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Sixth Order Butcher 'S RK Methods in MAGDM Using Intuitionistic Triangular Fuzzy Sets

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Abstract: - MAGDM is the one of the best choice out of alternative solutions. The Data set is adopted from the Intuitionistic Triangular Fuzzy Number Matrices. Some order of Runge Kutta methods is used to calculate the weights. We applied the solutions of Weighted Geometric operator and Hybrid Geometric operator by using decision making. New Extended Normalized Hamming Distance Formula is exerted for sort the alternatives. This paper provides a numerical illustration of elasticity and efficiency.

Keywords: Intuitionistic Triangular Fuzzy Set, Weighted Geometric operator, Hybrid Geometric operator, Butcher's Sixth Order Runge Kutta Method.

I. INTRODUCTION

Multiple attribute group decision making problems are frequently encountered in everyday situations. Finding a good answer from a limited number of variable alternatives that are evaluated on several quantitative and qualitative criteria is known as a MAGDM problem. Numerous devices and systems, including facial pattern recognition, air conditioning, washing machines, hoover cleaners, antiskid braking systems, gearbox systems, subway system control and more, use fuzzy logic. The notion of Intuitionistic Fuzzy sets and Operators over interval-valued intuitionistic fuzzy sets was first presented in fuzzy set theory by Atanassov K (1986). He has introduced the concept of Intuitionistic Fuzzy sets and characterized by a membership and non-membership function which is a generalization of the concept of fuzzy set [4]. Li, Chan and Zhao, Shup and Zhang (2014) presented Aggregation operators on Triangular Intuitionistic Fuzzy numbers and its application [14]. Alcantud, Khameneh & Kilicman (2020) [2] provided Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information. In order to clarify the numerical methods and their applications, Arumugham [3] presented the implementation of a numerical method with an appropriate convergence. lyengar and Jain's are defined Numerical Solution of Differential Equations Problems [9]. Robinson &Amirtharaj [20] calculated the correlation coefficient of Triangular Fuzzy Set. In the context of averaging operator, Wei, Zhaao, Lin, and Wang [29] have interesting research on Multiple Attributes Group Decision Making. Robinson & Akila [21] presented a number of methods for determining the weight of an attribute in MAGDM. A generalization of the fourth order method is the Runge Kutta Method. Butcher developed the Sixth Order Runge Kutta Method to solve numerical approaches. Numerical illustration Robinson and Akila. (2019) Attribute weight determination in MAGDM Problems using some Numerical Method Techniques [21]. A new method of combining numerous numerical values into a single value is to be acknowledged aggregation operators. Numerous applications, including unmanned helicopters, knowledge-based systems for multi objective optimization of power systems, weather forecasting systems, models for project risk assessment or new product pricing, medical diagnosis and treatment plans, and stock trading, have made use of fuzzy logic. In this study, weights of intuitionistic triangular fuzzy sets based on MAGDM problems were determined using various Runge Kutta methods. Prior to choosing or ranking options, it is frequently necessary to evaluate multiple attribute decision making simultaneously. Some numerical examples of the Intuitionistic Triangular Fuzzy Weighted Geometric (ITrFWG) and Intuitionistic Triangular Fuzzy Hybrid Geometric operators (ITrFHG) under the Intuitionistic Triangular Fuzzy Number utilised in this paper are provided below. The process for determining decisions using the ranking function for ITrFNs is also illustrated by some aggregation operators. A numerical example that explains the created decision-making model is given.

II. PRELIMINARIES

Definition: Intuitionistic Fuzzy Number (IFN)

An IFS R in Z is stated by $R = \{\langle z, \mu_P(z), \gamma_P(z)\rangle / z \in \mathrm{Z}\}$, here $\mu_R : \mathrm{Z} \to [0,1]$, $\gamma_R : \mathrm{Z} \to [0,1]$, $0 \leq \mu_R(z) + \gamma_R(z) \leq 1, \forall\, z \in \mathrm{Z}$. The $\mu_R(z)$ and $\gamma_R(z)$ represent the membership and non-membership degree of the element z to the set R properly. Let $\mathrm{Z} = \{z_1, z_2, \dots\dots, z_n\}$ is defined by Z.

Definition: Intuitionistic Triangular Fuzzy Number (ITrFN)

We defined the IFS in membership function $\mu_Q(z)$and non-membership function $\gamma_Q(z)$of the Intuitionistic Triangular Fuzzy Number:

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$$\mu_Q(z) = \begin{cases} \frac{z-a_1}{a_2-a_1}, for\ a_1 \le z \le a_2, \\ \frac{a_3-z}{a_3-a_2}, for\ a_2 \le z \le a_3, \\ 0, otherwise. \end{cases}, \gamma_Q(z) = \begin{cases} \frac{z-a_1}{a_2-a_1}, for\ a_1 \le z \le a_2, \\ \frac{a_3-z}{a_3-a_2}, for\ a_2 \le z \le a_3, \\ 0, otherwise. \end{cases}$$

Here$a_1 \le\ a_1 \le a_2 \le\ a_3 \le a_3$. This ITrFN is denoted by $(a_1 a_2, a_3;\ a_1 a_2 a_3)$. This ITrFN is also stated as: $Q =\ \langle([a_1 a_2, a_3]; \mu_\varrho), ([a_1 a_2, a_3]; \upsilon_\varrho)\rangle$.

Let $x_1 = ([a_1, b_1, c_1]; \overline{\upsilon_{a1}}, \overline{\nu_{a1}})$, $x_2 = ([a_2, b_2, c_2]; \overline{\upsilon_{a2}}, \overline{\nu_{a2}})$ be two Intuitionistic Triangular Fuzzy Numbers. Then,

1. $x_1 +\ x_2 = ([a_1 +\ a_2, b_1 + b_2, c_1 + c_2]; \overline{\upsilon_{a1}} +\ \overline{\upsilon_{a2}} - \overline{\upsilon_{a1}}.\overline{\upsilon_{a2}}, \overline{\nu_{a1}}.\overline{\nu_{a2}})$
2. $x_1.x_2 =\ ([a_1.a_2, b_1.b_2, c_1.c_2]; \overline{\upsilon_{a1}}.\overline{\upsilon_{a2}} - \overline{\upsilon_{a1}} +\ \overline{\upsilon_{a2}}, \overline{\nu_{a1}}.\overline{\nu_{a2}})$
3. $\lambda x_1 = ([\lambda a_1, \lambda b_1, \lambda c_1]; 1 - (1 - \overline{\upsilon_{a1}})\lambda, \overline{\nu_{a1}}\lambda), \lambda \ge 0$
4. $x_1^\lambda = ([a^\lambda, b^\lambda, c^\lambda]; (\upsilon_a^-)^\lambda, 1 - (1 - \nu_a^-)^\lambda), \lambda \ge 0$

Definition: New Normalized Hamming Distance Formula:

Let $x_1 = ([a_1, b_1, c_1]; \overline{\upsilon_{a1}}, \overline{\nu_{a1}})$, $x_2 = ([a_2, b_2, c_2]; \overline{\upsilon_{a2}}, \overline{\nu_{a2}})$ have two values. Then we defined the distance formula for $a_1$and a_2 as follows:

$$d(a_1, a_2) = \frac{1}{8}[(1 +\ \overline{\upsilon_{a1}} -\ \overline{\nu_{a1}} - \overline{\pi_{a1}})a_1 - (1 + \overline{\upsilon_{a2}} -\ \overline{\nu_{a2}} - \overline{\pi_{a2}})a_2 + (1 +\ \overline{\upsilon_{a1}} -\ \overline{\nu_{a1}} - \overline{\pi_{a1}})b_1 - (1 + \overline{\upsilon_{a2}} -\ \overline{\nu_{a2}} - \overline{\pi_{a2}})b_2 + (1 +\ \overline{\upsilon_{a1}} -\ \overline{\nu_{a1}} - \overline{\pi_{a1}})c_1 - (1 + \overline{\upsilon_{a2}} -\ \overline{\nu_{a2}} - \overline{\pi_{a2}})c_2]$$

ITrFWG Operator and ITrHG Operator for Decision Making

Definition:

Let $\beta_h(h = 1, \ldots.n)$ be a series of intuitionistic triangular fuzzy number, then ITrFWG: $\Omega^n \to \Omega$ operator of dimension m if $ITrFWG = \prod_{h=1}^{n}(\beta_{(h)})^{w_j}$

$$= \left(\left[\prod_{h=1}^{m}(a_{(h)})^{w_j}, \prod_{h=1}^{m}(b_{(h)})^{w_j}, \prod_{h=1}^{m}(c_{(h)})^{w_j}\right]; \prod_{h=1}^{m}(\mu_{\alpha_h})^{w_j}, 1 - \prod_{h=1}^{m}(1 - \nu_{\alpha_h})^{w_j}\right)$$

where $w = (w_1, w_2, \ldots \ldots.w_n)^t$ is the weight $\sum_{j=1}^{n} w_j = 1, w_j \in [0,1]$.

Definition:

Let $\beta_i(i = 1, 2, \ldots.n)$be a collection of intuitionistic triangular fuzzy numbers. A intuitionistic triangular fuzzy hybrid geometric (ITrHG) operator of dimension n is a mapping ITrHG: $\Omega^n \to \Omega$, if ITrHG$(\alpha_1, \alpha_2, \ldots \ldots.\alpha_n) = \prod_{q=1}^{x}(\alpha_{\sigma_{(q)}})^{w_j}$

$$= \left(\left[\prod_{q=1}^{x}\left(a_{\sigma_{(q)}}\right)^{w_j}, \prod_{q=1}^{x}\left(b_{\sigma_{(q)}}\right)^{w_j}, \prod_{q=1}^{x}\left(c_{\sigma_{(q)}}\right)^{w_j}\right]; \prod_{q=1}^{x}\left(\mu_{\alpha_{\sigma_{(q)}}}\right)^{w_j}, 1 - \prod_{q=1}^{x}\left(1 - \nu_{\alpha_{\sigma_{(q)}}}\right)^{w_j}\right)$$

where $(\sigma_1, \sigma_2, \ldots.\sigma_n)$ is a permutation of 1,2,…n ∋ $a_{\sigma_{(q-1)}} \ge a_{\sigma_{(q)}} \forall q$.

Algorithm for MAGDM Problems:

Step 1: To obtain the individual value $z_i^{\sim(k)}$, the Intuitionistic triangular fuzzy decision matrix is given and the weights are provided. Here $r_{ij}^{\sim(k)} = ([a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}]; \mu_{ij}^{(k)}, \gamma_{ij}^{(k)}) = ITrFWG_w(r_{i1}^{\sim(k)}, r_{i2}^{\sim(k)}, \ldots \ldots, r_{in}^{\sim(k)})$ $i = 1,2 \ldots n;\ k = 1,2, \ldots \ldots$

Step 2: Use the ITrFGH operator to obtain the collective Intuitionistic Triangular Fuzzy values $z_j^{\sim}, (j = 1,2, \ldots n)$ of the option.

Step 3: See how far apart the intuitional triangular fuzzy positive ideal value $z_i^{\sim+} = ([a^{\sim+}, b^{\sim+}, c^{\sim+}];\ \mu^{\sim+}, \gamma^{\sim+}) = ([1,1,1]; 1,0)$ and the collective total value $z_i^{\sim} =\ ([a_i, b_i, c_i];\ \mu_i, \gamma_i)$.

Step 4: Sort all the option and arrange in ascending order.

Numerical Illustration

An example is considered to perform the proposed method.

Butcher's Sixth Order RK Method:

The Initial Value Problem is given $\frac{dy}{dx} = f(x, y), y(x_l) = y_l$

$K_1 = hf(x_m, y_m)$

$$K_2 = hf(x_m + h, y_m + K_1)$$

$$K_3 = hf\left(x_m + \frac{1}{2}h, y_m + \frac{3K_1 + K_2}{8}h\right)$$

$$K_4 = hf\left(x_m + \frac{2h}{3}, y_m + \frac{8K_1 + 2K_2 + 8K_3}{27}\right)$$

$$K_5 = hf\left(x_m + (7-\sqrt{21})\frac{h}{14}, y_m + \frac{3[3(\sqrt{21}-7]K_1 - 8[7-\sqrt{21}]K_2 + 48(7-\sqrt{21})K_3 - 3[21-\sqrt{21}]K_4}{392}\right)$$

$$K_6 = hf\left(x_m + (7-\sqrt{21})\frac{h}{14}, y_m + \frac{-5(231+51\sqrt{21}]K_1 - 40[7+\sqrt{21}]K_2 - 320\sqrt{21}K_3 + 3[21+121\sqrt{21}]K_4 + 392(6+\sqrt{21})K_5}{1960}\right)$$

$$K_7 = hf\left(x_m + h, y_m + \frac{15\,(22+7\sqrt{21}]K_1 + 120K_2 + 40(7\sqrt{21}-5)K_3 - 63(3\sqrt{21}-2)K_4 - 14(49+9\sqrt{21})K_5 + 70(7\sqrt{21})K_6}{180}\right)$$

$$y_{(n+1)} = y_n + \frac{1}{180}(9K_1 + 64K_3 + 49K_5 + 49K_6 + 9K_7)$$

Problem suggested by the Decision Maker 1:

To solve the Initial Value Problem, use Butcher`s 6th order Runge Kutta method

$\frac{dy}{dx} = x + y^2, x = 0, y = 1\ with\ h = 0.1.$

Solution: Consider $\frac{dy}{dx} = x + y^2, x = 0, y = 1\ with\ h = 0.1$

For $j = 0, x = 0, y = 1$

$K_1 = 0.1$

$K_2 = 0.131$

$K_3 = 0.116065$

$K_4 = 0.121614$

$K_5 = 0.105315$

$K_6 = 0.1279$

$K_7 = 0.682196$

The approximate value of $Y_1 = y(0.1) = 1.14387$

Similarly, we have $Y_2 = y(0.2) = 1.37595$

$Y_3 = y(0.3) = 2.75270$

$Y_4 = y(0.4) = 3.48174$

Weights are $W = (0.13066, 0.15718, 0.31444, 0.39772)$

Problem suggested by the Decision Maker 2:

To resolve the initial value problem, use 4th Order Runge Kutta and Butcher`s 5th order RK Method. $\frac{dy}{dx} = (1 + xy), y(0) = 2\ with\ h = 0.1.$

Runge Kutta Fourth Order method:

$K_1 = 0.1(1 + 0.2) = 0.1$

$K_2 = 0.1(1 + 0.1025) = 0.11025$

$K_3 = 0.1(1 + 0.10276) = 0.110276$

$K_4 = 0.1(1 + 0.2110276) = 0.12110$

$Y_1 = y(0.1) = 2.11045$

Similarly, we have $Y_2 = y(0.2) = 2.243095$

$Y_3 = y(0.3) = 2.401213$

$Y_4 = y(0.4) = 2.588562$

Weights are $\gamma = (0.225879, 0.240075, 0.256998, 0.277050)$

Butcher`s Fifth Order Runge Kutta Method:

$K_1 = 1$

$K_2 = 1.050625$

$K_3 = 1.0506408$

$K_4 = 1.102627$

$K_5 = 1.15512045$

$K_6 = 1.21089$

$Y_1 = y(0.1) = 2.112892$

Similarly, we have $Y_2 = y(0.2) = 2.2491817752$

$Y_3 = y(0.3) = 2.41033084$

$Y_4 = y(0.4) = 2.6015429089$

Weights are $\delta = (0.225401, 0.239940, 0.257131, 0.219775)$

Four members are challenging for the town council in town. The town council has exactly
Four members: A1,A2,A3,A4. During one week, the council members vote on exactly four bills:

Z1 :Recreation Bill

Z2 :Subsidy Bill

Z3 :School Bill

Z4 :Tax bill.

Each council member votes either for or against each bill. Decision makers E1,E2,E3,E4 use the following four attributes for their assessment of alternatives. The following four attributes shall be marked:

Y1 : Each member votes at least one of the bills and against at least one of the bills.

Y2 : Exactly two members of the council vote for the recreation bill.

Y3 : Exactly one member of the council votes for the school bill.

Y4 : Exactly one member of the council votes for the tax bill.

Decision makers E1,E2,E3,E4 gives decision matrices R1,R2,R3,R4 respectively. Intuitionistic fuzzy numbers by weight vector are used by the decision makers such as $W = (0.13066, 0.15718, 0.31444, 0.39772)^t$ to express four attributes and the weight vector $\gamma = (0.22588, 0.24008, 0.25700, 0.27705)$ $\delta = (0.22540, 0.23994, 0.25713, 0.21978)$. The data set $R = (r_{2ij})^k_{4X4}$are

$$R_1 = \left(\begin{bmatrix} ([0.3,0.6,0.9]; 0.4,0.5)([0.2,0.4,0.5]; 0.4,0.3)([0.5,0.7,0.8]; 0.5,0.2)([0.2,0.6,0.9]; 0.4,0.2) \\ ([0.1,0.2,0.3]; 0.3,0.2)([0.2,0.7,0.9]; 0.3,0.6)([0.3,0.5,0.6]; 0.4,0.2)([0.2,0.4,0.6]; 0.5,0.4) \\ ([0.2,0.4,0.6]; 0.5,0.3)([0.4,0.6,0.8]; 0.2,0.4)([0.2,0.5,0.6]; 0.1,0.6)([0.3,0.4,0.8]; 0.1,0.2) \\ ([0.1,0.3,0.5]; 0.4,0.1)([0.2,0.3,0.5]; 0.1,0.5)([0.4,0.7,0.9]; 0.5,0.3)([0.7,0.8,0.9]; 0.2,0.4) \end{bmatrix}\right)$$

$$R_2 = \left(\begin{bmatrix} ([0.1,0.2,0.8]; 0.1,0.4)([0.2,0.4,0.5]; 0.4,0.3)([0.2,0.3,0.5]; 0.1,0.4)([0.1,0.2,0.4]; 0.7,0.2) \\ ([0.1,0.4,0.8]; 0.5,0.3)([0.1,0.3,0.7]; 0.2,0.6)([0.4,0.5,0.6]; 0.5,0.2)([0.2,0.5,0.7]; 0.6,0.3) \\ ([0.3,0.4,0.6]; 0.2,0.4)([0.3,0.5,0.6]; 0.7,0.2)([0.6,0.7,0.8]; 0.3,0.6)([0.2,0.6,0.9]; 0.5,0.2) \\ ([0.2,0.7,0.8]; 0.3,0.6)([0.7,0.8,0.9]; 0.2,0.6)([0.2,0.7,0.9]; 0.4,0.2)([0.4,0.5,0.6]; 0.4,0.3) \end{bmatrix}\right)$$

$$R_3 = \left(\begin{bmatrix} ([0.2,0.4,0.6]; 0.1,0.2)([0.1,0.2,0.3]; 0.1,0.2)([0.2,0.3,0.4]; 0.7,0.2)([0.3,0.4,0.5]; 0.5,0.4) \\ ([0.1,0.2,0.8]; 0.1,0.6)([0.3,0.4,0.5]; 0.2,0.3)([0.5,0.6,0.7]; 0.6,0.3)([0.1,0.2,0.3]; 0.2,0.3) \\ ([0.3,0.4,0.8]; 0.3,0.6)([0.5,0.6,0.7]; 0.3,0.4)([0.4,0.5,0.8]; 0.5,0.4)([0.7,0.8,0.9]; 0.2,0.6) \\ ([0.4,0.6,0.8]; 0.7,0.1)([0.7,0.8,0.9]; 0.4,0.5)([0.3,0.5,0.7]; 0.5,0.3)([0.3,0.5,0.7]; 0.5,0.3) \end{bmatrix}\right)$$

$$R_4 = \left(\begin{bmatrix} ([0.1,0.2,0.3];0.1,0.2) & ([0.3,0.4,0.7];0.4,0.1) & ([0.2,0.3,0.4];0.7,0.2) & ([0.3,0.4,0.5];0.5,0.4) \\ ([0.3,0.4,0.5];0.2,0.3) & ([0.2,0.5,0.8];0.5,0.3) & ([0.5,0.6,0.6];0.4,0.2) & ([0.1,0.2,0.3];0.2,0.3) \\ ([0.5,0.6,0.7];0.3,0.4) & ([0.1,0.4,0.9];0.3,0.6) & ([0.4,0.5,0.8];0.5,0.4) & ([0.7,0.8,0.9];0.2,0.6) \\ ([0.7,0.8,0.9];0.4,0.5) & ([0.2,0.4,0.8];0.1,0.5) & ([0.3,0.5,0.7];0.5,0.3) & ([0.3,0.5,0.7];0.5,0.3) \end{bmatrix} \right)$$

Step 1: Apply the ITrFWG operator

$$\tilde{r_{11}} = \big[[(0.3)^{0.13066} * (0.2)^{0.15718} * (0.5)^{0.314441} * (0.2)^{0.39772}, (0.6)^{0.13066} * (0.4)^{0.15718} * (0.7)^{0.314441} * (0.6)^{0.39772},\ (0.9)^{0.13066} * (0.5)^{0.15718} * (0.8)^{0.314441} * (0.9)^{0.39772}](0.4)^{0.13066} * (0.4)^{0.15718} * (0.5)^{0.314441} * (0.4)^{0.39772}, 1 - [(1-0.5)^{0.13066} * (1-0.3)^{0.15718} * (1-0.2)^{0.314441} * (1-0.2)^{0.39772}]\big]$$

$\tilde{r_{11}} = ([0.2812978002, 0.5909135855, 0.790412191]; 0.4290741747, 0.26327654642)$

Similarly,

$\tilde{r_{12}} = ([0.207523628, 0.447957377, 0.584112762]; 0.402383974, 0.360142215)$

$\tilde{r_{13}} = ([0.262048034, 0.457309686, 0.703849417]; 0.137607884, 0.395746720)$

$\tilde{r_{14}} = ([0.373886712, 0.578421143, 0.759914745]; 0.261923784, 0.354695974)$

$\tilde{r_{21}} = ([0.138666568, 0.253347152, 0.486516161]; 0.269615082, 0.310769352)$

$\tilde{r_{22}} = ([0.203721748, 0.448164607, 0.678616663]; 0.465493648, 0.331452441)$

$\tilde{r_{23}} = ([0.317498817, 0.580427802, 0.771741762]; 0.398277201, 0.380402553)$

$\tilde{r_{24}} = ([0.320827998, 0.625309709, 0.754264912]; 0.345477367, 0.378591834)$

$\tilde{r_{31}} = ([0.210741039, 0.327686357, 0.440527734]; 0.349725241, 0.286491169)$

$\tilde{r_{32}} = ([0.197141496, 0.315044497, 0.482342063]; 0.258061623, 0.349357110)$

$\tilde{r_{33}} = ([0.498458383, 0.602470865, 0.820954701]; 0.299808789, 0.515707046)$

$\tilde{r_{34}} = ([0.355864234, 0.551314487, 0.741021136]; 0.504464695, 0.313895209)$

$\tilde{r_{41}} = ([0.228775997, 0.333765679, 0.459705042]; 0.434869318, 0.273158851)$

$\tilde{r_{42}} = ([0.213521045, 0.357220844, 0.488391839]; 0.326290338, 0.300000000)$

$\tilde{r_{43}} = ([0.413762909, 0.596027241, 0.839261487]; 0.299808789, 0.520886718)$

$\tilde{r_{44}} = ([0.314429350, 0.513343427, 0.738710089]; 0.377088803, 0.364615602)$

Step 2: Use the Hybrid Geometric operator with the current intuitionistic triangular fuzzy matrix.

$z_1 = ([0.3284828023, 0.477439635, 0.628302258]; 0.4886370786, 0.4103830258)$

$z_2 = ([0.3265793472, 0.503914145, 0.6540532108]; 0.4750661724, 0.4605993456)$

$z_3 = ([0.4955755834, 0.664654909, 0.8440554939]; 0.3981705587, 0.5717623259)$

$z_4 = ([0.4663138224, 0.667242315, 0.8143145769]; 0.4955472506, 0.4770360197)$

Step 3: Compute the distance formula between $\tilde{z_i} = ([a_i, b_i, c_i]; \mu_i, \gamma_i)$ and $z^{\sim+} = ([1,1,1]; 1,0)$

$d(\tilde{z_1}, z^{\sim+}) = 0.5748007645660$

$d(\tilde{z_2}, z^{\sim+}) = 0.5736855199824$

$d(\tilde{z_3}, z^{\sim+}) = 0.5504880823870$

$d(\tilde{z_4}, z^{\sim+}) = 0.5086845058681$

Step 4: Sort the option, $Z_k (k = 1, 2, 3, 4)$

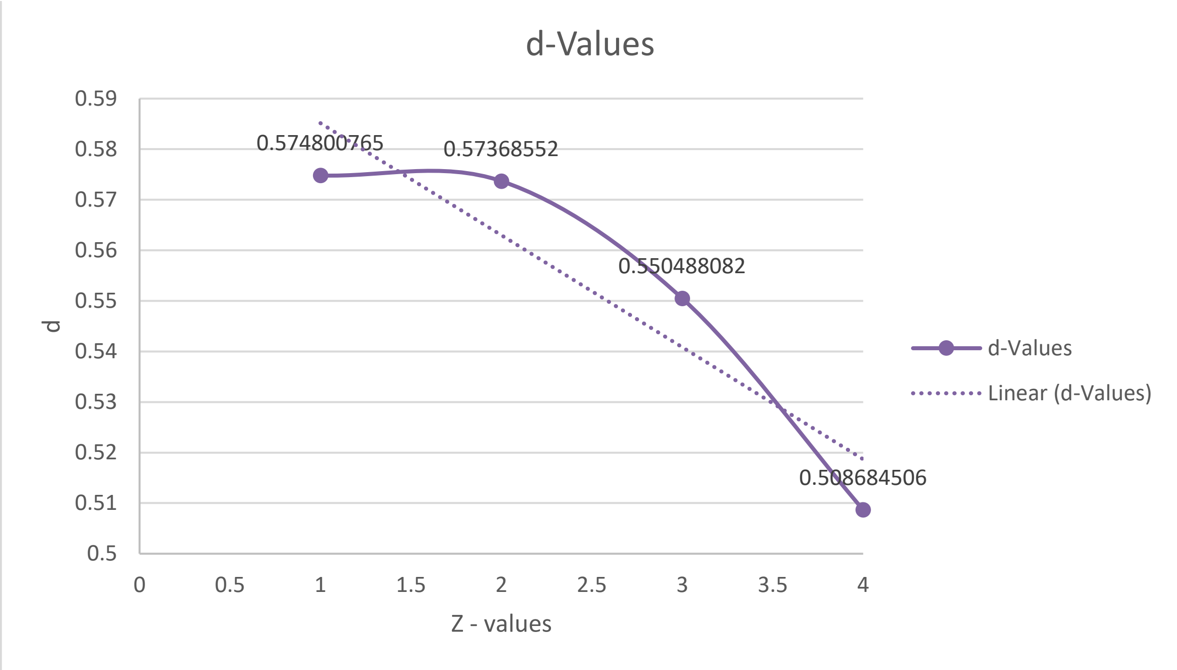
$Z_4 < Z_3 < Z_2 < Z_1$

As a result, the best option is Z_4

Z_4: Tax Bill is the finest option.

III. Results and Discussions

According to the findings of the report, the members are expected to vote on the tax bill. The members may influence it and not affect the other tax payers. With some limitations, the results should be interpreted with care. We used the decision values as weights under four attributes for this work so that we could select the best option. There are several recommendations for future research at the end of this report.



IV. Conclusion

The numerical solutions are obtained by applying different Runge Kutta methods. To select the finest option, a Decision Making Problem implemented to Multiple Attributes Groups. |Here the weights shall be derived from Runge Kutta method in the data set using the hybrid geometric operator and weighted geometric operator.The distance formula selects the best option. We gave an example to show how the suggested method could work.

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