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[1]Deepti Khimani
[2]Rakesh Barai
[3]Machhindranath Patil

Practical Implementation of Sliding Mode and Super-twisting Control for Flow Control System

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Abstract: - This work investigates robust control designs for one of the crucial controlled variables in process industries, the flow control system. Two widely-used control designs, sliding mode control (SMC) and super-twisting control (STC), are applied to regulate a flowrate in a laboratory experimental setup with an objective of setpoint tracking. The aim is to evaluate the performance of SMC with its known limitation of chattering, and assess whether the implementation of second-order sliding mode control (STC) mitigates this issue. The results highlight significant chattering in the process output during the SMC implementation, while STC effectively reduces these oscillations, achieving more stable setpoint tracking. for current disturbance mitigation. Simulations of MATLAB/SIMULINK environment of the present work shows the efficacy.

Keywords: Sliding mode control (SMC), Super-twisting control (STC), Flow control system, Robust Control.

I. INTRODUCTION

In process industries, nearly all processes are controlled using traditional approaches and often by PID control [1], [2]. These systems assume zero disturbance. Traditional state feedback based controls are always designed using a model of the plant in order to achieve the desired response specifications. However, in practice, the performance of the system is always degraded due to some mismatch between the plant and its model and the disturbances. Numerous modern control theories dealing with disturbances have been found in literature [3], [4], [5]. These modern control techniques involve complex mathematical design procedures. However, sliding mode control and higher order sliding mode control are relatively simpler to design but ideally, they can completely eliminate the disturbance.

Since the 1950s, sliding mode control (SMC) has attracted researchers for its ability to make systems robust against the disturbances that disturbs the system through input channel [6], [7], [8]. Designing an SMC involves two key steps: design of a sliding surface which governs the desired system behavior and design of a control law which enforces the trajectory of the system onto the sliding surface, where it becomes insensitive to disturbances. The tracking can be achieved using SMC by transforming the system into a stabilization of error between the trajectory of the system and the desired reference trajectory. The control law then aims to drive this error to zero, effectively making the system track the set point.

In theory, achieving perfect sliding mode control is possible because the switching frequency of the control signal can be assumed infinitely high. However, real-world actuators, like those based on electromechanical or electro-pneumatic principles, have limitations in responding to very high frequencies. This practical limitation prevents perfect implementation of the ideal SMC. To overcome this problem, the higher order sliding mode control can be used in practical applications [9].

To eliminate the chattering effect, the super-twisting control (STC) can be implemented. It is second order sliding mode control and preserves the robustness of first-order sliding mode control (SMC) for the matched uncertainties while being almost chattering-free [10], [11].

This paper discusses SMC design and higher-order SMC, simulation, and implementation on the laboratory experimental setup. The paper is organized as follows: Section 2 presents the dynamics of the flow control system along with details of the experimental setup. Section 3 outlines the procedure for system identification. In Section 4, a formulation of the control problem in error coordinates is presented. Section 5 and 6 provides

[1] Department of Robotics and Automation, V.E.S. Institute of Technology, Mumbai, India.

[2] Occipital Technologies Pvt. Ltd. Sector 08, Nerul, Navi Mumbai, rakesh@agrograde.com

[3] *Corresponding author: Department of Robotics and Automation, V.E.S. Institute of Technology, Mumbai, India.

a discussion on the design and implementation of both SMC and STC, respectively, for the flow control system; thereafter, a conclusion follows.

II. Flow Control System

A. Dynamics of a Flow Control System

The flow control system is typically regarded as the faster process among various process variables such as temperature, level, pressure, etc. The inherent delay in the process is minimal, and consequently, the response time of the entire process control system is primarily influenced by the dynamic behavior of the measurement system, control valve, and transmission lines that comprise the entire system. The models of different components within the flow control system are outlined below.

A dynamical behavior of the fluid flow can be modeled by considering variations in flowrate caused by bends, joints, pipe length and the flow transducer. Despite the very small process response time, it can be modeled as first order process with a smaller time constant.

$$G_f(s) = \frac{k_f}{\tau_f s + 1} \tag{1}$$

Where, τ_f is time constant and k_f is steady-state gain of the system.

The control valve is the next control system component that manipulates the process conditions. Being an electro-pneumatic component, its response time to the control signal is comparatively higher than that of other components in the control loop. A valve considered in this study is a equal percentage control valve, allowing it to be modeled as a first-order first order linear model in the given operating range as follows,

$$G_v(s) = \frac{k_v}{\tau_v s + 1} \tag{2}$$

Where, τ_v is valve time constant and k_v is gain of the valve.

So effectively, for a flow control system, a plant's dynamic can be given by,

$$G_p(s) = \frac{k_p}{(\tau_f s + 1)(\tau_v s + 1)} \tag{3}$$

Clearly, the poles of the system are real and negative, so it is expected to be the overdamped system or with damping factor close to unity.

B. An Experimental Setup and Data Acquisition

The flow loop process control begins with a setup involving a Data Acquisition (DAQ) system, where signals flow bidirectionally between a computer (acting as a controller) and the process. The system includes following major components.

1) Sensor: A magnetic flow meter measures the outlet flow rate, outputting a *4-20 mA* signal proportional to the flow rate. In this experimental setup, the measurement of flowrate is conducted using a Rosemount transmitter equipped with a magnetic flow meter FT06 model, capable of measuring within the range of *0* to *1000 LPH* (liters per hour). The magnetic flow meter's *4-20 mA* analog signal is converted into a digital signal for interfacing with the controller. This conversion occurs in two stages, an Analog Signal Conversion and Digital Signal Conversion.

2) Analog Signal Conversion: A current-to-voltage (I-to-V) converter converts the *4-20 mA* signal to *0-5 VDC*. The I-to-V converter used in setup ensures signal isolation, noise immunity, and protection against ground loops.

3) Digital Signal Conversion: An Arduino, serving as the core of the DAQ system, converts the *0-5 VDC* analog signal to a 10-bit digital signal (*0-1024* range). Scilab reads this digitized flow rate data via the serial port for further processing.

4) *Control Valve:* A control valve regulates the inlet flow rate. The control valve employed for throttling the fluid flow in the setup is an equal percentage valve with a valve coefficient (CV) of *3.2*.

To control the inlet flow rate, the system commands the valve position using Scilab. The control signal (in the *0-255* range) is transmitted to the Arduino via serial communication, where it is converted into a Pulse Width Modulation (PWM) signal. The PWM signal, proportional to the desired valve position, generates a *0-5 VDC* analog signal and converted to *4-20 mA*. Since the valve actuator operates on a *3-15 psi* input, a current-to-pressure (I-to-P) converter is used for the final stage of signal conversion. Thus, setup enables the control of the inlet flow rate.

The experimental setup of the flow control system is shown in Fig.1 and the functional block diagram is shown in Fig.2 The magnetic flow meter is responsible for accurately measuring the flow rate, while the Microcontroller board, along with the converters, performs the data acquisition and communicate it to PC. The designed control law, implemented in Scilab, adjusts the control signal through Microcontroller board to ensure effective control over the flow dynamics.

Fig. 1 Experimental Setup of Flow Control System.

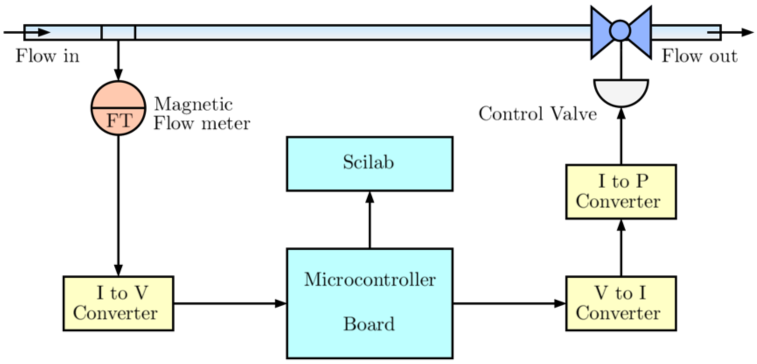

Fig. 2 Functional block diagram of the Flow Control System.

III. System Identification

Modeling the flow loop involves system identification to construct a mathematical model of the process. The step-response technique is employed, where a step change in the process input (valve position) is introduced, and the output response (flow rate) is observed until steady state. Since the system is linear time-invariant (LTI) and intended for setpoint tracking, step response data is deemed appropriate for system identification.

Starting with the flowrate at an arbitrary steady-state value of approximately 684 LPH, the control valve is closed to bring the flow rate to zero. After about 42 sec, the control valve is adjusted to 10%, resulting in a steady-state flow rate between 261 and 267 LPH. The data is collected using an Arduino Microcontroller board interfaced to Scilab on the PC.

The transfer function, using the system Identification toolbox in MATLAB with the collected input-output data is obtained as follows.

$$G(s) = \frac{2.1332}{s^2 + 0.5307s + 0.08025} \tag{4}$$

Selecting the state variables, $x_1 = y$ and $x_2 = \dot{y}$, the state space representation of the flow control system can be given by,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.08025x_1 - 0.5307x_2 + 2.133u \\ y &= x_1 \end{aligned} \tag{5}$$

The step response obtained from the experimental data and the corresponding model simulation performed in MATLAB is shown in Fig. 3(a) and, the input signal, represented by the control valve opening (%), is shown in Fig. 3(b).

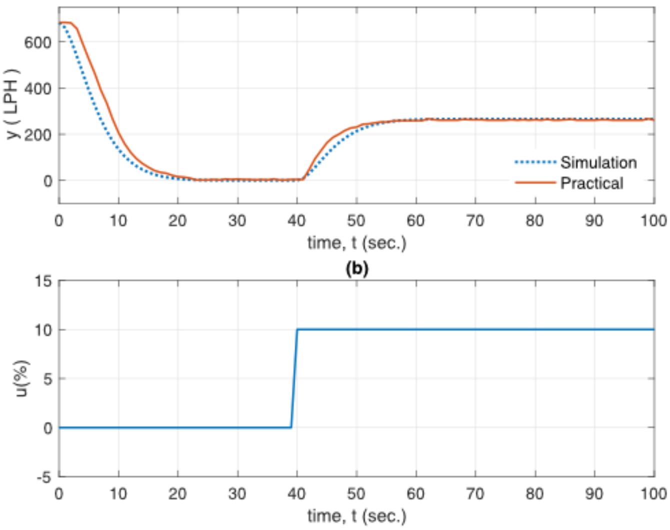

Fig. 3 (a) Open-loop step response of an experimental setup. (b) Step input u=10% valve opening.

Since in a flow control application, disturbances are typically observed as fluctuations in flow, often originating from pumps or compressors. These disturbances can also be classified as a matched disturbance since it originates from the input line. So, the plant model with such matched uncertainties can be written as,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.0802x_1 - 0.5307x_2 + 2.133u + 2.133\phi(t) \\ y &= x_1 \end{aligned} \tag{6}$$

Where, ϕ is a matched disturbance with $max\ \phi < \ell$. For the given experimental setup disturbances due to turbulence is assumed to be negligible as compare to the span of for the given range. So instead of neglecting it, we have considered the maximum disturbance input of *6%* of nominal control which is approximately, $\ell = 1.5$.

Remark 1: Note that, the MATLAB is used offline only for system identification and simulation, while inside the loop Scilab is used online to take control action.

IV. A SYSTEM REPRESENTATION IN ERROR COORDINATES

Consider a system in a regular form,

$$\dot{\mathbf{x}}_1(t) = A_{11}\mathbf{x}_1(t) + A_{12}x_2(t) \tag{7a}$$

$$\dot{x}_2(t) = A_{21}\mathbf{x}_1(t) + A_{22}x_2 + B_2u(t) + B_2\phi(t) \tag{7b}$$

Where, $\boldsymbol{x_1} \in \mathbb{R}^{n-1}$ and $x_2 \in \mathbb{R}$ denote the states of the system. Additionally, $u \in \mathbb{R}$ represents the input, and $\phi \in \mathbb{R}$ signifies a bounded disturbance, with $max\ \phi < \ell$. Let the output of the system be represented as, $y = C$ x with the state vector $x = (\boldsymbol{x_1^T}, x_2)^{\boldsymbol{T}}$.

Note that, the flow control system represented in (6) is already in regular form. Identifying the subsystem matrices as,

$$\begin{aligned} &A_{11} = 0,\ A_{12} = 1,\ A_{21} = -0.0802 \\ &A_{22} = -0.5307 \ \text{ and } \ B_2 = 2.133 \end{aligned} \tag{8}$$

SMC is the robust control technique for asymptotic stability, which ensures convergence of trajectory to the origin. However, in flow control system it is required to drive the output to the set point. In order to design the control for set point tracking, it is required to represent the system in error coordinates.

Suppose that, $x^* = (\mathbf{x_1^*}, x_2^*)$ represents the reference trajectory that satisfies the system (7), resulting in the desired output (set point), $y^* = C$ x$^* = R$ using the control u^*.

Define, $e_1 = \boldsymbol{x_1} - \boldsymbol{x_1}^*$, $e_2 = x_2 - x_2{}^*$ and $\Delta u = u - u^*$, then the system (7) in error coordinates can be given by,

$$\dot{\mathbf{e}}_1(t) = A_{11}\mathbf{e}_1(t) + A_{12}e_2(t) \tag{9a}$$

$$\dot{e}_2(t) = A_{21}\mathbf{e}_1(t) + A_{22}e_2 + B_2\Delta u(t) + B_2\phi(t) \tag{9b}$$

Clearly, an objective here is to design the control Δu such that, $\|e\| \to 0$ as $t \to \infty$. This in turn implies, $\|x - x^*\| \to 0$.

Remark 2: As the tracking signal is a set point, which is a constant, so the x^* can be considered to be the steady state values of the trajectory with the control u^*. This implies, $y = Cx^*$, which is the desired output R.

V. SMC DESIGN FOR FLOW CONTROL SYSTEM

SMC design involves two key steps, designing the sliding surface and formulating the control law that drives the trajectory to the surface and maintains it there until it converges to the origin.

First, consider the stabilization of the subsystem (9a). Note that, a pair (A_{11}, A_{12}) is controllable as the system is controllable. If e_2 is considered as an input, then (9a) can be made stable using some state feedback gain σ_1.

By selecting $e_2 = -\sigma_1\ \boldsymbol{e_1}$, the dynamics of the subsystem (9a) can be written as,

$$\dot{\mathbf{e}}_1(t) = (A_{11} - A_{12}\sigma_1)\mathbf{e}_1(t) \tag{10}$$

In other words, if we restrict the trajectory on the manifold $e_2 = -\sigma_1 \boldsymbol{e_1}$, then $\|\boldsymbol{e_1}\| \to 0$ as $t \to \infty$. This further implies, $\|e_2\| \to 0$ as $t \to \infty$. This asymptotically stable convergence of trajectory $(\boldsymbol{e_1}, e_2)$ to the origin is said to be the sliding motion on the sliding surface. Such a sliding surface can be given by,

$$s(t) = \sigma^\top e(t) = \begin{bmatrix}\sigma_1 & 1\end{bmatrix} e(t) = 0 \tag{11}$$

For the Flow control system, we have, $A_{11} = 0,\ A_{12} = 1$. So, we select, $\sigma_1 = 1$, so that the eigenvalue of the $A_{11} - A_{12}\sigma_1$ can be placed at *-1* for the asymptotically stable sliding motion of the subsystem (10).

The second step in the SMC technique is the design of the control that converges the trajectory on the sliding surface in finite time and enforces the sliding motion. One of the popular ways to design the SMC is the design based on constant-plus-proportional reaching law. It is typically given by,

$$\dot{s} = -ks - Q\ \mathbf{sgn}(s) \tag{12}$$

Where Q is the positive constant, which should be greater than the maximum absolute value of the disturbance and, $k > 0$ is the constant that is selected as per desired rate of convergence of trajectory towards the sliding surface $s(t)=0$ during reaching phase.

Follow from (11), we can write (12) as,

$$\sigma^\top \dot{e} = -ks - Q\ \mathbf{sgn}(s) \tag{13}$$

Define,

$$A := \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{12} \end{pmatrix},\ B := \begin{pmatrix} 0 \\ B_2 \end{pmatrix}$$

So, from (13), the control law can be given by,

$$\Delta u = -B_2^{-1}\big(\sigma^\top A + ks + Q\ \mathbf{sgn}(s)\big) \tag{14}$$

In the flow control experimentation, $\ell = 1.5$, therefore, $Q=2$ is selected that satisfies the inequality with upper bound on the disturbance. The other tuning parameter, $k=0.5$ is selected arbitrarily. Thus, the sliding mode control for the flow control system can be given by,

$$\Delta u = -0.4688\big(-0.0802 e_1 + 0.4693 e_2 + 0.5s + 2\mathbf{sgn}(s)\big) \tag{15}$$

As $u = \Delta u + u^*$, so to achieve the tracking of the set point of *325 LPH*, it is required to compute the u^*, As the system is open loop stable, and it is forming the chain of integrators, clearly the steady state trajectory can be given by $x^* = [325, 0]^T$. This implies, $u^* = -0.0802\ x_1^* + 0.4693 x_2^* = -26.0809$.

$$u = -\ 0.4688\big(-0.0802 e_1 + 0.4693 e_2 + 0.5s + 2\mathbf{sgn}(s)\big) - 26.0809 \tag{16}$$

Fig. 4 shows the response of the system for the set point of *325 LPH* which can be achieved with the control (16) in about *10 sec*. Fig. 5(b) shows the SMC expressed as percentage opening of valve while Fig. 5(a) is sliding variable, which undergoes sliding motion at about *3 sec*.

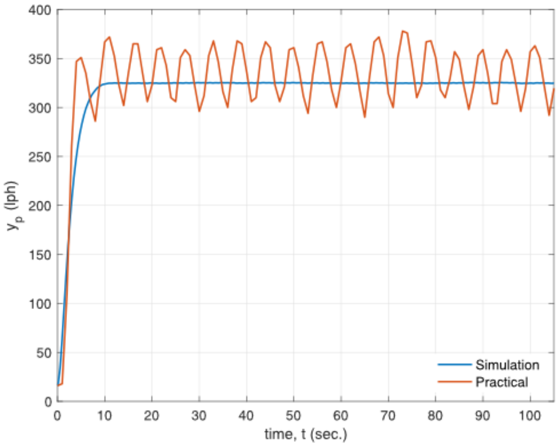

Fig. 4 Response of the flow control experimental setup and simulation using the SMC designed in (16)

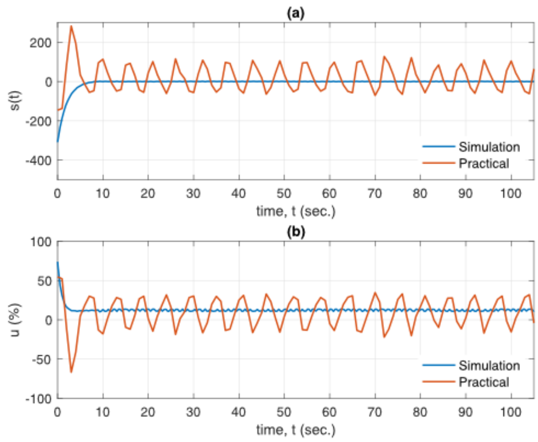

Fig. 5 (a) Sliding variable as designed in (11). (b) Reaching law based SMC as in (16)

VI. Super-twisting Control for Flow Control System

It is evident from the Fig. 4 and Fig. 5 that SMC has a limitation of chattering of an actuator. This is due to the fact that SMC is discontinuous control, however, ideally this discontinuous nature due to sign (*sgn*) function only makes it robust control technique. To overcome the problem of chattering, the second or higher order sliding mode control can be used. One of the popular second order SMC is super-twisting control (STC). Typically, the Super-twisting algorithm is given by,

$$\dot{s} = -k_1|s|^{\frac{1}{2}}\ \mathrm{sgn}(s) - \int_0^t k_2\,\mathrm{sgn}(s)dt + \phi(t) \tag{17}$$

Where,

$$k_1 > 1.5\sqrt{\ell}, \quad \text{and } k_1 > 1.1\ell \tag{18}$$

for the given bound $max\ \phi < \ell$. If these inequalities are satisfied then, the trajectory $s(t)$ and also, $\dot{s}(t)$ converges to origin in finite time. See [10], [11] for proof.

Clearly, if the control is designed such that (17) is satisfied then, the trajectory of the system can be driven to the origin. To obtain the control law, differentiate the sliding surface (110 w.r.t. time *t*, we get,

$$\begin{aligned}\dot{s} &= \sigma_1 \dot{\mathbf{e}}_1 + \dot{e}_2 \\ &= \sigma_1(A_{11}\mathbf{e}_1 + A_{12}e_2) + A_{21}\mathbf{e}_1 + A_{22}e_2 \\ &\quad + B_2\Delta u + B_2\phi(t) \\ &= \sigma^\top e + B_2\big(\Delta u + \phi(t)\big)\end{aligned} \tag{19}$$

Comparing (17) and (19), the super-twisting control (STC) can be obtained as,

$$\Delta u = -B_2^{-1}\left(\sigma^\top e + k_1|s|^{\frac{1}{2}} \ \mathbf{sgn}(s) + \int_0^t k_2 \ \mathbf{sgn}(s)dt\right) \tag{20}$$

Note that, the STC in (20) is looks like a continuous unlike SMC as given in (14) yet it may exhibit the chattering because of sign function. However, the chattering in the STC is significantly lesser as compared to the SMC.

For the flow control system with,

$$A_{11} = 0, \ \ A_{12} = 1, \ \ A_{21} = -0.0802$$

$$A_{22} = -0.5307 \ \ \mathbf{and} \ \ B_2 = 2.133$$

the surface designed $\sigma_1 = 0.2$ that place the eigenvalue of $A_{11} - \sigma_1 A_{12}$ at -0.2. For the STC, parameters that satisfies the bound in (18) are chosen to be,

$$k_1 = 1.4 \ \ \mathbf{and} \ k_2 = 1.8 \tag{21}$$

So, the STC for the flow control system can be given by,

$$\Delta u = -0.4688\left(-0.0802\mathbf{e}_1 - 0.033e_2 + 1.4|s|^{\frac{1}{2}} \ \mathbf{sgn}(s) + \int_0^t 1.8 \ \mathbf{sgn}(s)dt\right) \tag{22}$$

As $u = \Delta u + u^*$, where $u^* = -24.0747$ for the steady state trajectory $x^* = [300, 0]^T$. Thus, the STC for the flow control application is given by,

$$\Delta u = -0.4688\left(-0.0802\mathbf{e}_1 - 0.033e_2 + 1.4|s|^{\frac{1}{2}} \ \mathbf{sgn}(s) + \int_0^t 1.8 \ \mathbf{sgn}(s)dt\right) - 24.0 \tag{23}$$

Fig. 6 shows the response of the system for the set point of *300 LPH* which can be achieved with the STC as given in (23) in about *10 sec*. Fig. 7(b) shows the STC expressed as percentage opening of valve while Fig. 7(a) is sliding variable, which undergoes sliding motion at about *11 sec*.

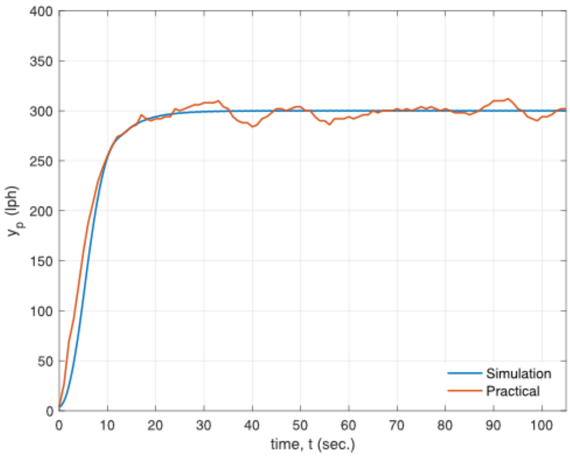

Fig. 6 Response of the flow control experimental setup and simulation using the STC as designed in (23)

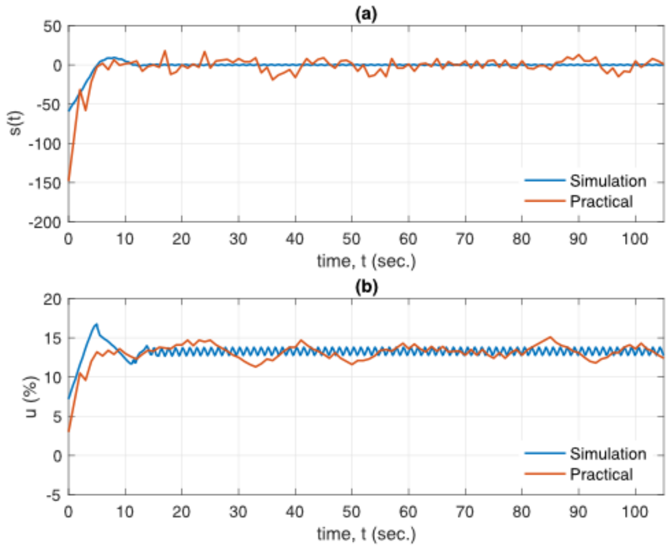

Fig. 7 (a) sliding variable s(t). (b) STC as in (23).

VII. CONCLUSION

The study demonstrates the effectiveness of robust control designs in addressing the challenges of flowrate regulation in process industries. Sliding mode control (SMC) was observed to achieve rapid setpoint tracking but exhibited significant chattering. This limitation can impact system stability and lifespan of the actuator. On the other hand, the implementation of Super-twisting control (STC) not only achieved stable setpoint tracking but also effectively circumvents the chattering issue by ensuring relatively better control actions with significantly reduced oscillations. The comparative results clearly indicate the advantages of second-order sliding mode control in enhancing control performance while reducing undesired oscillations.

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