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A Comprehensive Review of Time-Domain Approaches for Chatter Dynamics and Forced Vibration Analysis in Thin-Walled Milling



Abstract: - Chatter is a prevalent self-excited vibration phenomenon that occurs in most milling procedures and substantially affects surface quality, tool life, and productivity. This paper discusses time-domain methods for modeling chatter dynamics and forced vibration in thin-walled milling, mainly focusing on prediction, detection, and control techniques. To deal with the chatter phenomenon, theoretical factors affecting chatter, such as process damping, tool run-out, and gyroscopic effects, are discussed to gain insight into their influence on stability prediction. This paper reviews models for milling force prediction with the prospective application of these models to thin-walled structures. It further questions the established experimental methods, such as finite element modal analysis or structural coupling, for obtaining frequency response functions of workpieces and tools. Advances and performances of hardware in the loop simulation study deal with the reflection of mirror-milling technology development aimed at application to performing thin-walled part components. The review discusses the root causes of chatter and provides the path for future work to further improve prediction accuracy and suppression methodologies. This contributes to the sustainable planning of successful thin-walled milling processes with improved characteristics and machine output.

Keywords: Chatter Dynamics, Thin-Walled Milling, Time-Domain Analysis, Stability Prediction, Chatter Suppression

I. INTRODUCTION

Thin walls are essential in aerospace, automotive, and precision engineering due to their lightweight design, complex geometries, and high-performance applications [5]. However, their low rigidity makes them very sensitive to deformation and vibration during machining processes, resulting in decreased surface quality, dimensional accuracy, and tool wear. Such challenges reduce the efficiency and quality of manufacturing and spur interest in advanced techniques to help stabilize the process of thin-walled part machining.

Mirror milling is a cutting technique in which the machining path is optimally arranged for minimizing cutting forces and controlling deformation along a plane of symmetry; therefore, mirror milling is recognized as a relatively new specialized technique [10, 11]. Mirror milling has some advantages, but the stability of this innovative process depends on various factors, including cutting depth, spindle speed, feed rate and support mechanisms. Due to these complex interactions, choosing appropriate process parameters is crucial for thin-walled components' stability and surface characteristics, necessitating designs of experiences (DoE) for reproducible and high-quality machining.

The thin-walled part is a typical example of milling; this review papers focus on the research progress of theoretical models for chatter prediction in thin-walled milling. Chatter, a self-excited vibration during machining operations, remains an essential problem, adversely affecting tool life and surface quality. This paper reviews empirical, finite element, and analytical models for predicting milling forces in thin-walled structures and compares the benefits and drawbacks of each modeling approach. Second, it reviews the dynamic behavior of thin-walled components in mirror milling, comparing frequency-domain and time-domain methods for stability prediction. The development of experimental techniques in recent years, such as finite element modal analysis and structural coupling, can obtain precise frequency response functions of workpieces and tools to improve the accuracy of stability prediction [9]. Many innovative components have been introduced to mirror milling technology to counteract deformation and suppress chatter, such as fluid-lubricating supports in angular positioning systems or flexible multi-point following support systems [25]. This has led to the subsequent synthesis of advanced machining accuracy for significant components.

This paper aims to provide a high level of existing methodologies, compare different modeling approaches, and highlight the primary challenges in chatter prediction and suppression when applied to thin-walled part milling.

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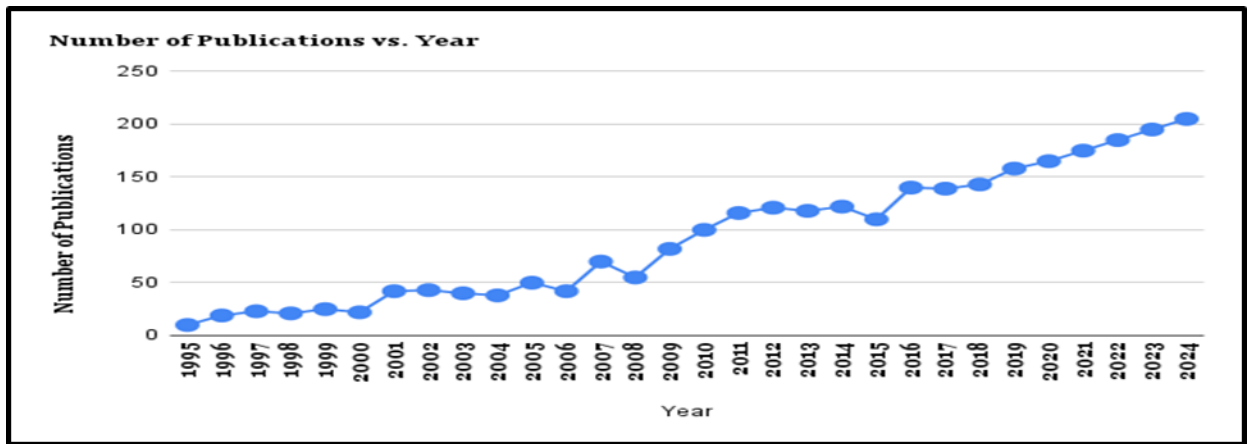


Figure 1. Publications on chatter in milling over time

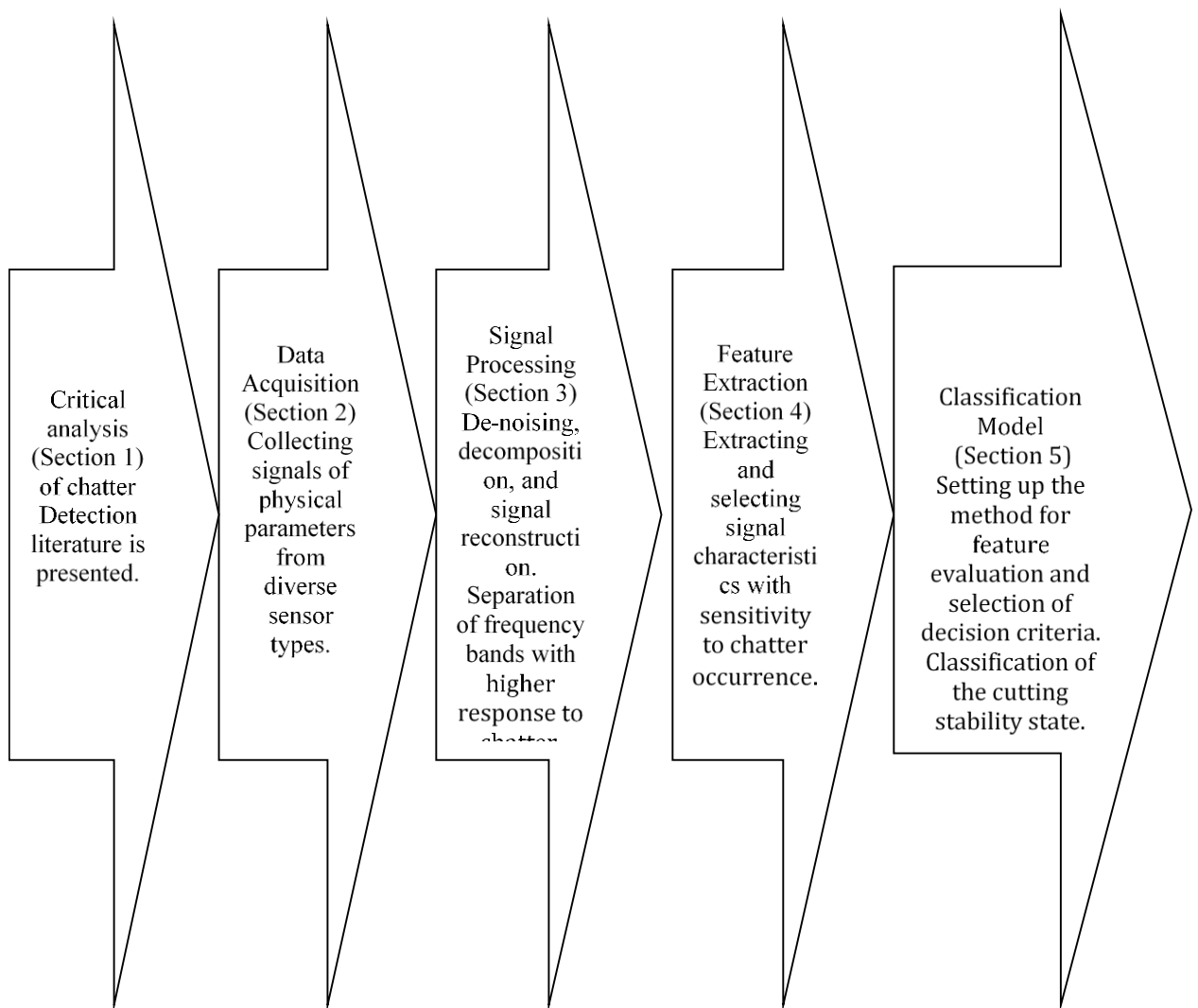


Figure 2. A standard method in chatter detection

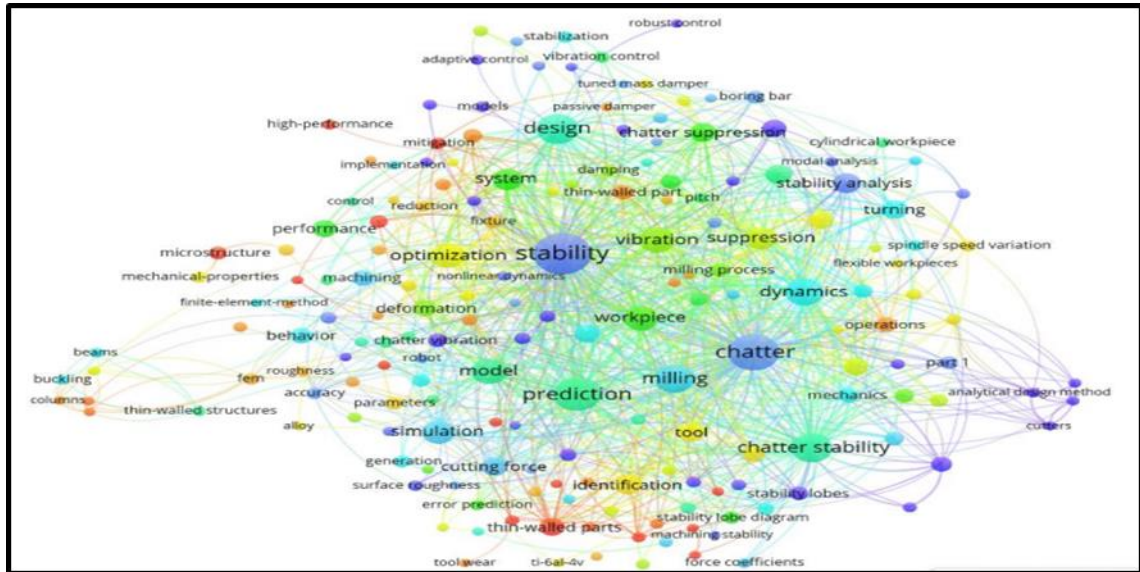


Figure 3. Research topics in thin-wall machining dynamics

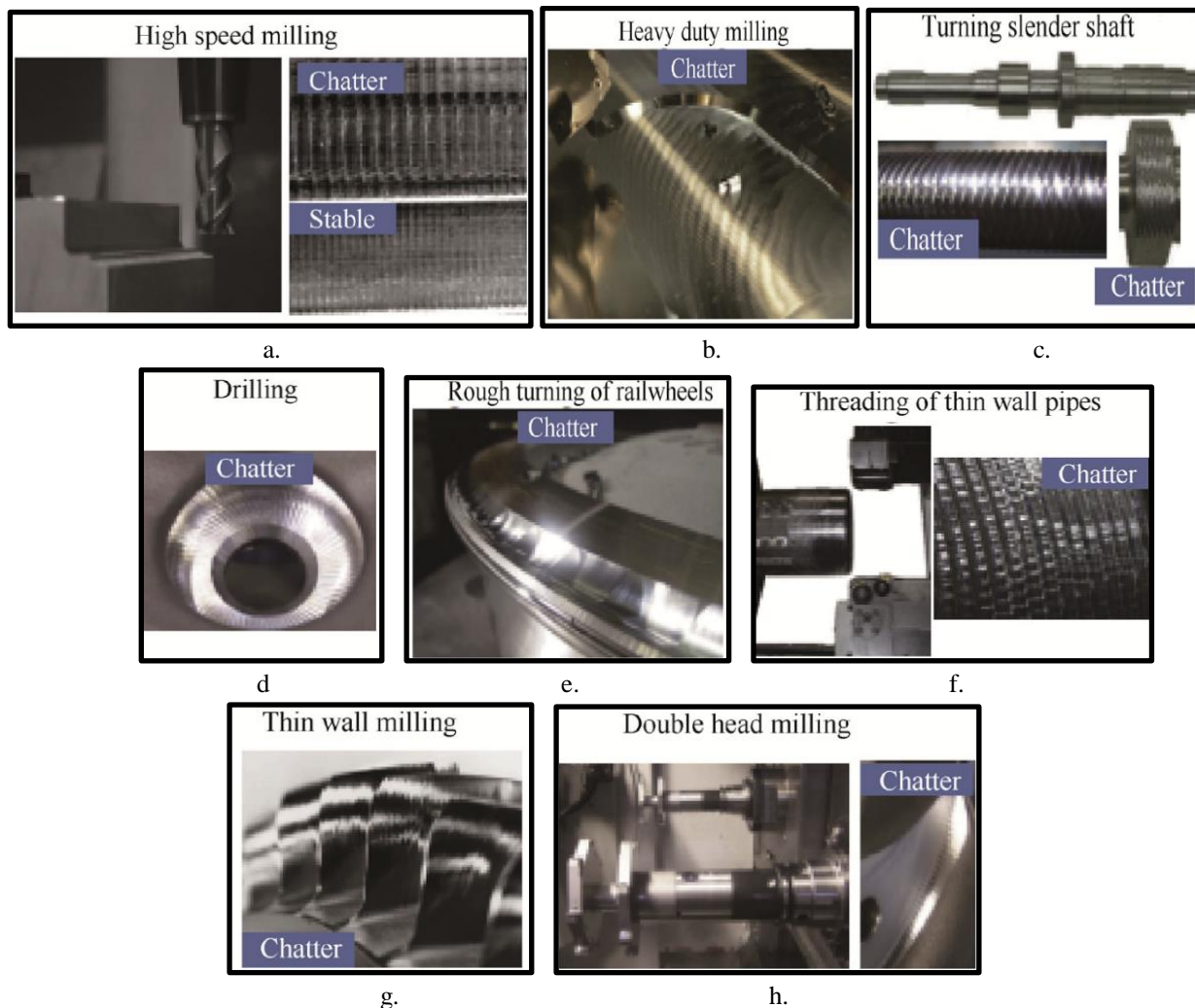


Figure 4. (a, b, c, d, e, f, g, h) Chatter problem in cutting processes.

Chatter during cutting is one of the most progressive issues in machining, among many other issues, due to self-excited vibrations that occur upon exiting specific dynamic modes [1, 2]. This mechanism adversely impacts the quality of the surface, tool life, and productivity [3]. Chatter Speeds As mentioned before, chatter is one of the prevalent issues we face in high-speed milling, resulting from the runnable cutting speeds that impose more

dynamic forces on basically every machine and reduce system damping at these rapid cut rates where any setup becomes prone to instabilities. In part, the high cutting depth and its associated forces cause milling vibrations, which would be difficult to avoid. However, this is common in heavy-duty milling, especially for machines with low structural rigidity. Such conditions might result in tool wear or failure and poor surface finish. It is strict with drilling operations, typically executed with long, flexible drill bits that can deflect and resonate, resulting in lousy hole quality and propagating tool failure.

On the contrary, slender shafts are very complicated to turn as their low structural rigidity allows deflection and vibration of long workpieces under machining cutting forces, affecting prompt dimensional accuracy. For example, the rough turning of rail wheels is an operation in which a large amount of material is removed; thus, it has high cutting forces and conditions, which lead to chatter loss of the roundness and surface quality degradation. Again, chatter is also a problem when making thin-walled pipe threads due to their low rigidity: bad threads and destroyed pipes.

Tools used for boring, such as flexible boring bars, are subject to chatter and deflection and often end up with a poor finish on the surface. With long, boring bars used for turning slender features and bars, flexibility is an even bigger issue. The low stiffness of the thin-walled part causes its severe deformation and vibration, which makes it more complex to conduct precise thin-wall milling. The simultaneous operations of both cutters in double-head milling results in misalignment tendencies and uneven force loading about the spindle, which promotes chatter. Adaptive control, vibration suppression, and optimal path planning can solve these problems more efficiently than traditional methods, which makes machining more efficient and cost-effective.

II. MECHANISM-BASED MODELLING OF CUTTING FORCES FOR WEAR PREDICTION

Cutting forces are crucial for thin-walled milling because they greatly influence workpiece deformation, surface accuracy, and stability [1]. Several researchers have performed studies to establish force models and their applications for the enhanced machining of such thin-walled parts. Ge M J et al. [2] explored the milling force model for titanium alloy thin-walled components in detail. They established a milling force model through experimental analysis, which explains the correlation between cutting forces and the deformation of titanium alloy workpieces in milling. Such a model supports improving machining precision, limited by thin-walled parts' material properties and structural flexibility. Similarly, Wu Kai et al. [3] investigated on deformation mechanisms of thin-web structures, in the end, milling. The research determined the underlying factors resulting in these machining deformations due to lack of rigidity and cutting forces. They suggested ways to lessen these deformations with optimum cutting parameters, support systems, and deformation compensations, hence contributing to increasing machining accuracy.

He Yongqiang and Cao Yan concentrated on building a cutting force model of thin-walled parts, with an emphasis on predicting and preventing the deformation occurring as the result of machining [12]. Here, they emphasized the impact of choosing the right cutting parameters to improve the part's stability and quality. Qiao F et al. [13] Modeling and experimental verification of a mechanical model for end milling thin-walled parts with workpiece deformation prediction capability. The results highlighted the value of this model as a tool to enhance prediction for thin-walled machining.

On the other hand, Zheng Jinxing utilized a mixed model that connected particle swarm optimization and artificial neural networks to predict the cutting forces in HSM [14]. When tested experimentally, this model could predict the force accurately and be a valuable tool for optimizing high-speed milling processes. Tsai J S and Liao C L adopted a finite-element model for static surface error investigation in peripheral milling, which shows the deformation mechanisms imposed by an insufficient stiffness of the workpiece [15]. They also suggested using the force model to optimize machining accuracy as part of practical implications.

Ratchev S proposed a parametric force model for low-rigidity thin-walled parts, considering the deformation change in real-time during milling. Such an adaptive model delivered enhanced stability and better accuracy for low-stiffness component machining. Moreover, M. Meshreki investigated the dynamic behavior of thin-walled aerospace structures for fixture design in multi-axis milling, which has established guidelines to improve the stability and accuracy of fixtures [10] due to its high-precision requirement for fixed aerospace parts during machining processes.

In summary, various modeling strategies for predicting the evolution of cut material deformations in thin-walled milling are addressed, and potential paths or possibilities toward more robust and effective control using force-sensing data are indicated.

III. METHODS FOR SOLVING STABILITY LOBE DIAGRAMS

The stability lobe diagram (SLD) is one of the most powerful tools for evaluating and predicting chatter in milling processes, first pioneered by Merritt. It is usually obtained by recourse to the solution of dynamic equations that describe the system's behavior, with methods falling into frequency-domain and time-domain categories. Frequency-domain-based methods that consider an analytical framework utilize stable boundary estimates as averages of cutting force coefficients with Fourier series expansion of the periodic matrix. There are zero-order and higher-order methods: the first allows us to get very quick (but also rough) stability estimates.

Feng J L et al. proposed an approach based on variable dynamic parameters to predict chatter stability in high-speed thin-walled parts made of titanium alloy. The model proposed in this paper includes dynamic features important for obtaining more accurate chatter predictions, which is critical to improving machining precision (looseness between the load and fixture, etc.) They researched process damping influences regarding chatter stability and optimized machining parameters from numerical predictions of the experimental model (Zhu L D et al. [19]). Based on the frequency and time domain chatter mechanisms examined, Altintas Y et al. [16], Stepan G et al., and Merdol D [17] developed predictive models that optimize milling parameters and improve stability.

Multi-frequency Approach to Chatter Prediction in Low Immersion Milling Merdol S D, Altintas Y - Mill chatter is often characterized by chatter limits at various frequencies. Tang A J and Liu Z Q created a three-dimensional stability lobe diagram to maximize material removal rates during machining by optimizing parameters while guaranteeing that tool vibrations (amplitude and frequency) were negligible during the end milling of thin-walled plates. Wang M H et al. proposed a regenerative chatter model of titanium alloy thin-walled part high-speed milling. They provided parameter optimization (i.e., stable process enhancement) prediction capability to support process stability [24].

An improved multi-frequency solution for the milling stability analysis of thin-walled parts enables the connection between cutting forces and workpiece vibrations Yan B L, Zhu L D, Zhang Z et al. Centre of interest for chatter mitigation in thin-walled milling, with some approaches such as optimized cutting parameters and improved support methods resulting in up to 90% reduction in the occurrence of chatter.

In this development, more advanced methods, such as the recent semi-discretization method by Insperger T and Stepan G, have improved computational efficiency in solving periodic delay-differential equations for stability analysis. Song Q H et al. also proposed a time-varying parameter model for predicting dynamic stability limits for high-speed milling simulation of thin-walled workpieces. They configured this to improve process parameters for enhanced machining efficiency and stability. Together, these methods wrap a comprehensive architecture for vibration chatter dynamics analysis and control, thus improving thin-walled milling accuracy and quality.

IV. STATE OF THE ART IN STABILITY PREDICTION

A Stability Lobe Diagram (SLD) is popularly recognized as an efficient way to predict stability in milling dynamics. While avoiding any structural changes on the machine tools or tool necks, the SLD could assist in identifying optimal cutting parameters. Fig 5: The SLD indicates the stable/unstable range of the cutting process for optimizing Material. This explains why chatter due to process damping is rare at low spindle speeds, and thus, SLD analysis becomes vital for proper high-speed milling optimization.

Specific input parameters are identified, which are necessary to define a consolidated and accurate SLD (for Example – Cutting force coefficients, system dynamics, process parameters, and tool geometry). Abstract: The cutting force coefficient describes a cutting tool-workpiece pair's material strength and friction mechanism, which is fundamental to establishing a relationship between cutting force and stability. It can be obtained mainly by orth: original cutting database with oblique cutting transformation and mechanical calibration via average and instantaneous force methods. The latter generally gives much higher fidelity and uses global nonlinear optimization paired with simulated and experimental force spectra.

Many research studies have reported that the cutting force coefficient varies concerning spindle speed, feed per tooth, and cutting depth, as shown by Campatelli et al. Budak et al. found a trend in which the cutting force coefficient first decreases and then increases at high speeds. (2004) and Grossi et al. proposed a more robust method based on smoothing the cutting force coefficient concerning spindle speed. Yue et al. combined a comprehensive dynamic milling force model with an improved semi-discretization method for stability prediction, resulting in higher precision predictions [2]. In this sense, dynamic modeling better resolves the time-varying nature of cutting force coefficients and provides more reliable chatter predictions and process optimizations than classical thin-walled milling work.

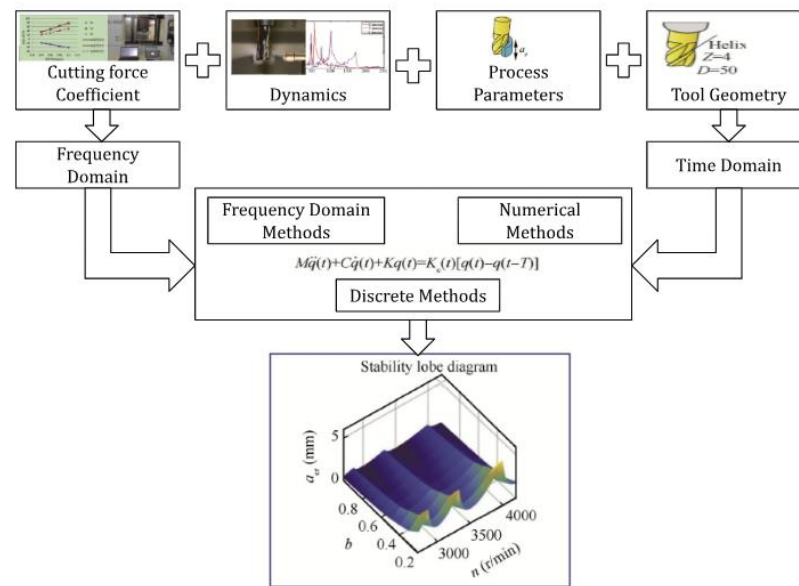


Figure 5. Procedure to obtain SLDs

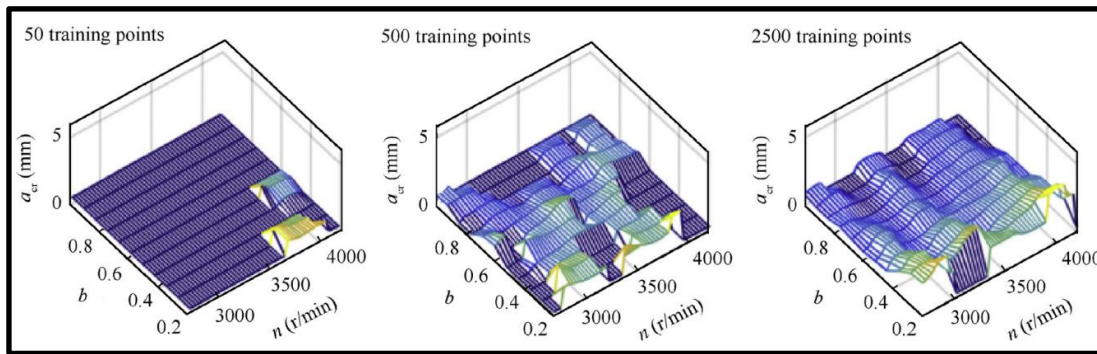
V. EXPERIMENTAL TECHNIQUES FOR CHATTER DETECTION

Generally, the three dynamic equations in the milling process are solved with a time-frequency domain approach to give rise to the Stability Lobe Diagram (SLD) [1]. In addition, it helps determine optimum feasible cutting parameters by considering minimum chatter and maximal surface finish and material removal rates. On the other hand, it is not appropriate to idealize any of these as the methodology demands a detailed description of dynamic processes and materials for parameters that are hard or impossible to determine in normal workshop conditions accurately. Moreover, machining is more complicated than turning due to different types of machine tools/workpieces and cutting tool combinations, leading to difficulty in accurately predicting stability.

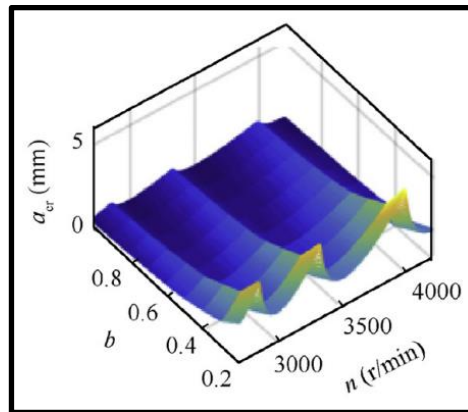
In this regard, some research works have focused on vibration signal acquisition and processing for real-time chatter detection and parameter regulation [9-11]. Some frequently used monitoring signals [7] are acceleration, cutting force, and acoustic. Multi-variable stability prediction through extended support vector machines and continual learning neural networks (MI-flush-RANEKF) has been demonstrated to identify the boundaries between stable and unstable-possibly-survivable flight conditions more clearly when using reliability measures.

Chatter detection is performed by point blanks (signal processing methods), e.g., By wavelet packet transforms [3], Ensemble Empirical Mode Decomposition (EEMD), and Aggregation Empirical Mode Decomposition (AEMD). Cao et al. To identify chatter, wavelet packet transform, and Hilbert-Huang analysis to reconstruct interested vibration components. Fu et al. applied Ensemble Empirical Mode Decomposition (EEMD) technology to divide the vibration signals into Intrinsic Mode Functions (IMFs) parts. After that, they measured IMFs using Hilbert spectral analysis in time–frequency. Ji et al. introduced AEMD-based power spectrum entropy and fractal dimensions features extraction for milling state prediction robustness increase.

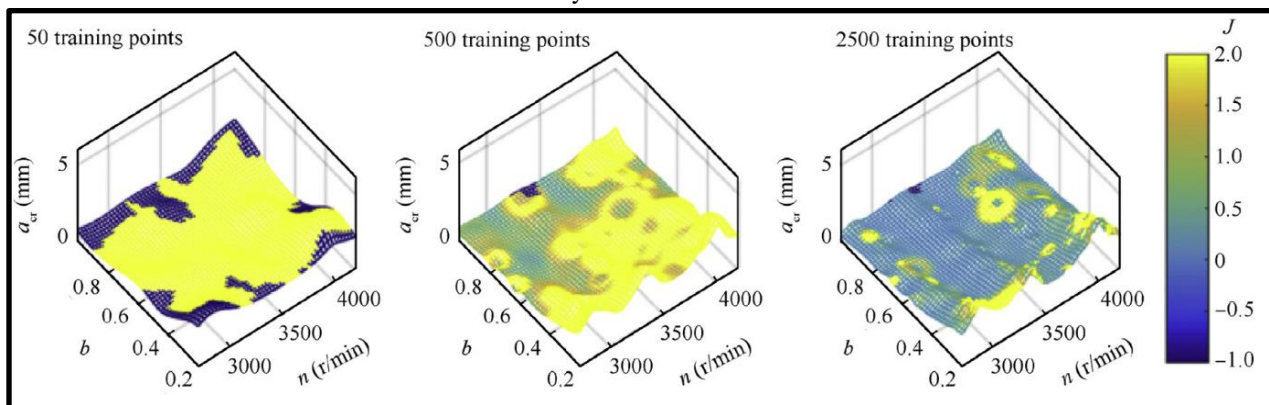
Recent years have seen an impressive improvement in the classification of stable and unstable cutting states for real-time chatter detection using advanced machine learning models, such as Radial Basis Functions (RBF), Multi-Layer Perceptron (MLP), or Convolutional Neural Networks (CNN). Progress in both theoretical and practical regions of vibration signal analysis could yield a systematic approach for real-time monitoring of thin-walled milling, refinement, and adaptation of existing principles to guide machining stability.



a. CSVC with MSLD



b. Analytical solution with MSLD



c. MI-flush-RANEKF with MSLD

Figure 6. (a, b, c) Multivariable stability lobe diagram (MSLD) of a milling process.

In natural, multi-tooth milling processes, the tool runout is an angle that results in a deviation of the axis of spindle rotation from the geometric axis of the tool and thus induces change to the actual cutting radius. This misalignment, often due to slight variations introduced during manufacturing, can create uneven forces along the cutting edge. It is then subsequent that the vibration frequency fluctuates using both larva sound variety and spindle pivot frequency, influencing the machining process steadiness.

The influence of tool runout on chatter dynamics has been documented [1], although runout usually does not change the global stability boundary [2]. Integrated eccentricity into the direction coefficient, discovering that even if tool runout influences the nature of chatter frequency, it does not alter stability boundaries. Otto et al. studied the influence of cutter runout in both slot milling and 25% axial immersion milling process with conventional and variable-helix tools. While their findings showed little impact on the stability of traditional tools, they revealed significant effects when variable-pitch end mills were tested with low axial immersion depths.

Wan et al.'s research developed a multi-delay dynamic model that includes cutter runout to study the impacts of radial immersion, feed direction, feed per tooth, and helix angle. The study also revealed that reduced radial depths of cut, feeds per tooth, and helix angles lead to less stable vibration motion. However, this effect decreases with an increase in these parameters.

Zhang et al. Tool path trochoids based varying-time delay model considering tool runout. The simulated results correlate closely with the experimental data, and the researchers concluded that tool runout could improve stability during milling under some conditions. Ma et al. verified that local runout of a tool could isolate stable region expansion, most notably at the lower end of feasible feed rates. This implies that the helical angle might play a role in stability under normal circumstances, but its effects become relatively small when tool runout and feed rate are accounted for.

In summary, thin-walled milling processes are highly dynamic, and runout can drastically alter cutting dynamics. Tool runout is a critical damping setup to increase the stable cutting region. Proper consideration during the analysis will only promote better chatter mitigation strategies

VI. EFFECT OF TOOL RUNOUT ON STABILITY LOBE DIAGRAMS (SLDs)

In the actual multi-tooth milling processes, the tool runout is a deviation of interaction between the axis of rotation of the spindle drive tool's geometric axis, influencing the tool, which influences the practical cutoff radius. This deviation, attributed to minor manufacturing imperfections, results in an uneven force distribution along the cutting edge. This leads to the difference between tooth passing frequency and spindle rotation frequency being changeable, influencing the stability of the machining process.

Although it has been shown in several studies that runout affects the chatter dynamics, it generally will not change the essential stability boundary. For example, the non-conventionality in the bearing area force coefficient (e.g., tool runout chatter frequency) does influence the stability limit, while upload shoe support is negligible [2]. Otto et al. An analysis of cutter runout under various milling conditions (slotting and 25% axial immersion) with conventional and variable-helix tools was performed [13]. They found that in the case of traditional tools, runout does not have a significant effect on stability; however, with variable-pitch end mills at low axial immersion depths, this can be highly affected by runout.

This led Wan et al. to do more research. Proposed a dynamic model based on multi-delay, which includes the cutter runout and has been studied to assess the effects of (i) radial immersion, (ii) feed direction, (iii) radial depth of cut, and (iv) helix angle. This study found that reducing radial depths of cut or feeds per tooth or relaxing helix angles reduces stability, but that effect falls off with increased parameter values.

Zhang et al. developed a time-varying delay model, including tool runout with trochoidal engagement paths. They found that, for some conditions where the runout occurs over a considerable distance, it can improve stability during milling, with predicted results from simulations closely matching those observed experimentally. Ma et al. also suggested that runout of tool-localized, more robust stable zones can lead to locally expanded stable zones, particularly at low feed rates. However, this implies that even though helical angle could affect stability under normal circumstances, it has a lesser contribution towards stability when tool runout and feed are included.

In summary, including tool runout in milling analysis is essential for accurately predicting cutting dynamics because it may change the shape of the thin-walled milling process by expanding the stable cutting area and helping with chatter mitigation strategies.

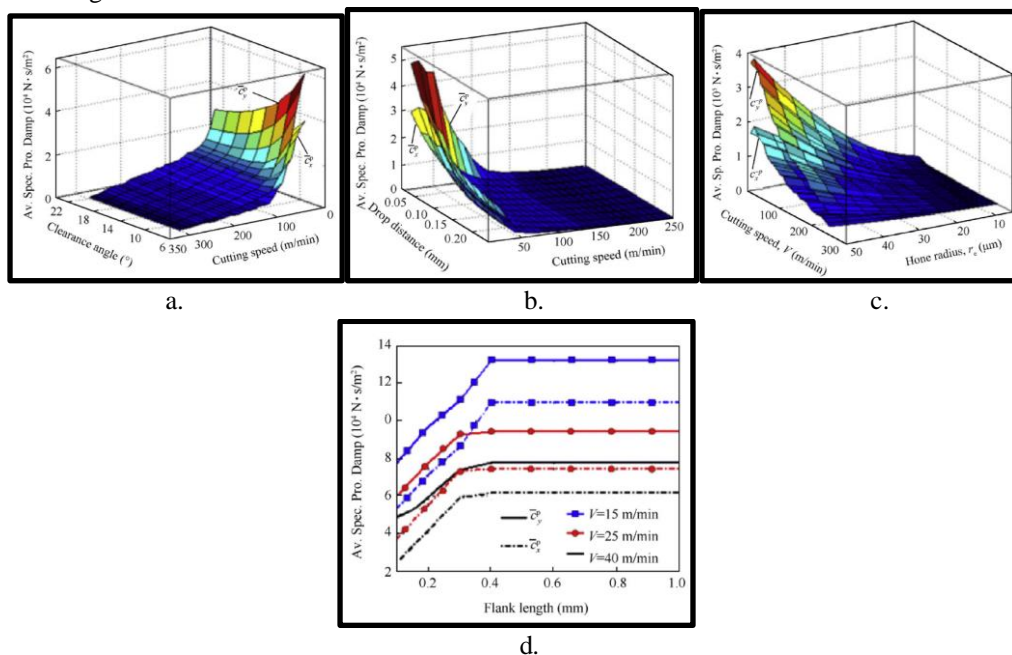


Figure 7. (a, b, c, d) Effects of tool parameters on process damping in milling.

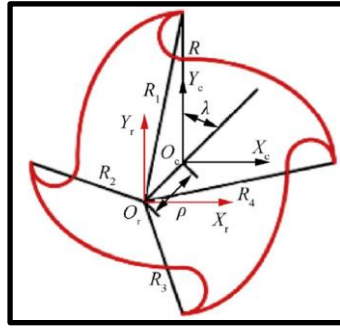


Figure 8. Tool geometry with cutter runout

VII. DISCUSSION

Time-Domain Analysis of Chatter Dynamics and Forced Vibration in Thin-Walled Milling Over the last 2 decades, there has been an emerging research effort towards developing time-domain analysis for chatter dynamics and forced vibration in thin-walled milling driven by development such as continuous-time models that will further improve behavior evaluation of stability lobes at manufacturing conditions. Dynamics, including tool runout and cutting forces fundamentals, are critical in chatter to accurately incorporate the constant force coefficients and machining parameters associated with grinding into a cutting process model, leading to a better basis for modeling the physics of chatter [1]. Despite the promise shown by conventional approaches based on dynamic force models or adaptive chatter detection techniques, there are still limitations for deployment in industrial practice where either the tool condition can change as parts are being produced or the material properties can be uncertain.

Even though these models may represent simplified physics of the machining process, there is a demand for more generalizable models, which could vary at each instant machining cycle depending on minute environment conditions. Integrating advanced signal processing with machine learning algorithms and real-time monitoring systems could offer improved predictions and chatter suppression. Future work that incorporates hybrid models using time-domain and frequency-domain analysis to detect more complex dynamic behaviors is encouraged. In addition, more emphasis should be placed on experimental validation and tool development for ease of implementation in industry practice. It provides a more efficient and precise enhancement to the thin-walled milling process, signaling a shot across the bow for manufacturers to shift their processes to a more sustainable and flexible method.

VIII. Conclusion

The present work is a comprehensive review that summarizes the recent and promising advances in time-domain approaches for chatter dynamics and forced vibration analysis of thin-walled milling processes. Due to the reliability of chatter prediction and stability analysis, time-domain methods that can model complex, nonlinear behaviors and account for various dynamic factors, including tool runout, cutting force fluctuations, and time-varying material properties, are imperative. Advances in signal processing, adaptive control, and real-time monitoring have progressively allowed just-in-time detection of chatter together with compensation actions to enhance sophistication while minimizing the time domain over which performance degradation occurs, consequently enhancing machining performance and surface quality.

However, industrial applications still face troubles as there is no predictability in the tool conditions, workpiece dynamics, or environmental factors that affect the stability of the milling process. Current models fail to incorporate these differences, highlighting the need for a more flexible and adaptive approach. Integrating time-domain methods with machine learning algorithms and hybrid modeling techniques can provide a future roadmap for assessing real-time chatter prediction and control.

In height, the evolution of time domain analysis continues to promise potential for improvements in operational safety and efficiency as process motivation and metrics become more reliant on monitoring data [15], leading pathways towards assuring the stable operation of thin-walled milling processes alongside both the prediction more generally applied in manufacturing environments, improved productivity and diminished tool wear.

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