} particle at the next iteration, v_i^t is the velocity of the i^{th} particle at the current iteration, X_i^t is the position of the i^{th} particle at the current iteration, w_i^t is the inertial weight at the current iteration, C_1 and C_2 are two positive factors that give relative weights that attract the particle towards the personal best or global best positions, while r_1 and r_2 are two randomly generated values between [0,1].

Step 10: Update the particles' positions according to the following equation

$$X_i^{t+1} = X_i^t + v_i^{t+1}, i = 1, \ldots, NIND \quad (19)$$

where X_i^{t+1} is the position of the i^{th} particle at the next iteration.

Step 11: Apply the reconstruction operators to the updated positions to ensure satisfying their corresponding constraints

If $X_{i,j}^{t+1} > X_{i,j,max}$

$$X_{i,j}^{t+1} = X_{i,j,max} \quad (20)$$

Else if $X_{i,j}^{t+1} < X_{i,j,min}$

$$X_{i,j}^{t+1} = X_{i,j,min} \quad (21)$$

Step 12: Update the inertial weight according to the following equation

$$w^{t+1} = w_{max} - \left(\frac{w_{max} - w_{min}}{itermax}\right) * t \quad (22)$$

where w^{t+1} is the inertial weight at the next iteration, w_{max} and w_{min} are the upper and lower limit of the inertial weight respectively, itermax is the maximum number of iterations and t is the iteration counter. The inertial weight balances global and local exploration and it decreases linearly from w_{max} to w_{min}.

Step 13: Return to step (5) until reaching the maximum number of iterations.

After reaching the maximum number of iterations, this means that the OPF problem is solved and the optimal setting of the generators' active power and voltages are obtained. Then, the OCAS is dealt with.

5.2 The Second Phase

In this phase, buses are chosen for capacitive compensation and their sizes are determined in order to improve the voltage profile of the system.

Step 1: Perform the initial power flow (or load flow) analysis using Gauss Seidel or Newton Raphson algorithm without capacitor compensation to calculate the loss sensitivity factors which are defined in equation (23), and the values are arranged in descending order for all the lines of the given system. Then, determine the candidate buses requiring the capacitor placement.

$$\frac{\partial P_{loss_k}}{\partial Q_{pq}} = \frac{2Q_{pq}}{V_p^2} R_{pq} \qquad (23)$$

where P_{loss_k} is the power loss through line k, Q_{pq} is the reactive power through line k, R_{pq} is the resistance of line k and V_p is the voltage of the supply bus that gives the power to line k.

Step 2: Select the suitable range of capacitors from Table 1 such that

$$Q_i^c \leq Q_C{}^{max} \leq Q_{total} \qquad (24)$$

Table 1: Yearly cost of fixed capacitors

Capacitor size (KVAR)	150	300	450	600	750	900	1050	1200	1350	1500	1650	1800	1950	2100
Capacitor cost ($/year)	0.5	0.35	0.253	0.22	0.276	0.183	0.228	0.17	0.207	0.201	0.193	0.187	0.211	0.176
Capacitor size (KVAR)	2250	2400	2550	2700	2850	3000	3150	3300	3450	3600	3750	3900	4050	
Capacitor cost ($/year)	0.197	0.17	0.189	0.187	0.183	0.18	0.195	0.174	0.188	0.17	0.183	0.182	0.179	

Step 3: Randomly initialize the position of particles. Each position is a two-dimensional matrix. Thus, the positions of the particles are represented by the following three-dimensional matrix.

$$X_{JxRxNIND} = [Q_1^c, \ldots, Q_j^c, \ldots, Q_J^c] \quad (25)$$

where J is the number of candidate buses for reactive compensation, NIND is the number of particles, R is the number of allowable capacitor sizes (e.g. if the available Q^c sizes that satisfy (24) are [150, 300, 450, 600], then $R = 4$) and Q_i^c is the reactive power installed at bus i. To select the capacitor size Q_i^c to be placed at bus i, a combination of capacitor sizes is chosen from Table 1,

$$Q_i^c = b_1\, sz_1 + b_2\, sz_2 + \ldots + \ldots + b_R\, sz_R \qquad (26)$$

where b_k between {0 , 1} and sz_k is capacitor size.

Step 4: Initialize the velocity of the particles.

Step 5: Perform the power flow (or load flow) analysis using Gauss Seidel or Newton Raphson algorithm for each particle to get the following

a) The active power losses (P_{loss}).
b) The voltage at each bus (V_{bus}).
c) Transmission line flows to determine the overloaded lines.

Step 6: Calculate the total annual cost function for each particle.

Step 7: Calculate the fitness function (FF) for each particle.

$$FF = \text{total annual cost} + \sum_{i=1}^{N_{load}} (\text{penalty factor}) * (V_i - V_{max})^2$$
$$+ \sum_{i=1}^{N_{load}} (\text{penalty factor}) * (V_{min} - V_i)^2$$
$$+ \sum_{i=1}^{N_{line}} (\text{penalty factor})\left(Flow_i - Flow_{i,max}\right)^2 \quad (27)$$

where the penalty factor is selected as follows

$$\text{penalty factor} = \begin{cases} 0, & \text{if the constraints are not violated} \\ (0.5 * \text{cost} * \text{iteration}^2), & \text{if the constraints are violated} \end{cases} \quad (28)$$

Step 8: Compare particles' fitness values and determine the personal (individual) best ($pbest_i$) for each particle and global best (gbest) through all the particles.

Step 9: Update the particles' velocities according to equation (18).

Step 10: Update the particles' positions. Kennedy and Eberhart have adapted the PSO to search in binary spaces by applying a sigmoid transformation to the velocity component Equation (29) to squash the velocities into a range [0,1], and force the component values of the locations of particles to be 0's or 1's. The equation for updating positions is

$$\text{sigmoid}\left(v_{i,j}^{t+1}\right) = \frac{1}{1 + e^{-v_{i,j}^{t+1}}} \quad (29)$$

$$X_{i,j}^{t+1} = \begin{cases} 1, & \text{if rand} < sigmoid\ (v_{i,j}^{t+1}) \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

where $X_{i,j}^{t+1}$ is the j^{th} component of the position vector of the i^{th} particle at the next iteration and rand is a random number between [0, 1].

Step 11: Update the inertial weight according to equation (22).

Step 12: Return to step (5) until reaching the maximum number of iterations.

After reaching the maximum number of iterations, this means that the OCAS problem is solved and the optimal locations and sizes of the capacitors are obtained.

6. Simulation Results

The proposed algorithm is tested on the 26-bus power system and on the standard IEEE 30-bus test system. The data of these systems are given in [16] and [10], respectively.

6.1 The 26-bus Power System

The main parameters of the PSO are shown in Table 2 for the OPF algorithm.

Table 2: Parameters of the PSO

C_1	C_2	W_{max}	W_{min}	Itermax	NIND
2.1	2.1	0.9	0.4	50	100

After performing 10 independent runs, the simulation results are summarized in Table 3, which include the average, maximum, minimum and standard deviation of the power generation cost.

Table 3:Statistics for the generator cost of the 26-bus system

Average ($)	Minimum ($) (best)	Maximum ($) (worst)	Standard deviation
15 545	15 492	15 571	20.2404

The optimal settings for the control variables are shown in Table 4. All generators' powers are in MW and all voltages are in p.u. values. The total generating cost is 15 492 $/hr which is better than that obtained in [1] and [16].

Table 4:The optimal settings of the continuous control variables for the 26-bus system

P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	P_{G6}	V_{G1}	V_{G2}	V_{G3}	V_{G4}	V_{G5}	V_{G6}
445.93	164.39	256.98	122.88	164.3	120	1.05	1.05	1.0489	1.05	1.05	1.05

Table 5 shows a comparison between the obtained results and those in [1] and [16]. It is clear that the proposed algorithm gives good results with regard to the generating cost and the total power losses.

Table 5: A comparison between the obtained results and those in [1] and [16]

Method	Proposed algorithm	MIGA [16]	PSO in [1]
Cost ($/hr)	15 492	15 598	15 523
Losses (MW)	11.5161	12.5007	12.7685

After the OPF problem is solved, the loss sensitivity factors are calculated. After running an initial load flow without any capacitor compensation, the total power losses is 12.9105 MW and the annual cost function of the power losses is $2168963. The loss sensitivity factors are calculated from this initial load flow and the values of the loss sensitivity factors are calculated for each transmission line. Because the 26-bus test system is an interconnected system not a radial system, then a modification is done on the criteria used to determine the candidate buses for capacitor compensation. The candidate buses that have the highest loss sensitivity factor are selected. It is found that selecting the highest 13 buses (i.e. half of the system buses) gives good results. The candidate buses for capacitor compensation are 25, 12, 11, 6, 21, 24, 20, 22, 16, 23, 19, 10 and 26. Now, sizing of capacitors at buses listed in the 'rank bus' vector is done by using discrete PSO. The parameters of the discrete PSO algorithm are shown in Table 6.

Table 6: Discrete PSO parameters

C_1	C_2	w_{max}	w_{min}	Itermax	NIND
2	2	0.9	0.4	100	100

The simulation results of 10 independent runs are summarized in Table 7, which include the average, maximum and minimum active power losses and the standard deviation of the results of the various runs. Table 8 shows the average, maximum and minimum total cost (capacitor cost plus cost of power losses) and the standard deviation of the results of the 10 runs. Table 9 shows the average, maximum and minimum net saving and the standard deviation. Table 10 shows the optimal sizing of capacitors for the 26-bus system. The total MVAR installed is 227.1 MVAR. The cost of the installed capacitors is 43169 $/year. The maximum voltage is 1.05 p.u. and the minimum voltage is 0.9987p.u.. Total power losses is 11.623 MW. This means that the proposed algorithm reduces the total power losses by 10%. The net saving is 173135 $/year, which is the difference between the cost of the reduced power losses and the capacitor cost.

Table 7: Statistics for the power losses for the 26-bus system

Average (MW)	Minimum (MW) (best)	Maximum (MW) (worst)	Standard deviation
11.6381	11.6105	11.6657	0.0159

Table 8: Statistics for the total cost for the 26-bus system

Average ($/year)	Minimum ($/year) (best)	Maximum ($/year) (worst)	Standard deviation
1996858	1995828	1998104	804

Table 9: Statistics for the net saving for the 26-bus system

Average ($/year)	Minimum ($/year) (worst)	Maximum ($/year) (best)	Standard deviation
172105	170860	173135	804

Table 10: Optimum sizing of capacitors for the 26 bus

Bus No	25	12	11	6	21	24	20	22	16	23	19	10	26
Q^c in MVAR	10.8	8.85	22.2	16.35	19.95	24.45	11.4	18.15	27.6	20.1	19.05	18.45	9.75

If the candidate buses for capacitor compensation are raised to 17 buses (i.e. 2/3 of all the buses), this will give better results. The maximum net saving becomes 210689 $/year which is better than 173135 $/year which was obtained when the candidate buses for capacitor compensation were 13 buses. Table 11 shows the optimal sizing of capacitors for the 26-bus system by compensating 2/3 of all the buses.

Table 11: Optimum sizing of capacitors for the 26 bus by compensating 2/3 of all buses

Bus No	25	12	11	6	21	24	20	22	16
Q^c in MVAR	12.75	10.8	12.15	11.7	15.3	30.45	17.1	10.5	22.95
Bus No	23	19	10	26	2	17	7	18	
Q^c in MVAR	20.55	9	19.2	10.8	17.1	32.55	22.2	20.55	

6.2 The Standard IEEE 30-bus Test System

After performing 10 independent runs, each run takes about 3 minutes. The simulation results are summarized in Table 12, which include the average, maximum, minimum and standard deviation of these 10 runs results.

Table 12: Statistics for the total cost for the 30-bus system

Average ($)	Minimum ($) (best)	Maximum ($) (worst)	Standard deviation
802.5770	801.6954	803.3557	0.4726

The optimal settings for the control variables are shown in Table 13. All generators' powers are in MW and all voltages are in p.u. values. The total generating cost is 801.6954 $/hr, which is better than that obtained in [3], [10], and [11]. However, the obtained cost is not better than that in [13]. This is because the maximum number of iteration used in [13] is 500 compared with 50 in the proposed algorithm. Table 14 shows a comparison between the obtained results and those in [3], [10], [11] and [13]. It is clear that the proposed algorithm gives good results with regard to the generating cost and the total power losses using a relatively small number of iterations.

Table13:The optimal settings of the continuous control variables for the 30-bus system

P_{G1}	P_{G2}	P_{G3}	P_{G4}	P_{G5}	P_{G6}
173.48	48.68	20.55	23.31	14.1	12.24
V_{G1}	V_{G2}	V_{G3}	V_{G4}	V_{G5}	V_{G6}
1.1	1.0758	1.0561	1.0447	1.075	1.0523

Table14: A comparison between the obtained results and those in [3], [10], [11] and [13].

Method	Proposed algorithm	GA [3]	Gradient method in [10]	ABC in [11]	PSO in [13]
Cost ($/hr)	801.6954	802.06	802.4	801.88	800.41
Losses (MW)	9.412	9.39	9.48	9.49	9

After the OPF problem is solved, then the loss sensitivity factors are calculated. After running an initial load flow without any capacitor compensation, the total power losses is found to be 9.412 MW and the annual cost function of the power losses is

$1581208. The loss sensitivity factors are calculated from this initial load flow and the values of the loss sensitivity factors are calculated for each transmission line. The candidate buses that have the highest loss sensitivity factors are selected. It is found that selecting the highest 15 buses (i.e. half of the system buses) gives good results. The candidate buses for capacitor compensation are 26, 30, 24, 5, 15, 25, 29, 21, 22, 20, 23, 14, 3, 16 and 18. Now, sizing of capacitors at buses listed in the 'rank bus' vector is done by using discrete PSO. The parameters of the discrete PSO algorithm are shown in Table 8.

After performing 10 independent runs, the simulation results are given in Table 15, which include the average, maximum and minimum active power losses and the standard deviation of the 10 runs results. Table 16 shows the average, maximum and minimum total cost (capacitor cost plus cost of power losses) and the standard deviation of the results. Table 17 shows the average, maximum and minimum net saving and the standard deviation of these 10 results. Table 18 shows the optimal sizing of capacitors for the 26-bus system. The total MVAR installed is 28.65MVAR. The cost of the installed capacitors is 5220 $/year. The maximum voltage is 1.0858 p.u. and the minimum voltage is 0.9984 p.u. Total power losses is 9.1225MW. This means that the proposed algorithm reduces the total power losses by 3%. The net saving is 43414$/year, which is the difference between the cost of the reduced power losses and the capacitor cost.

Table 15: Statistics for the power losses for the 30-bus system

Average (MW)	Minimum (MW) (best)	Maximum (MW) (worst)	Standard deviation
9.1389	9.1225	9.1581	0.0115

Table 16: Statistics for the total cost for the 30-bus system

Average ($/year)	Minimum ($/year) (best)	Maximum ($/year) (worst)	Standard deviation
1540407	1537794	1543225	1712

Table 17: Statistics for the net saving for the 30-bus system

Average ($/year)	Minimum ($/year) (worst)	Maximum ($/year) (best)	Standard deviation
40800	37983	43414	1712

Table 18: Optimum sizing of capacitors for the 30 bus

Bus No	26	30	24	5	15	25	29	21	22	20	23	14	3	16	18
Q^c in MVAR	1.95	0	6.15	0	0.45	0	2.1	7.65	0	2.85	3.75	3.75	0	0	0

If the candidate buses for capacitor compensation are raised to 20 buses (i.e. 2/3 of all the buses), this will yield better results. The maximum net saving becomes 45120 $/year which is better than 43414 $/year which was obtained with 15 candidate buses for capacitor compensation. Table 19 shows the optimal sizing of capacitors for the 30-bus system by compensating 2/3 of all the buses.

Table 19: Optimum sizing of capacitors for the 30-bus by compensating 2/3 of all the buses

Bus No	26	30	24	5	15	25	29	21	22	20	23
Q^c in MVAR	2.85	0	3.3	0	7.95	0.3	2.85	7.05	0	0	0
Bus No	14	3	16	18	17	4	19	6	7		
Q^c in MVAR	0	0	2.7	0	1.65	1.95	1.8	0	2.7		

7. Discussion

Some comments on the proposed algorithm are now provided.

- Unlike the gradient methods, PSO is a population based search algorithm, i.e. PSO has implicit parallelism. This property ensures PSO to be less susceptible to be trapped in local minima [13].
- Unlike the traditional methods, the solution quality of the proposed algorithm does not rely on the initial population. Starting anywhere in the search space, the algorithm ensures the convergence to the optimal solution [13].
- Unlike GA, PSO has the flexibility to control the balance between the global and local exploration of the search space. This property enhances the search capabilities of the PSO technique [13].
- The reconstruction operators ensure that all particles represent a possible solution satisfying the units' constraints, while looking for the optimal solution only within the feasible space reducing the computing time and improving the quality of the achieved solution.
- The OCPS problem was tested on a radial distribution system, but in this paper the OCPS is tested on both radial and interconnected electric power systems. The results of the radial distribution system are not shown because these systems have only one generator and no need for OPF and the proposed algorithm performs both OPF and OCPS on the same electric power system.

8. Conclusion

In this paper, the optimal power flow (OPF) problem is and the optimal capacitor placement and sizing (OCPS) are solved. The PSO technique is used as the optimization tool, with its discrete version being used to solve the OCPS problem. The proposed algorithm has been tested on the 26-bus and the 30-bus systems and it gives superior results compared with those reported in literature with a relatively small number of iterations. The integration between the OPF and the OCPS problems takes the advantages of using capacitors, which improve the system power factor, improve the system voltage profile, decrease the flow through cables and transformers and reduce losses due to the compensation of the reactive component of power flow. By decreasing the flow through cables, the systems' loads can be increased without adding any new cables or overloading the existing cables.

References

[1] Y. Pablo O˜nate, Juan M. Ramirez and Carlos A. Coello "Optimal Power Flow Subject to Security ConstraintsSolved With a Particle Swarm Optimizer", *IEEE transactions on power systems*, vol. 23, no. 1, February, 2008.

[2] Y. Pablo O˜nate, Juan M. Ramirez and Carlos A. Coello Coello "An optimal power flow plus transmission costs solution," *Electric Power Systems Research* ,pp. 1240–1246, 2009.

[3] Anastasios G. Bakirtzis, Pandel N. Biskas, Christoforos E. Zoumas, and Vasilios Petridis "Optimal Power Flow by Enhanced Genetic Algorithm", *IEEE transactions on power systems*, vol. 17, no. 2, May 2002.

[4] T.S. Chug and Ge Shaoyun, "A recursive LP-based approach for optimal capacitor allocation with cost-benefit consideration", *Electric Power Systems Research* ,pp. 129–136, 1997.

[5] J. B. V. Subrahmanyam, "Optimal capacitor placement in unbalanced radial distribution networks", *Journal of Theoretical and Applied Information Technology* vol. 16, pp 106-115 2005-2009.

[6] K.S.Pandya and S.K. Joshi, "A survey of optimal power flow methods", *Journal of Theoretical and Applied Information Technology* , 2005- 2008.

[7] N. Rugthaicharoencheep and S. Auchariyamet "Optimal Capacitor Placement in Distribution Feeders" , World Academy of Science, *Engineering and Technology*, 2012.

[8] Jamshid Aghaei, Afshin Lashkar Ara and Meisam Shabani "Fuzzy multi-objective optimal power flow considering UPFC", International Journal of Innovative Computing, *Information and Control*, Volume 8, Number 2, February 2012.

[9] Hermann W. Dommel and William F. Tinney, "Optimal Power Flow Solutions", *IEEE transactions on power apparatus and systems*, vol. no. 10, October 1968.

[10] O. Alsac and B. Stott"Optimal Load Flow with Steady-State Security", *IEEE transactions on power apparatus and systems*, vol. no. 93, pp 745 – 751, May 1974.

[11] C. Sumpavakup, I. Srikun, and S. Chusanapiputt "A Solution to the Optimal Power Flow Using Artificial Bee Colony Algorithm", *International Conference on Power System Technology*, 2010

[12] Madhu Garg, "GA BASED OPTIMAL POWER FLOW SOLUTIONS", *Master of Engineering in Power Systems and Electric drives*, Thapar University, Patiala, June 2008.

[13] M.A. Abido, "Optimal power flow using particle swarm optimization", *Electric Power and Energy Systems*, 2002.

[14] K. Prakash and M. Sydulu, "Particle Swarm Optimization Based Capacitor Placement on Radial Distribution Systems", *IEEE Power Engineering Society General Meeting*, June 2007

[15] R. Srinivasa Rao, "Capacitor Placement in Radial Distribution System for Loss Reduction Using Artificial Bee Colony Algorithm", *International Journal of Engineering and Natural Sciences*, pp. 84 – 88, 2010.

[16] Zwe-Lee Gaing and Rung-Fang Chang ,"Security-Constrained Optimal Power Flow by Mixed-Integer Genetic Algorithm with Arithmetic Operators ", *IEEE Power Engineering Society General Meeting*, 2006.