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Elasticity Coefficient Calculation for the Active Volume of Soil Layer Compacted by a Vibratory Road Roller

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Abstract: - article discusses process of evaluation for the stiffness (elasticity coefficient) of soil mass of special shape being determined by stress distribution caused by surface force. It is important for model description of interaction between road roller and soil during vibratory compaction in the framework of lumped parameters approach. The main part of the modelling procedure for dynamic deformation during compaction is a choice of initial stress distribution. The problem of calculation of the soil mass stiffness under assumption of Westergaard normal stress distribution is solved and recommendation further application of the results for numeric experiments are done.

Keywords: stiffness, elasticity, Westergaard stress distribution, compaction, sandy loam, lumped parameters, vibratory roller, linear approximation, Flaman problem.

I. Introduction

Soil compaction efficiency generally depends on normal stress which is produced in depth of the soil layer by surface pressure due to the action of work tool of compacting machine. The accuracy of predictions and recommendations researcher may do concerning the features and design of compacting machines and parameters of their work patterns demand for the methods of correct description of interaction of machine and soil layer.

Evaluation of characteristics for the soil area involved in interaction with the work tool of the compactor within the framework of the interacting columns approach does not cause any difficulties. However, the real shape and size of the this so called active volume of the soil significantly depend on both the type of contact area of the work tool with the surface and the operating mode of the compaction machine. Their determination should take into account the distribution of mechanical stresses provided by the compacting force in the depth of the soil layer.

Initial data of soil-object interaction problem demand for special tools for its solution. The set of parameters includes at the first place the velocity of deforming object and the initial state of the material which is usually determined by its density. In some special situations one must implement methods close to those of hydrodynamics [1] but industrial soil compaction definitely is not of this case.

The basic mechanical approach to description of deformative processes in soil layer which is being compacted is well established method of lumped parameters approximation. The latter allows to take into consideration inertial, viscous and elastic parts of resistance force of the soil. Since that effective volume of the soil media can be presented by a deforming massive elastic viscous plastic body. Although the approximation is quite obvious its application may cause several problems and one of the principal ones is quantitative assessment of mechanical properties of active volume of the soil.

II. Westergaard Approach To Normal Stress Distribution And Assessment Of Effective Volume Of Soil Layer

In order to determine geometric properties of soil mass we will use Westergaard approach usually implemented to study stress distribution for cohesive soils. Westergard stress distribution is well established model widely used in soil mechanics. Its aspects were investigated from different points of view and for different applications as well. For instance, Westergard stress distribution often is used as a reference for comparison and justification of usability for numeric methods in engineering of soils [3]. Another set of application for Westergard distribution is presented by the problems of fracture mechanics [4],[5],[6] as well as for problems with dislocations [7]. Also, here must be mentioned applications of Westergaard distribution for solutions of foundation mechanics problems under special boundary conditions (corners, for instance) [8]. Theoretical investigations of Westergard stress distribution problems attract interest as well and calls for special methods of the well-established theory of integral transformation and special functions [9],[10], differential equations integration [11] and potential theory [12]. All

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of mentioned allowed to obtain interesting and important results but the problem of calculation for integrated characteristics of soil mass stated above still remains.

First of all, it's necessary find the shape and size of constant stress surfaces (isobars) produced in the layer by a surface force. At first it is necessary to obtain an expression of produced stress for load which is distributed along the line with linear density q by integration of contributions from elementary point on a line directed along the Y-axis. Each of the elementary point contributes qdy to the total load. The result of integration over the variable gives

$$\sigma_z = \frac{q\gamma}{2\pi}\int_{-\infty}^{+\infty}\frac{zdy}{(\gamma^2 z^2 + x^2 + y^2)^{3/2}} = \frac{q\gamma}{\pi}\frac{z}{(\gamma^2 z^2 + x^2)} \tag{1}$$

Calculations for the surface load, distributed with surface density σ_0 on a strip of width $2b$, oriented along the Y-axis, with the center at the origin, are made by integration of contributions from each elementary line, represented by a strip of width dt, passing through the point with coordinate t along the X-axis providing a load $\sigma_0 dt$ as follows (variable t changes from $-b$ to b)

$$\sigma_z = \int_{-b}^{+b} d\sigma_z(t) = \frac{\sigma_0\gamma}{\pi}\int_{-b}^{+b}\frac{zdt}{(\gamma^2 z^2 + (x-t)^2)} = \frac{\sigma_0}{\pi}\left(\arctan(\frac{x+b}{\gamma z}) - \arctan(\frac{x-b}{\gamma z})\right) \tag{2}$$

Similarly to the previous case, for analyzing the load distribution in the depth of the soil, it is convenient to switch to dimensionless variables, related to the half-width of the strip where the load is applied $\zeta = z/b, \xi = x/b$. In this case, the stress distribution will take following form

$$\sigma_z(\xi,\zeta) = \frac{\sigma_0}{\pi}\left(\arctan(\frac{\xi+1}{\gamma\zeta}) - \arctan(\frac{\xi-1}{\gamma\zeta})\right) \tag{3}$$

The family of cross-sections of surfaces of constant stress by a vertical plane perpendicular to the soil surface with the normal vector oriented parallel to the axial line of the load application strip is shown in Fig. 1. in respect to the same coordinates $\zeta = z/b, \xi = x/b$.

Unlike the Boussinesq model [2] of stress distribution, the Westergaard approach allows an analytical representation of the family of curves of constant stress (3). They are represented by ellipses passing through the points (-1,0) and (1,0). Thus, the surfaces of constant stress will be intersecting elliptical cylinders.

Let's present the equation of the line of constant stress in the form

$$\frac{\pi\sigma_z}{\sigma_0} = \arctan(\frac{\xi+1}{\gamma\zeta}) - \arctan(\frac{\xi-1}{\gamma\zeta})$$

and transform it, considering that on the line of constant stress, taking the tangent function of both sides of the equation and introducing a dimensionless constant which parameterizes the line of constant stress, $C = \tan\frac{\pi\sigma_z}{\sigma_0} = const, \frac{\sigma_z}{\sigma_0} > \frac{1}{2}$, $C = \tan\pi\left(1 - \frac{\sigma_z}{\sigma_0}\right) = const, \frac{\sigma_z}{\sigma_0} < \frac{1}{2}$ where - σ_0 the maximum value of contact pressure on the contact area

$$\frac{2\gamma\zeta}{\gamma^2\zeta^2 + \xi^2 - 1} = C,$$

which after reducing the equation of the ellipse to canonical form gives

$$\frac{(\zeta - \frac{1}{\gamma C})^2}{\frac{1}{\gamma^2}(1+\frac{1}{C^2})} + \frac{\xi^2}{(1+\frac{1}{C^2})} = 1 \quad (4)$$

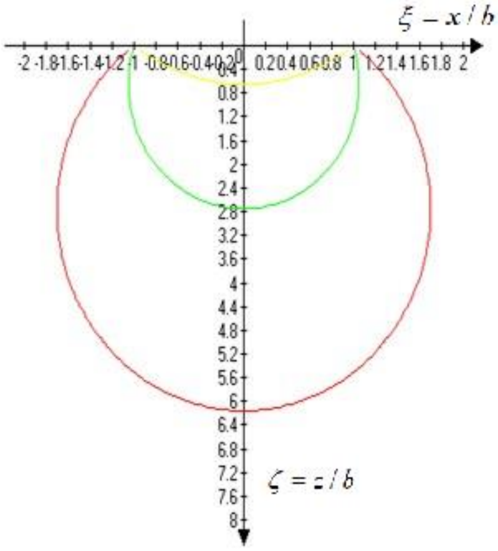

Fig.1. Distribution of sections of surfaces of constant stress during the operation of the roller in the depth of the soil layer according to Westergaard model ($\mu = 1/3, \quad \gamma = 1/2$) (case of uniform load distribution over an infinite strip of known width)

Among the elliptical sections of surfaces of constant stress defined by expression (4) in the thickness of the layer, the isobar corresponding to half the contact pressure $\frac{\sigma_z}{\sigma_0} = \frac{1}{2}$ occupies a special position. It corresponds to an ellipse with the center at the origin with the equation

$$\gamma^2 \zeta^2 + \xi^2 = 1 .$$

Its analysis allows obtaining an important result about the distribution of stresses in the thickness of the layer – stresses fall by half relative to the initial contact pressure at a depth equal to γb . $\gamma = 1/2$ is usually approximated usual for real soil media which corresponds to 50% decrease in stress in respect to the contact pressure at depth equal to the length of the contact area.

For applications, considering the stress-strain behavior of soil media in the framework of Westergaard approach to the distribution of normal stresses is more acceptable.

Information about the type of distribution of surfaces of constant stress taking into account the "stress - strain" curve, allows conclusions about the development of deformation in the soil thickness with changes in surface stress and describe its nature.

In this case, the soil medium can be considered as sequentially deformable elliptical cylinders, each of which has a known resistance character to the impact of operational stresses. The right side of the inequality in the dynamic relationship, which defines the contact stresses during effective compaction, must not be violated

$$\sigma_{pl}(t) \le \sigma(t) < \sigma_{destr}(t) ,$$

where σ_{pl} - the yield limit of the soil, $\sigma(t)$ - the current value of contact stress, σ_{destr} - the strength limit of the soil. Each of these cylinders is defined by the magnitude of normal stress at its boundary.

For the cylinder closest to the surface, the contact pressure may equal the strength limit σ_{destr} and for it the constant $C_{destr} = \frac{\pi \sigma_{destr}}{\sigma_0}$, it experiences only plastic deformations under a constant resistance force to deformation. The next cylinder corresponds to the plasticity limit σ_{pl} and for it $C_{pl} = \frac{\pi\sigma_{pl}}{\sigma_0}$ and its resistance force increases with increasing stress, non-linearly. The third cylindrical area is in a state of inelastic deformation with weak recovery, its lower boundary corresponds to the elasticity limit $C = \frac{\pi\sigma_{el}}{\sigma_0}$. Outside this cylinder lies an area experiencing elastic deformations, insufficient for compaction.

The results obtained for the shape of the cylindrical area, bounded by surfaces of constant stress in the case of surface load distribution over a flat strip of known width (Flaman's problem) can be generalized to the contact spot in the form of a side surface section of a circular cylinder. The results of its solution can be used to calculate the stiffness and viscosity of the active deformable area, and also serve to verify the correspondence of the theoretical approach used with existing experimental results. Surfaces of constant normal stress (isobars) in the deformable volume of the soil layer can be constructed taking into account both the load caused by the normal surface force and the load caused by the traction effort, providing translational movement of the roller.

Direct application of the mathematical model of interaction of the work tool with the soil medium under dynamic impact, requires the calculation of lumped parameters of the model which describe the deformative and inertial characteristics of the active area. The masses, elasticity (stiffness), and viscosity (coefficient of viscous friction) defined in this way will depend primarily on both the current stress-strain state of the material and the magnitude of the contact pressure applied by the work tool of the roller on soil surface and generating the distribution of normal stresses in the depth of the compacted layer.

It should be particularly noted that in the case of considering the operating modes of the roller, significant for the application of the concentrated parameters model, the condition of quasi-stationarity of dynamic processes, which must be stipulated before the start of modeling. The condition of quasi-stationarity in the case of impact on a layer of medium with a thickness h periodic force with a period T can be considered as follows $\frac{T}{8} >> \frac{h}{c}$, where - c the speed of propagation of elastic longitudinal waves in the medium. Considering the known values for the speeds of sound propagation in soil media, ranging from several hundred meters per second for sands to more than one and a half thousand meters per second for heavy clays, it can be assumed that for thickness values of soil layers compacted by vibratory rollers, making up 20-60 cm, roller frequencies of 15-100 Hz will ensure a quasi-stationary deformation mode.

III. Properties Of Soil In Framework Of Lumped Parameters Method

Known results concerning the transition from distributed systems to concentrated parameters relate to a wide class of problems: problems of mechanics, geology, electrodynamics, etc. In all cases, the transition occurs by integrating over the volume of the area affected by the disturbing factor (force, temperature, substance source, stress, etc.). Using this approach, it is possible to establish the values of geometric, mechanical, and deformative characteristics of the active soil layer area under surface force impact, acting within the framework of the model approach as described above. In this regard, it is worth mentioning the work by Z.G. Ter-Martirosyan with co-authors [14] and the series of articles by C.-F. Dobrescu [15,16] on the optimization of vibratory soil compaction, based on his work. The authors consider the compacted medium under the foundation of the structure or under the working organ of the compactor as a set of a finite number of inequal nonlinear oscillators with constant coefficients of mass, viscosity, and stiffness.

Calculation of mass, coordinate of the center of mass, boundary area, elasticity coefficient (stiffness), and viscosity () may be performed by integration over elementary areas under the following assumptions:

1) the medium is homogeneous;
2) defining «stress-strain» relations for all points of the medium are known;
3) corresponding concentrated dynamic characteristics are determined relative to the coordinates and velocities of the center of mass of the area.

The mass of the cylindrical area is calculated in a standard way

$$m_0 = \int_V \rho dV = \rho l \int_S dS , \qquad (5)$$

here integration is carried out over the area of the section by a vertical plane of the cylinder under constant normal stress. Similarly, the coordinate of the center of mass of the area is determined as

$$z_0 = \frac{\int_V z\rho dV}{\int_V \rho dV} = \frac{\rho l \int_S z dS}{\int_V \rho dV}, \quad (6)$$

which plays a role of the initial position during the movement of the concentrated mass under the action of the surface compacting force.

The length of the boundary section $L_0 = \int_{dV} dl$, which is necessary for calculating the area of the lateral surface of the cylindrical area, essential for determining the forces acting on sequentially interacting cylindrical bodies of the model. It is considered that during the operation of the roller, their shape and size are determined by the type of corresponding isobars. The stiffness of the massive area which is characterized by the modulus of deformation E relative to normal stress is calculated according to the following expression

$$c_{01} = \int_V dc = \int_0^l dy \int_{-b}^{b} \frac{\sigma dx}{\int_0^{z(\sigma)} \frac{\sigma(x,z)}{E} dz} = El \int_{-b}^{b} \frac{\sigma_1 dx}{\int_0^{z(\sigma_1)} \sigma(x,z) dz}, \quad (7)$$

where - the length of the cylindrical area, and - the constant value of stress defining the boundary of the area. Accordingly, its viscosity can be calculates as

$$b_0 = \int db, \quad (8)$$

but assessment of viscosity coefficient, anyhow, is not a subject of this paper and needs additional research.

The results of the calculations are presented in Table 1. in dimensionless numeric form, not allowing simple analytical representation in the form of approximate formulas. The calculations were done in dimensionless coordinates $\zeta = z/b, \xi = x/b$, in which the equation of the boundary section takes the form $\frac{\pi\sigma_z}{\sigma_0} = \arctan(\frac{\xi+1}{\gamma\zeta}) - \arctan(\frac{\xi-1}{\gamma\zeta})$, where b – the half-width of the load distribution area (contact strip).

Table 1: Mechanical properties of soil mass bounded by a surface of constant normal stress.

σ_z/σ_0	S_0	z_0	L_0	
0,04	399,8205	15,8383	75,28929	2,017762
0,08	101,255	7,815822	36,90849	2,165012
0,12	45,85263	5,109581	24,2049	2,275756
0,16	26,37896	3,739077	17,9034	2,371972
0,2	17,30174	2,906891	14,14731	2,464689
0,24	12,32008	2,346233	11,64875	2,562304
0,28	9,274494	1,942071	9,853292	2,673009
0,32	7,262545	1,636419	8,48234	2,807216
0,36	5,852793	1,396751	7,383235	2,979746
0,4	4,817751	1,203348	6,4738	3,213102
0,44	4,028144	1,043529	5,719222	3,551463
0,48	3,406018	0,908757	5,106585	4,11608
0,52	2,902037	0,793065	4,601019	5,716693
0,56	2,483683	0,692154	4,14393	7,634173

σ_z/σ_0	S_0	z_0	L_0	
0,6	2,128768	0,602841	3,713237	9,279882
0,64	1,821683	0,522719	3,324278	11,23644
0,68	1,551141	0,449926	2,992406	12,18541
0,72	1,30877	0,382992	2,720727	15,11531
0,76	1,088204	0,320736	2,504331	16,63483
0,8	0,884481	0,262188	2,33566	20,9419
0,84	0,693628	0,206534	2,20739	24,59477
0,88	0,512377	0,153077	2,113493	32,42907
0,92	0,337951	0,101201	2,04946	47,90363
0,96	0,167905	0,05035	2,01222	86,02082

The length of its boundary line is L_0 and the coordinate of its center of mass is z_0 (in dimensionless units, related to the half-width of the contact patch), stiffness is c_0, corresponding to surfaces of constant stress for various ratios σ_z/σ_0 in dimensionless units (values having the dimension of length are related to the half-width of the contact strip).

As an example of successful application of suggested method Figure 1. presents dependence of elasticity coefficient of near-surface area in graphic form. Parameters of the contact strip taken into consideration are given are typical for the medium size roller.

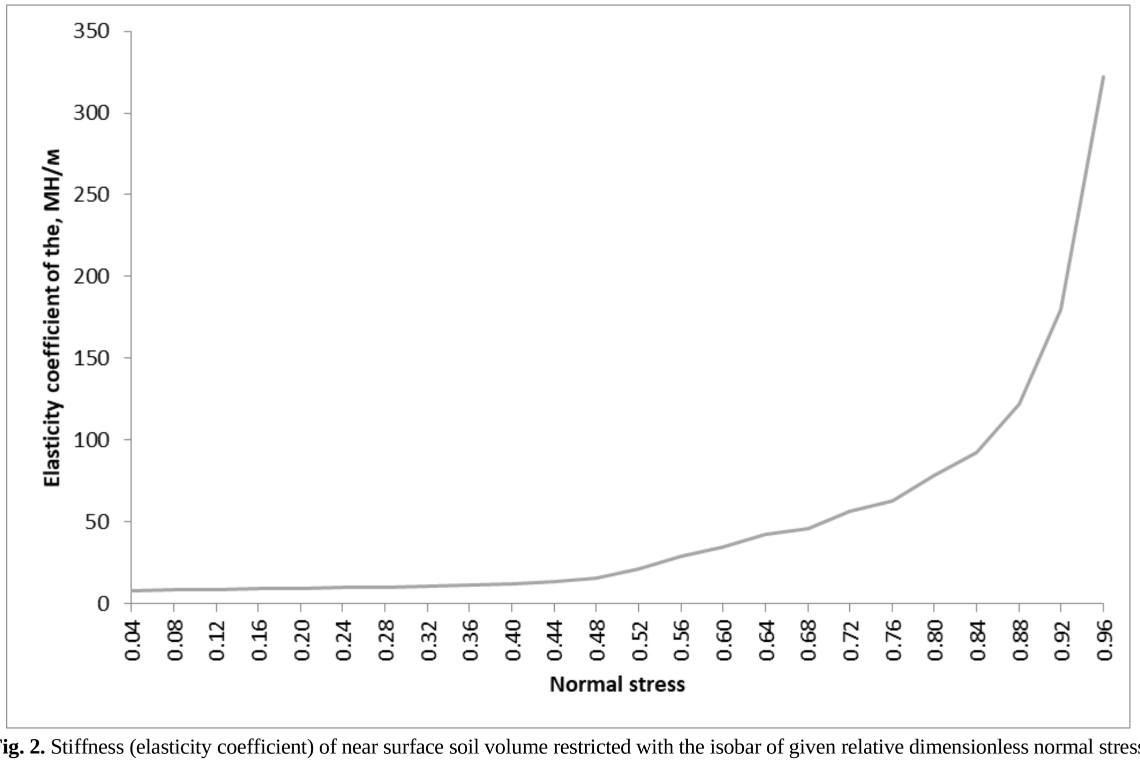

Fig. 2. Stiffness (elasticity coefficient) of near surface soil volume restricted with the isobar of given relative dimensionless normal stress σ_z/σ_0 in case of medium size roller b=0,125 m, l= 2 m)

The note which must be made concerns assumption which had been proposed when the points of resulting graph on Fig.3 had been calculated. Basically, it corresponds to the assumptions done in theory of electricity while capacity of infinite systems (flat and cylindrical capacitors). In given case we consider the side effects influencing the shape of constant stress surface insignificant because length of the contact strip is much greater than its width. All the numeric calculations had been performed in Maple package.

IV. Electrical Analogy And Further Research Perspectives

Equation of motion of elastic and viscous massive body with mass m, coefficient of viscous friction b and elasticity c in respect to coordinate z under action of external force $F(t)$

$$m\ddot{z} + b\dot{z} + cz = F(t), \tag{9}$$

and processes in circuits with lumped parameters

$$L\ddot{q} + R\dot{q} + Cq = U(t) \tag{10}$$

electrical resistance R, capacity C and inductivity L under voltage $U(t)$ are described with similar equations of oscillator with dissipation of energy.

Thorough investigation of behavior for both electrical and mechanical systems can be performed in similar way. For instance, consideration of resonance properties of soil mass [16] could reveal the most efficient work patterns of compacting machines. Since that one can use methods of investigation and simulation of electrical systems to reveal the properties of corresponding mechanical systems.

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