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Nonlinear Model Predictive Control for Landing Guidance of Reusable Rocket Using Thruster Inputs

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Abstract: - This paper proposes a control system design method based on nonlinear model predictive control for automatic landing of reusable rockets with considering the thruster inputs and the manipulation of gimbal angles and aerodynamic coefficients. Model predictive control is a kind of optimal feedback control in which the control performance over a finite future is optimized and its performance index has a moving initial time and a moving terminal time. This paper provides a numerical solution method based on the C/GMRES algorithm to solve the nonlinear model predictive control problem of automatic landing of reusable rockets. The effectiveness of the proposed method is verified by numerical simulations.

Keywords: Nonlinear Control, Optimal Control, Space Engineering, Control Systems, Automatic Landing.

I. Introduction

Launch rockets have an important role in space missions, especially in the field of space science and engineering. Space technology has a crucial role in our standard services like communication, weather forecasting, remote sensing, etc. The cost to access the space is one of the factors which holds back space utilization [1]. In recent years, the idea of the reusable rocket has attracted much attention in space engineering.

The development of reusable rocket has expected to save the space mission cost. However, the technological complexities make it challenging to implement. Especially, the technology of control and guidance of vertical landing needs to be developed. Thus, this paper examines the design problem of the control system for automatic landing of rocket.

Model predictive control (MPC), also known as receding horizon control [2]-[7], is a useful control methodology where the control input is determined at each sampling time so as to minimize a given performance index. MPC is a useful control method that enables a control performance to be optimized with considering some constraints on the system state and the control inputs [8]-[11]. MPC method for the automatic landing of nonlinear rotational dynamics of rocket has been proposed in [12]. However, the dynamics of landing rocket was restricted to the plane motion. The dynamical complexities make it challenging to implement. Apart from [12], this paper considers the 6 degree of freedom (6-DoF) motion for the system model of rocket. To be more specific, 6-DoF system model of landing rocket is addressed with considering the thruster inputs and the manipulation of gimbal angles and aerodynamic coefficients. Thus, the nonlinearity and time-variance of rocket dynamics are considered to design the control system.

The objective of this paper is to propose a control system design method based on nonlinear model predictive control for automatic landing of reusable rockets with considering the thruster inputs and the manipulation of gimbal angles and aerodynamic coefficients. This paper provides a numerical solution method based on the C/GMRES algorithm to solve the nonlinear model predictive control problem of automatic landing of reusable rockets. The effectiveness of the proposed method is verified by numerical simulations.

II. Notations And System Model

In this section, we introduce the system model [13] that represents the dynamics of a rocket capable of vertical take-off and landing. The notations used in this study are defined in Table 1. The motion of the rocket is expressed using a ground-fixed coordinate system and a body-fixed coordinate system, as illustrated in Fig. 1. The origin of the body-fixed coordinate system is set at the rocket's center of mass. In the following, we derive the system model for the rocket.

Considering the rotation of the rocket, the application of Newton's second law yields the translational equations of motion:

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$$\dot{\boldsymbol{v}} = \frac{1}{m}\left(\boldsymbol{F_g} + \boldsymbol{F_p} + \boldsymbol{F_a}\right) - \boldsymbol{\omega} \times \boldsymbol{v} \tag{1}$$

where $\boldsymbol{v} = [U \quad V \quad W]^{\mathrm{T}}$ represents the velocity vector in the rocket's body-fixed frame, and $\boldsymbol{\omega} = [p \quad q \quad r]^{\mathrm{T}}$ represents the angular velocity vector. $\boldsymbol{F_g}, \boldsymbol{F_p}, \boldsymbol{F_a}$ denote gravitational force, propulsion force, and aerodynamic force, respectively, given by:

$$\boldsymbol{F_g} = \boldsymbol{A_{\overrightarrow{eb}}}\begin{bmatrix} -mg \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$\boldsymbol{F_p} = \begin{bmatrix} T\cos\eta_p\cos\eta_y \\ -T\cos\eta_p\sin\eta_y \\ -T\sin\eta_p \end{bmatrix} \tag{3}$$

$$\boldsymbol{F_a} = \begin{bmatrix} -\frac{1}{2}\rho {V_{total}}^2 S_x C_A \\ -\frac{1}{2}\rho {V_{total}}^2 S_y C_N \\ -\frac{1}{2}\rho {V_{total}}^2 S_z C_N \end{bmatrix} \tag{4}$$

Here, $\boldsymbol{A_{\overrightarrow{eb}}}$ denotes the transformation matrix from the Earth-fixed coordinate system to the body-fixed coordinate system. The airspeed V_{total} is given by:

$$V_{total} = \sqrt{U^2 + V^2 + W^2} \tag{5}$$

The rotational equation of motion for the rocket is given by:

$$\boldsymbol{M_a} + \boldsymbol{M_p} = \boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega} \tag{6}$$

where $\boldsymbol{J}$ is the inertia matrix, and $\boldsymbol{M_a}$, $\boldsymbol{M_p}$ represent the moments generated by aerodynamic forces and propulsion force, respectively, where $\boldsymbol{M_p}$ is the moment caused by the thrust from both the engine and gas thrust, expressed as:

$$\boldsymbol{J} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} = \begin{bmatrix} \frac{1}{8}md^2 & 0 & 0 \\ 0 & m\left(\frac{d^2}{16} + \frac{l^2}{12}\right) & 0 \\ 0 & 0 & m\left(\frac{d^2}{16} + \frac{l^2}{12}\right) \end{bmatrix} \tag{7}$$

$$\boldsymbol{M_a} = \begin{bmatrix} \frac{1}{2}\rho {V_{total}}^2 S_x d C_l \\ \frac{1}{2}\rho {V_{total}}^2 S_y d C_m \\ \frac{1}{2}\rho {V_{total}}^2 S_z d C_m \end{bmatrix} \tag{8}$$

$$\boldsymbol{M_p} = \begin{bmatrix} 0 \\ -T\sin\eta_p\,(x_g - x_{cg}) + T_{gy}(x_{cg} - x_g) \\ T\cos\eta_p\sin\eta_y\,(x_g - x_{cg}) + T_{gz}(x_{cg} - x_g) \end{bmatrix} \tag{9}$$

To express the position and attitude of the rocket, the velocity and angular velocity in the body-fixed coordinate system need to be transformed into the Earth-fixed coordinate system. Thus, the translational velocity of the rocket is given by:

$$\begin{bmatrix} \dot{x_e} \\ \dot{y_e} \\ \dot{z_e} \end{bmatrix} = \boldsymbol{A}_{\overrightarrow{\boldsymbol{be}}} \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{10}$$

where $\boldsymbol{A}_{\overrightarrow{\boldsymbol{be}}}$ is the transformation matrix from the body-fixed coordinate system to the Earth-fixed coordinate system, defined as the transpose of $\boldsymbol{A}_{\overrightarrow{\boldsymbol{eb}}}$. The angular velocity of the rocket is expressed as:

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi/\cos\theta & \cos\varphi/\cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{11}$$

Let the state vector $\boldsymbol{x}(t)$ consist of position, velocity, angles, angular velocity, and mass, while the control input $\boldsymbol{u}(t)$ comprises propulsion force, gimbal angles, and aerodynamic coefficients. Thus, using the equations of motion derived above, the system model for the rocket can be described by equation (12). For notational convenience, the trigonometric functions sin, cos, and tan are represented as s, c, and t, respectively:

$$\boldsymbol{x}(t) = [x_e, y_e, z_e, U, V, W, \varphi, \theta, \psi, p, q, r]^{\mathrm{T}}$$

$$\boldsymbol{u}(t) = \left[T, \eta_p, \eta_y, T_{gy}, T_{gz}, C_A, C_N, C_l, C_m\right]^{\mathrm{T}}$$

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t))$$

$$= \begin{bmatrix} (\mathrm{c}_{x_8}\mathrm{c}_{x_9})x_4 + (\mathrm{s}_{x_7}\mathrm{s}_{x_8}\mathrm{s}_{x_9} - \mathrm{c}_{x_7}\mathrm{s}_{x_9})x_5 + (\mathrm{c}_{x_7}\mathrm{s}_{x_8}\mathrm{s}_{x_9} + \mathrm{s}_{x_7}\mathrm{s}_{x_9})x_6 \\ (\mathrm{c}_{x_8}\mathrm{s}_{x_9})x_4 + (\mathrm{s}_{x_7}\mathrm{s}_{x_8}\mathrm{s}_{x_9} + \mathrm{c}_{x_7}\mathrm{c}_{x_9})x_5 + (\mathrm{c}_{x_7}\mathrm{s}_{x_8}\mathrm{s}_{x_9} - \mathrm{s}_{x_7}\mathrm{c}_{x_9})x_6 \\ (-\mathrm{s}_{x_8})x_4 + (\mathrm{s}_{x_7}\mathrm{c}_{x_8})x_5 + (\mathrm{c}_{x_7}\mathrm{c}_{x_8})x_6 \\ -g\mathrm{c}_{x_8}\mathrm{c}_{x_9} - \dfrac{\rho S_x u_6}{2m}{V_{total}}^2 + \dfrac{u_1}{m}\mathrm{c}_{u_2}\mathrm{c}_{u_3} - x_{11}x_6 + x_{12}x_5 \\ -g(\mathrm{s}_{x_7}\mathrm{s}_{x_8}\mathrm{c}_{x_9} - \mathrm{c}_{x_7}\mathrm{s}_{x_9}) - \dfrac{\rho S_y u_7}{2m}{V_{total}}^2 - \dfrac{u_1}{m}\mathrm{c}_{u_2}\mathrm{s}_{u_3} - x_{12}x_4 + x_{10}x_6 \\ -g(\mathrm{c}_{x_7}\mathrm{s}_{x_8}\mathrm{c}_{x_9} + \mathrm{s}_{x_7}\mathrm{s}_{x_9}) - \dfrac{\rho S_z u_7}{2m}{V_{total}}^2 - \dfrac{u_1}{m}\mathrm{s}_{u_2} - x_{10}x_5 + x_{11}x_4 \\ x_{10} + (x_{11}\mathrm{s}_{x_7} + x_{11}\mathrm{c}_{x_7})\mathrm{t}_{x_8} \\ x_{11}c_{x_7} - x_{12}s_{x_7} \\ \dfrac{1}{c_{x_8}}(x_{11}s_{x_7} + x_{12}c_{x_7}) \\ J_x^{-1}\left(\dfrac{\rho S_x d u_8}{2}{V_{total}}^2\right) \\ J_y^{-1}\left(\dfrac{\rho S_y d u_9}{2}{V_{total}}^2 - u_1 s_{u_2}(x_g - x_{cg}) + u_4(x_{cg} - x_g) - x_{12}x_{10}(J_x - J_z)\right) \\ J_z^{-1}\left(\dfrac{\rho S_z d u_9}{2}{V_{total}}^2 + u_1 c_{u_2} s_{u_3}(x_g - x_{cg}) + u_5(x_{cg} - x_g) - x_{10}x_{11}(J_y - J_x)\right) \end{bmatrix} \tag{12}$$

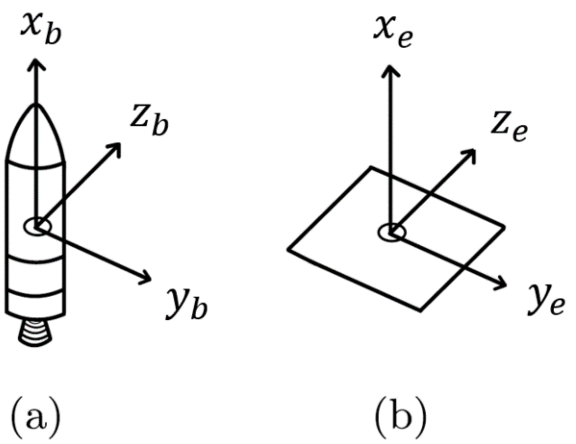

Fig. 1 (a) Body-fixed and (b) inertial reference frames

Table I. Definition of Notations

Definition	Symbol	Unit
Position	x_e, y_e, z_e	m
Position (body axis)	x_b, y_b, z_b	m
Speed	U, V, W	m/s
Airspeed	V_{total}	m/s
Angle	φ, θ, ψ	rad
Angular velocity	p, q, r	rad/s
Gravitational acceleration	g	$\mathrm{m/s^2}$
Body mass	m	kg
Density of air	ρ	$\mathrm{kg/m^3}$
Body cross-sectional area	S_x, S_y, S_z	$\mathrm{m^2}$
Body diameter	d	m
Overall length	l	m
Engine thrust	T	N
Gimbal angle	η_p, η_y	rad
Gas thrust	T_{ay}, T_{az}	N
Moment of inertia	J_x, J_y, J_z	$\mathrm{kg \cdot m^2}$
Gimbal position	x_a	m
Center of gravity	x_{ca}	m
Aerodynamic force coefficient	C_A, C_N	−
Aerodynamic moment coefficient	C_l, C_m	−

III. MODEL PREDICTIVE CONTROL

In this section, the nonlinear model predictive control problem of system model (12) is considered. First, the optimal control problem of nonlinear vehicle dynamics is considered. The control input at each time t is determined so as to minimize the following performance index:

$$J = \frac{1}{2}\left(x^{\mathrm{T}}(t+T)Px(t+T)\right)$$

$$+\int_{t}^{t+T} \frac{1}{2}\left\{x^{\mathrm{T}}(\tau)Qx(\tau) + \left(u(\tau)-u_f\right)^{\mathrm{T}} R\left(u(\tau)-u_f\right)\right\} d\tau, \tag{13}$$

where T is the evaluation interval of the performance index, and P, Q, R are weighting coefficients. The optimization problem of (12) subject to equality constraint (13) can be reduced to minimizing the following performance index $\bar{J}$ introduced by using the costate λ associated with the equality constraint.

$$\bar{J} = \frac{1}{2}\left(x^{\mathrm{T}}(t+T)Px(t+T)\right)$$

$$+\int_{t}^{t+T} \left\{x^{\mathrm{T}}(\tau)Qx(\tau) + \left(u(\tau)-u_f\right)^{\mathrm{T}} R\left(u(\tau)-u_f\right) + \lambda^{\mathrm{T}}(\tau)(f(x,u)-\dot{x})\right\} d\tau. \tag{14}$$

On the basis of the variational principle, we obtain the necessary conditions for a stationary value of $\bar{J}$ over the horizon $(t \le \tau \le t+T)$ as follows.

$$\dot{x}(\tau) = f\left(x(\tau), u(\tau)\right) \tag{15}$$

$$\dot{\lambda}(\tau) = -\left(\frac{\partial H}{\partial x}\right)^{\mathrm{T}} \tag{16}$$

$$\lambda(t+T) = \left(\frac{\partial \varphi}{\partial x}\right)^{\mathrm{T}} \tag{17}$$

$$\frac{\partial H}{\partial u} = 0 \tag{18}$$

Conditions (15)-(18) are called the stationary conditions or Euler-Lagrange equations that must be satisfied for the performance index (14) to be minimized. A well-known difficulty of the nonlinear optimal control is that it results in a nonlinear two-point boundary-value problem that cannot be solved analytically in general. Then, a fast algorithm, called the C/GMRES, for numerically solving stationary conditions has been proposed in [14]. In this study, we apply the C/GMRES algorithm to solving the obtained stationary conditions.

IV. Numerical Simulation

In this section, an illustrative example is provided to verify the effectiveness of the proposed method. We consider the situation where a rocket lands perpendicular to the ground at the origin of the ground-fixed coordinate system. The simulation parameters used here are listed in Table 2, where "diag" denotes a diagonal matrix.

In the following, we provide the simulation results to verify the effectiveness of the proposed method. Figs. 2-7 show the time responses of state variables using nonlinear model predictive control based on the C/GMRES method. It is seen that all state variables converge to the target state. Figs. 8-12 show the time responses of control inputs and optimality error. It is seen that the control input and optimality error converge to target input and zero, respectively. Consequently, the effectiveness of the proposed method was verified by the simulation results.

TABLE II. Simulation Parameters

Symbol	Value
g	$9.81\ \mathrm{m/s^2}$
ρ	$1.251\ \mathrm{kg/m^3}$
J_x	$24.5\ \mathrm{kg \cdot m^2}$
J_y, J_z	$220.5833\ \mathrm{kg \cdot m^2}$
S_x	$1.5394\ \mathrm{m^2}$
S_y, S_z	$7\ \mathrm{m^2}$
d	$1.4\ \mathrm{m}$
x_g	$5\ \mathrm{m}$
x_{cg}	$3.25\ \mathrm{m}$
$T(t)$	$1.2 - e^{-0.5t}$
$x(0)$	$[300, 185, 185, -1, 0, 0, 0, 0, 0, 0, 0, 0]^{\mathrm{T}}$
x_f	$[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^{\mathrm{T}}$
$u(0)$	$[0, 0, 0, 0, 0, 0, 0, 0, 0]^{\mathrm{T}}$
u_f	$[mg(1 - e^{-0.14t}), 0, 0, 0, 0, 0, 0, 0, 0]^{\mathrm{T}}$
P	$100 \cdot \mathrm{diag}[3, 2.4, 2.4, 90, 45, 30, 0, 300, 300, 0.1, 600, 600]$
Q	$1000 \cdot \mathrm{diag}[3, 2.4, 2.4, 90, 45, 30, 0, 300, 300, 0.1, 600, 600]$
R	$10 \cdot \mathrm{diag}[2, 1000, 1000, 10, 10, 10000, 10000, 10000, 10000]$

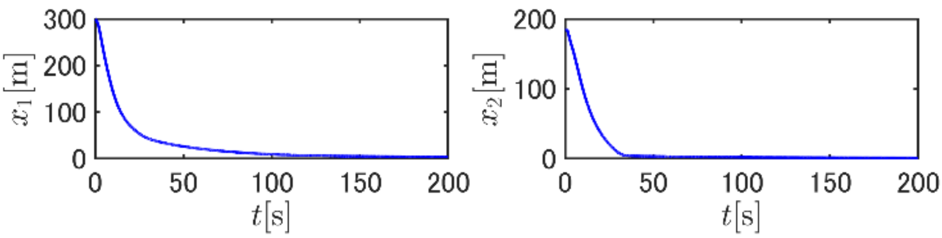

Fig. 2 Time responses of $x_1(t)$ and $x_2(t)$.

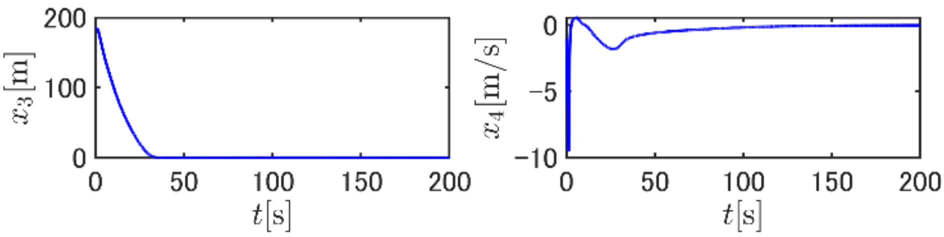

Fig. 3 Time responses of $x_3(t)$ and $x_4(t)$.

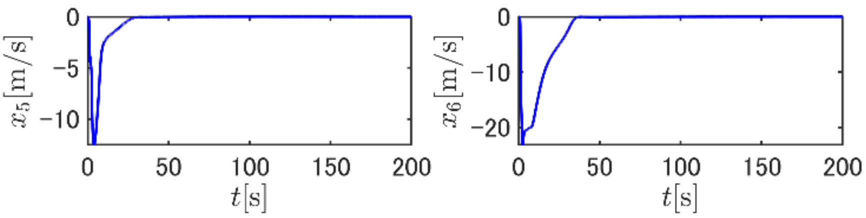

Fig. 4 Time responses of $x_5(t)$ and $x_6(t)$.

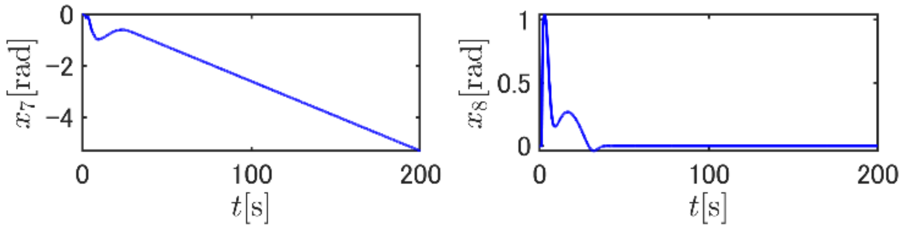

Fig. 5 Time responses of $x_7(t)$ and $x_8(t)$.

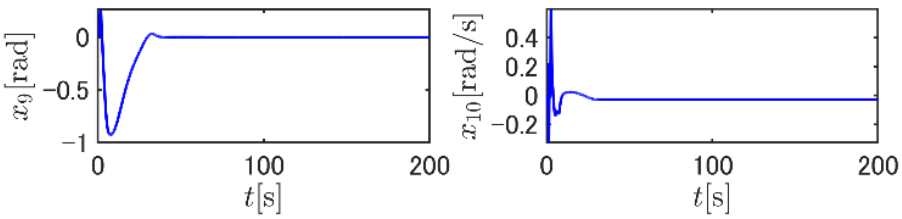

Fig. 6 Time responses of $x_9(t)$ and $x_{10}(t)$.

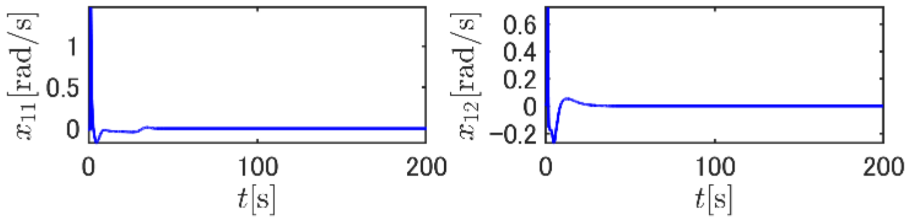

Fig. 7 Time responses of $x_{11}(t)$ and $x_{12}(t)$.

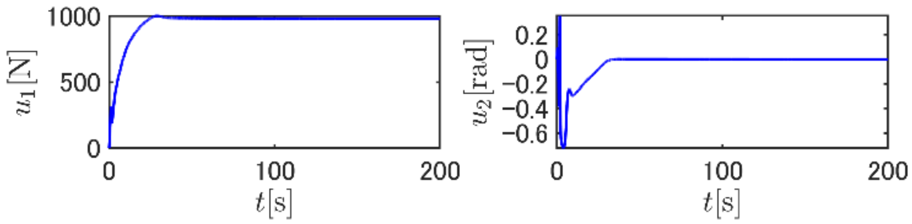

Fig. 8 Time responses of $u_1(t)$ and $u_2(t)$.

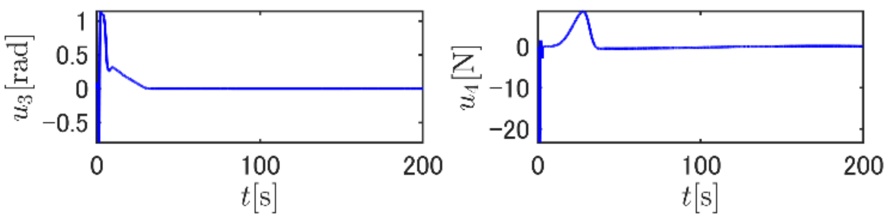

Fig. 9 Time responses of $u_3(t)$ and $u_4(t)$.

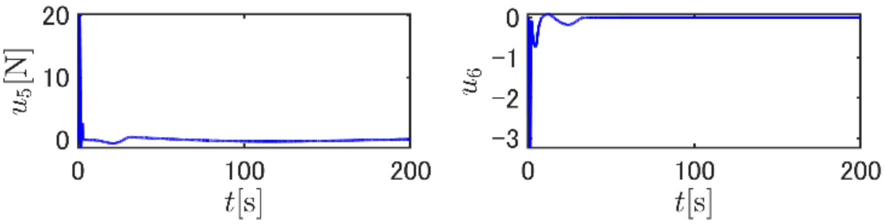

Fig. 10 Time responses of $u_5(t)$ and $u_6(t)$.

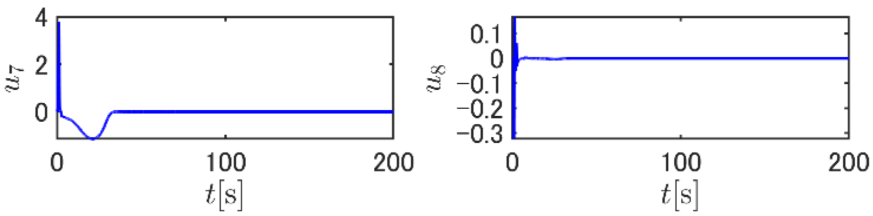

Fig. 11 Time responses of $u_7(t)$ and $u_8(t)$.

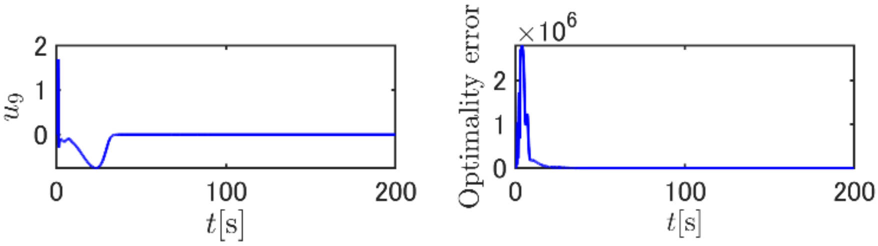

Fig. 12 Time responses of $u_9(t)$ and Optimality error.

V. CONCLUSIONS

In this study, 6-DoF system model of landing rocket was introduced with considering thruster inputs and the manipulation of gimbal angles and aerodynamic coefficients. Thus, the nonlinearity and time-variance of rocket dynamics were considered to design the control system. The control system design method based on nonlinear model predictive control for automatic landing of reusable rockets with considering thruster inputs and the manipulation of gimbal angles and aerodynamic coefficients has been established. The numerical solution method based on the C/GMRES algorithm to solve the nonlinear model predictive control problem of automatic landing of reusable rockets was provided. The effectiveness of the proposed method was verified by numerical simulations.

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