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Comparative Analysis of Controllers Based on Integer and on Fractional Order Operators in Control System of a Robot-Manipulator FANUC M-430iA/4FH

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Abstract: - In this article is propose a comparative analysis of the functional capabilities (for control of a wide class industrial plants) of controllers with algorithms based on operators for integration (differentiation) of integer order and of fractional controllers based on operators of fractional order.

Keywords: controllers of integer and of fractional order, characteristics and time characteristics, comparative analysis.

I. INTRODUCTION

The purpose of the research is a comparative analysis of the functional capabilities (for control of a wide class industrial plants) of controllers with algorithms based on operators for integration (differentiation) of integer order and of fractional controllers based on operators of fractional order. The tasks set in the research are systematizations of the time and frequency characteristics of the specified controllers for a wide parametric range of their setting parameters and their parallel comparative analysis

II. SYSTEMATIZATION OF DYNAMIC CHARACTERISTICS

■***The frequency characteristics*** of a fractional $I^{\alpha}\mathcal{D}^{\beta}$-controller $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$ of fractional α, β-order (1) and a PID-controller R_{PID} of integer order (2) are explored, as well as their time characteristics (4), (5). They are configured by a sequential structure with rational approximations (1), (2) of integer and fractional order operators [5-10].

● ***The frequency characteristics*** $\mathcal{R}_{I\mathcal{D}}(j\omega)$, $R_{PID}(j\omega)$ with their components: real $Re_{\mathcal{R}}(\omega)$; imaginary $Im_{\mathcal{R}}(\omega)$; module $Mod_{\mathcal{R}}(\omega)$; phase $Arg_{\mathcal{R}}(\omega)$, illustrate the response of the system μ to harmonic sinusoidal input impacts ε with frequency ω. The analytical relationship between the components in $\mathcal{R}(j\omega)$ is shown by (3).

■ ***The time characteristics*** of $R_{PID}(t)$ and $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}(t)$ are represented by:

● ***step responses*** $h_{\mathcal{R}}(t)$ (reactions of the systems to a single jump input impact) $\varepsilon(t)=1(t), \mathcal{E}(p)=p^{-1}$ (4),

● ***transfer characteristics*** $\mu_{\mathcal{R}}(t)$ (reactions of the systems to arbitrary input impact) $\varepsilon(t), \mathcal{E}(p)$ (5).

The following designations are used**:** $I^{\alpha}, \mathcal{D}^{\beta}$ - fractional operators (original functions)**;** $I^{\alpha}_{app}, \mathcal{D}^{\beta}_{app}$ - approximating operators (rational functions)**;** α, β - order of the fractional operators (fractional numbers)**;** n -

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positive integer; M, N - number of forcing units in the approximating polynomial (integers); i, j - counters (integers); ω_i^{-1} - time constants of the units in the approximating polynomial; ω_u - unit frequency of the approximating operator in the controller algorithm; $\omega_b \ ,\omega_h$ - lowest and highest approximation frequency; $\omega_A \ ,\omega_B$ - lower and upper frequency of the approximation range; $\mathcal{k}$ - proportional gain; $\Delta\omega$ - bandwidth of rational approximation.

III. Parameterization Of The Characteristics

Taking into account dynamic setting parameters of:

■ ***fractional controllers*** (1) $\left(\alpha, \beta, \mathcal{k}_{\mathcal{R}}, \Delta\omega_{\mathcal{R}}\right)$-

● order of operators α, β (6) ;

● proportional gain $\mathcal{k}_{\mathcal{R}}$ (7);

● bandwidth of rational approximation (8) $\Delta\omega_{\mathcal{D}}$ (8) ;

■ ***first integer order controllers*** (2) $\left(k_p, T_I, T_D\right)$-

● time constants of integration T_I and differentiation T_D;

● proportional gain k_p (2)

the dynamic characteristics of $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$ and R_{PID} are parameterized (9).
For the specified (Table I., Table II.) ranges of change of the dynamic setting parameters, the parameterized characteristics $\mathcal{R}_{I\mathcal{D}}\left(\alpha, \beta\right)$ and $R_{PID}\left(T_I, T_D\right)$ are illustrated in parallel (figure1, figure2). Such approach provides the possibility of a comparative analysis of the functional capabilities of $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$ and R_{PID}.

IV. 4. Comparative Analysis

The comparative analysis of the functional capabilities according to the results of the parallel visualization for the parametric range (10) of the characteristics of $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$ -controllers and R_{PID} - controllers (figure1, figure2) is based on the following indicators:

● ***dephasing*** of $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$ and R_{PID};

$$\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}\left(p\right) = I^{\alpha}_{app}\left(p\right)\mathcal{D}^{\beta}_{app}\left(p\right) \triangleq I^{\alpha}_{app}\left(p\right).\mathcal{D}^{\beta}_{app}\left(p\right) =$$

(1.a) $$= \left(\frac{\omega_{Iu}}{\omega_{Ih}}\right)^{\alpha}\prod_{i=1}^{N}\left(\frac{1+p\left(\omega_{Ii}\right)^{-1}}{1+p\left(\omega'_{Ii}\right)^{-1}}\right).\left(\frac{\omega_{\mathcal{D}u}}{\omega_{\mathcal{D}h}}\right)^{\beta}\prod_{j=1}^{M}\left(\frac{1+p\left(\omega'_{\mathcal{D}j}\right)^{-1}}{1+p\left(\omega_{\mathcal{D}j}\right)^{-1}}\right)$$

$$\left(n-1<\alpha<n,\ \omega_{Ii}>\omega'_{Ii}\ ;\ n-1<\beta<n,\ \omega_{\mathcal{D}j}<\omega'_{\mathcal{D}j}\ ;\ n\in M, N:=\left\{1,2,3,..\right\}\right)$$

(1.b) $$\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}\left(j\omega\right) \triangleq \left(\frac{\omega_{Iu}}{\omega_{Ih}}\right)^{\alpha}\prod_{i=1}^{N}\left(1+j\frac{\omega}{\omega'_{Ii}}\right)\left(1+j\frac{\omega}{\omega_{Ii}}\right)^{-1}.\left(\frac{\omega_{\mathcal{D}u}}{\omega_{\mathcal{D}h}}\right)^{\beta}\prod_{j=1}^{M}\left(1+j\frac{\omega}{\omega'_{\mathcal{D}j}}\right)\left(1+j\frac{\omega}{\omega_{\mathcal{D}j}}\right)^{-1},$$

$$\left(\omega_{Ii}>\omega'_{Ii}\ ;\ \omega_{\mathcal{D}j}<\omega'_{\mathcal{D}j}\ ;\ T_{Ii}>T'_{Ii}\ ;\ T_{\mathcal{D}j}<T'_{\mathcal{D}j}\ ;\ n-1<\alpha<n\ ;\ n-1<\beta<n\right)$$

(2.a) $$R_{PID}\left(p\right) = k_p\left(1+\frac{1}{T_I p}+T_D p\right) \triangleq k_p\left(\frac{\left(T_I p+1\right)}{T_I p}\frac{\left(T_D p+1\right)}{\left(T_f p+1\right)}\right)_{app},\ \left(T_f = 0.15T_D\right)$$

(2.b) $$R_{PID}\left(j\omega\right) \triangleq \left(k_p^2+\left(k_p T_D\omega - k_p\left(T_I\omega\right)^{-1}\right)^2\right)^{0.5} exp\left(-Arctg\left(T_D\omega-\left(T_I\omega\right)^{-1}\right)\right)$$

(3.a) $$\mathcal{R}\left(j\omega\right) = Re_{\mathcal{R}}\left(\omega\right)+j\,Im_{\mathcal{R}}\left(\omega\right) = Mod_{\mathcal{R}}\left(\omega\right)exp\left(-j\,Arg_{\mathcal{R}}\left(\omega\right)\right)$$

(3.b) $Re_{\mathcal{R}}(\omega) = Mod_{\mathcal{R}}(\omega) Cos(Arg_{\mathcal{R}}(\omega))$

(3.c) $Im_{\mathcal{R}}(\omega) = Mod_{\mathcal{R}}(\omega)\ Sin(Arg_{\mathcal{R}}(\omega))$

(3.d) $Mod_{\mathcal{R}}(\omega) = \left(Re^2_{\mathcal{R}}(\omega) + Im^2_{\mathcal{R}}(\omega)\right)^{0,5}$

(3.e) $Arg_{\mathcal{R}}(\omega) = ArcTg\left(-Im_{\mathcal{R}}(\omega)/Re_{\mathcal{R}}(\omega)\right)$

(4) $h_{\mathcal{R}}(t) = \mathcal{L}^{-1}\left\{\mathcal{R}(p)\big|_{\mathcal{E}(p)=p^{-1}}\right\} = \mathcal{L}^{-1}\left\{\mathcal{R}(p)\ p^{-1}\right\}$

(5) $\mu(t) = \mathcal{L}^{-1}\left\{\mathcal{R}(p)\big|_{\mathcal{E}(p)}\right\} = \mathcal{L}^{-1}\left\{\mathcal{R}(p)\mathcal{E}(p)\right\}$

(6.a) $\alpha = \left|arg I^{\alpha}(\omega)\right|_{max} / 90, [deg] = 2\left|arg I^{\alpha}(\omega)\right|_{max} / \pi, [rad]$,

(6.b) $\beta = \left|arg \mathcal{D}^{\beta}(\omega)\right|_{max} / 90, [deg] = 2\left|arg \mathcal{D}^{\beta}(\omega)\right|_{max} / \pi, [rad]$,

(7) $k_{\mathcal{R}} = min\left(d\left(mod\ \mathcal{R}(\omega)\right)/d\omega\right), [-] = 10^{\frac{(20\ log_{10} k_{\mathcal{R}})[dB]}{20}}, [-]$,

(8) $\Delta\omega_{I\mathcal{D}} = \left(\omega_{\mathcal{D}h} - \omega_{Ib}\right), [rad/s]$,

(9.a) $\mathcal{R}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}});\ Re_{\mathcal{R}}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}}); Im_{\mathcal{R}}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}});$
$Mod_{\mathcal{R}}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}});\ Arg_{\mathcal{R}}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}});$
$h_{\mathcal{R}}(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}});\ \mu(\alpha,\beta,k_{\mathcal{R}},\Delta\omega_{\mathcal{R}})$,

(9.b) $R(k_p,T_I,T_D,\Delta\omega_R);\ Re_R(k_p,T_I,T_D,\Delta\omega_R); Im_R(k_p,T_I,T_D,\Delta\omega_R);$
$Mod_R(k_p,T_I,T_D,\Delta\omega_R);\ Arg_R(k_p,T_I,T_D,\Delta\omega_R);$
$h_R(k_p,T_I,T_D,\Delta\omega_R);\ \mu(k_p,T_I,T_D,\Delta\omega_R)$,

Table 1. Order of the fractional operators for integration α and for differentiation β in $\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$

order $\alpha = ,(\beta =)$	0,111111	0,222222	0,333333	0,444444
phase correspondence in radians	$(1/9)(\pi/2)$	$(2/9)(\pi/2)$	$(3/9)(\pi/2)$	$(4/9)(\pi/2)$
phase correspondence in degrees	10^o	20^o	30^o	40^o
order $\alpha = ,(\beta =)$	0,555556	0,666667	0,777778	0,888889
phase correspondence in radians	$(5/9)(\pi/2)$	$(6/9)(\pi/2)$	$(7/9)(\pi/2)$	$(8/9)(\pi/2)$
phase correspondence in degrees	50^o	60^o	70^o	80^o
order $\alpha = ,(\beta =)$	1,111111	1,222222	1,333333	1,444444
phase correspondence in radians	$(10/9)(\pi/2)$	$(11/9)(\pi/2)$	$(12/9)(\pi/2)$	$(13/9)(\pi/2)$
phase correspondence in degrees	100^o	110^o	120^o	130^o
order $\alpha = ,(\beta =)$	1,555556	1,666667	1,777778	1,888889
phase correspondence in radians	$(14/9)(\pi/2)$	$(15/9)(\pi/2)$	$(16/9)(\pi/2)$	$(17/9)(\pi/2)$
phase correspondence in degrees	140^o	150^o	160^o	170^o

Table 2.

$T_I = ,(T_D =), s$	5	50	150	200
$T_I = ,(T_D =), s$	250	300	350	400
$T_I = ,(T_D =), s$	450	500	550	600
$T_I = ,(T_D =), s$	650	700	750	825

● ***advancement*** of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ and R_{PID} of (Table 3);

● ***frequency range*** of the operators used for integration and differentiation of integer and fractional order of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ and of R_{PID} (Table 4);

● ***efficiency domain*** in controlling classes of plants.

The results of the analysis (Table 3, Table 4) unequivocally confirm the advantages of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ controllers over R_{PID} - controllers. In this context, the functional capabilities (dephasing, advancement, frequency range) of the $\mathcal{R}_{I^{\alpha}D^{\beta}}$ - controllers are repeatedly wider (figure 1.a, figure3) than those of the R_{PID} -controllers, depending on the relations (11) (13) validity in which n is a positive number.

The efficiency domain of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ -controllers and R_{PID} -controllers in controlling classes of plants is analyzed with their capabilities (through their optimal settings) to achieve satisfaction of the desired quality criteria of the control systems (figure 4) with $\mathcal{R}_{I^{\alpha}D^{\beta}}$ and R_{PID} .

As an example, a system for (figure 4) control of a robot-manipulator [6-10] is considered. Here, the control (input) variables to the electric motors are $\mu_{Ji}\ (i \in [1,5])$ and the process (output) variables are the positions $y_{Ji}\ (i \in [1,5])$ of the axes of the 5-DOF-manipulator [1-4].

Let the properties of the analyzed controllers $\mathcal{R}_{ID}$ and R_{PID} are marked by $\mathcal{R}_{ID}(\alpha, \beta, \omega_{I\delta}, \omega_{Dh}, \Delta\omega_{ID})$, $R_{PID}(T_I, T_D, \omega_I, \omega_D, \Delta\omega_R)$, and let the properties of a generalized model G of industrial plants in control systems (figure 4) with the considered $\mathcal{R}_{ID}$ and R_{PID} are marked by $G(k_G, T_G, \tau_G, \omega_{u,G}, \omega_{res,G}, \omega_{c,G}, \omega_{\pi,G})$.

The properties of the generalized model are determined analytically as a function of the parameter multitude $\mathcal{G}$ (14), where: k_G is the static gain coefficient; T_G - dominant time constant; τ_G - delay; $\omega_{u,G}$ - single frequency; $\omega_{res,G}$ - resonant frequency; - $\omega_{c,G}$ cut off frequency; $\omega_{\pi,G}$ - "π"-frequency of the plant. Being dynamic systems, industrial plants are low-pass filters with essential frequencies in the multitude $\Omega_{\mathcal{G}}$, located in the range $(0 \div 20), [rad/s]$ and in ratio (15).

$$\begin{gathered}\alpha = \beta\ ;\ \alpha \in [0.1, 1.38]\ ;\ \beta \in [0.1, 1.38]\ ; \\ T_I = T_D\ ;\ T_I \in [5, 825]\ ;\ T_D \in [5, 825]\end{gathered} \tag{10}$$

$$\begin{gathered}(-\alpha_i 20\, dB/dec)\ /\ (-20\, dB/dec) = \alpha_i\ , \\ (\alpha_i \in [n-1;\ n]\ , n = 1, 2, 3, \ldots)\end{gathered} \tag{11}$$

$$\begin{gathered}(+\beta_i 20\, dB/dec)\ /\ (+20\, dB/dec) = \beta_i\ , \\ (\beta_i \in [n-1;\ n]\ , n = 1, 2, 3, \ldots)\end{gathered} \tag{12}$$

$$\begin{gathered}(\Delta\omega_{ID}, [rad/s])\ /\ (\Delta\omega_{R_{PID}}, [rad/s]) \gg n\ , \\ (n = 1, 2, 3, \ldots)\end{gathered} \tag{13}$$

Table 3.

$\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$	R_{PID}	$\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$	R_{PID}
$-\alpha_i\, 20\,dB/dec$	$-20\,dB/dec$	$+\beta_i\, 20\,dB/dec$	$+20\,dB/dec$
$\alpha_i \in [n-1;\ n]$, $n = 1, 2, 3, \ldots.$	*constante*	$\beta_i \in [n-1;\ n]$, $n = 1, 2, 3, \ldots.$	*constante*
dephasing		advancement	

Table 4. Frequency range of the operators used

$\mathcal{R}_{I^{\alpha}\mathcal{D}^{\beta}}$	R_{PID}
$\Delta\omega_{I\mathcal{D}} = (\omega_{\mathcal{D}h} - \omega_{Ib}), [rad/s]$	$\Delta\omega_{R_{PID}} = (\omega_{D_{PID}} - \omega_{I_{PID}}), [rad/s]$
$\Delta\omega_{I\mathcal{D}}, [rad/s] \gg \Delta\omega_{R_{PID}}, [rad/s]$	

$$\mathcal{G} = \{k_G, T_G, \tau_G, \omega_{u,G}, \omega_{res,G}, \omega_{c,G}, \omega_{\pi,G}\}$$
$$\mathcal{G} = \{k_G, T_G, \tau_G, \Omega_{\mathcal{G}}\}, \tag{14}$$

$$\Omega_{\mathcal{G}} = \{\omega_{u,G}, \omega_{res,G}, \omega_{c,G}, \omega_{\pi,G}\}$$
$$\omega_{u,G} < \omega_{res,G} < \omega_{c,G} < \omega_{\pi,G},\ (\omega_i \in [0, 20), rad/s). \tag{15}$$

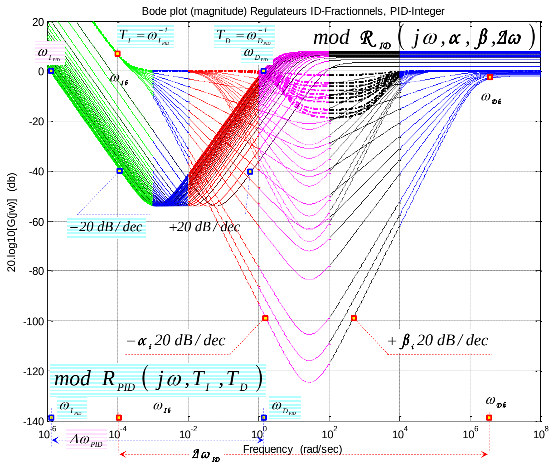

Figure 1.a.

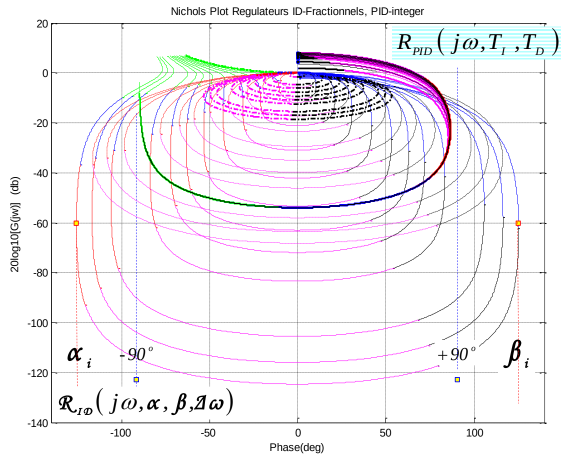

Figure 1.c.

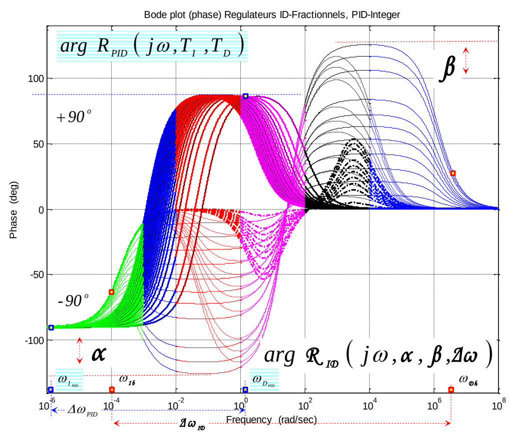

Figure 1.b.

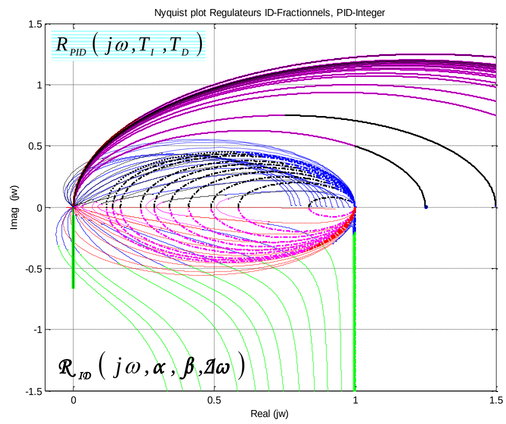

Figure 1.d.

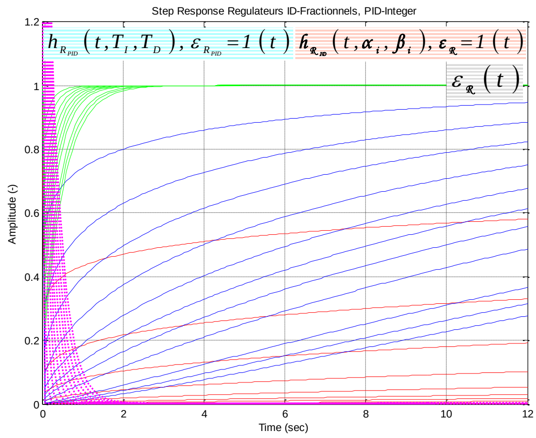

Figure 2.a.

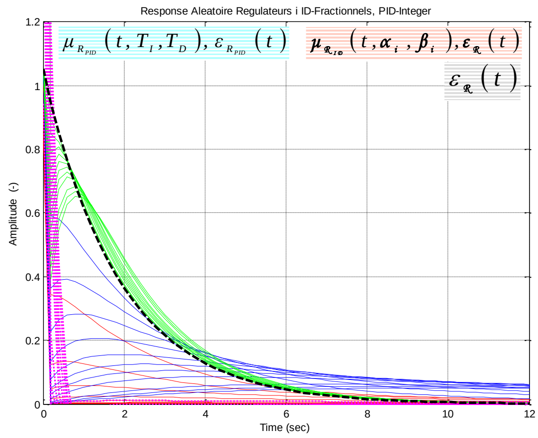

Figure 2.c.

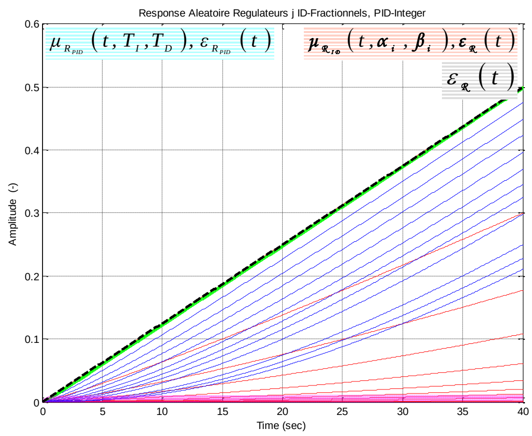

Figure 2.b.

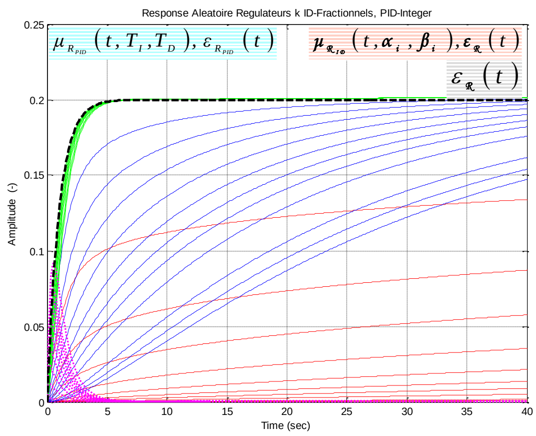

Figure 2.d.

The synthesis of the control systems (figure4) and the optimal setting of the controllers $\mathcal{R}_{I\mathcal{D}}$ or R_{PID} are subject to relevant functional dependencies $\mathcal{F}_i$ (16) between: • the dynamic setting parameters ($\alpha, \beta, \omega_{Ib}, \omega_{\mathcal{D}h}, \Delta\omega_{I\mathcal{D}}$ or $T_I, T_D, \omega_I, \omega_D, \Delta\omega_R$); • the essential parameters in the characteristics of $\mathcal{G}$ (14); • the requirements to the quality indicators of the control system σ (17).

The aim of the synthesis is the analytical determination of such values of the parameters (16) for which the requirements σ (17) are satisfied. In general, σ (17) is a function of: gain stability margin GM; phase stability margin PM; time of controlling t_{reg}; robust stability RS; robust performance RP and other indicators of the system quality (figure4). The properties of the controllers $\mathcal{R}_{I\mathcal{D}}\left(\alpha, \beta, \omega_{Ib}, \omega_{\mathcal{D}h}, \Delta\omega_{I\mathcal{D}}\right)$, $R_{PID}\left(T_I, T_D, \omega_I, \omega_D, \Delta\omega_R\right)$, are analytically determined as functions of the parametric multitudes (18) and (19), respectively.

The parametric multitude $\mathcal{R}_{I\mathcal{D}}$ (18.a) is considered as consisting of two non-intersecting parametric sub-multitudes $\mathcal{R}^{\blacksquare}_{I\mathcal{D}}$ (18.b) and $\mathcal{R}^{\checkmark}_{I\mathcal{D}}$ (18.c). The first of them is $\mathcal{R}^{\blacksquare}_{I\mathcal{D}}$ (18.b), for which the requirements σ (17) are not fulfilled and for which the evaluated controller $\mathcal{R}_{I\mathcal{D}}$ operates with an algorithm that is not able to satisfy quantitatively the demanded norms σ (17) for optimality to the quality indicators of the control system. The second multitude $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ (18.c) completely satisfies the requirements σ (17) and defines ***the efficiency domain*** of $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ (18.d).

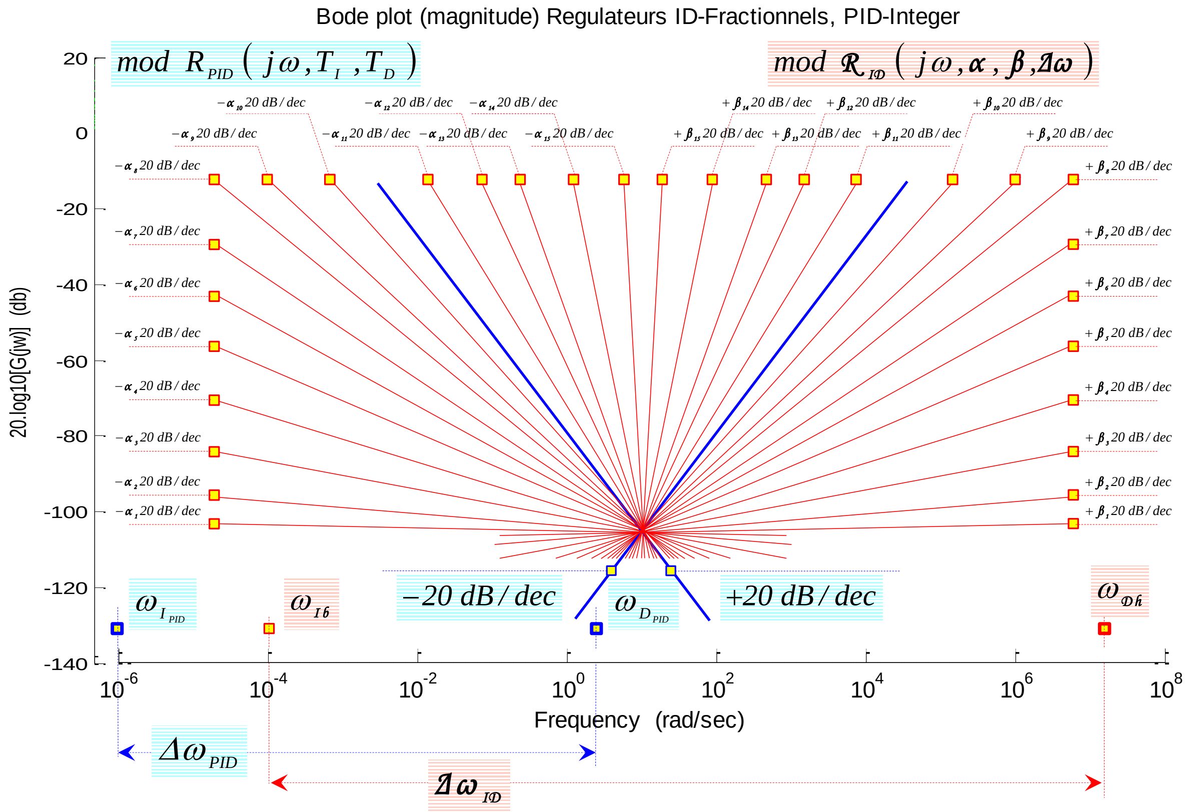

Figure 3.

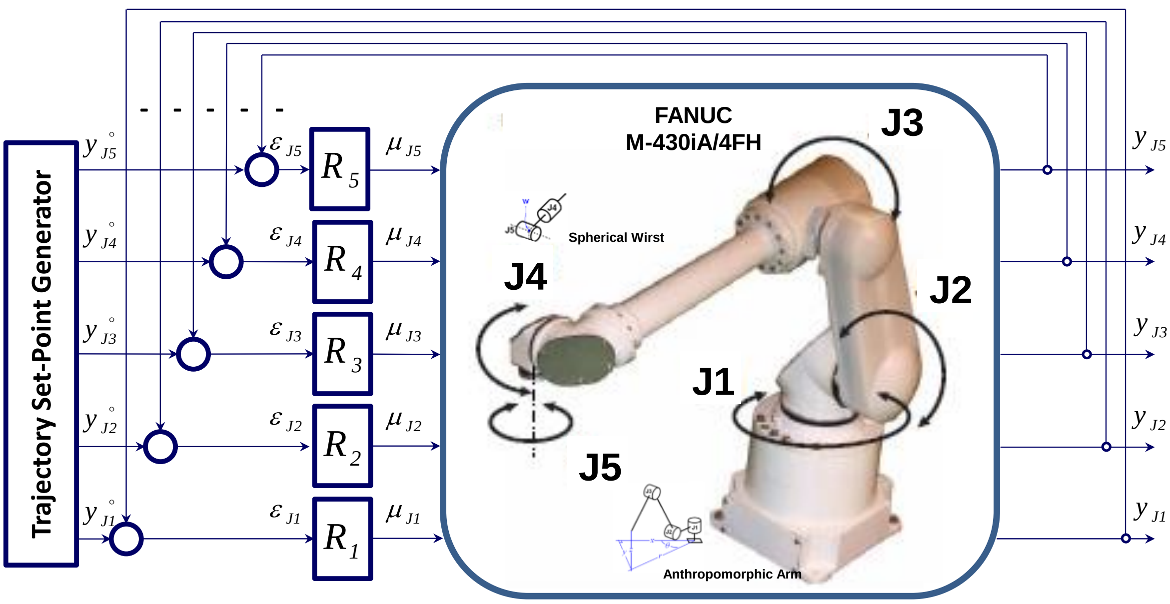

Figure 4.a

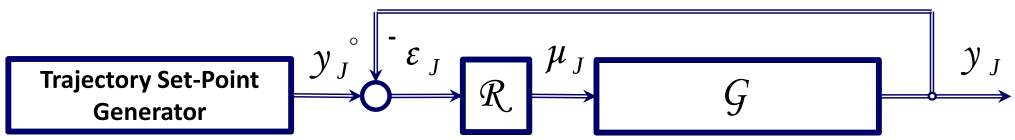

Figure 4.b

The parametric multitude R_{PID} (19.a) is considered as consisting of two non-intersecting parametric sub-multitudes $R_{PID}^{\blacksquare}$ (19.b) and $R_{PID}^{\checkmark}$ (19.c). The first of them is $R_{PID}^{\blacksquare}$ (19.b), for which the requirements of $R_{PID}^{\blacksquare}$ (17) are not fulfilled, and for which the evaluated controller R_{PID} operates with an algorithm that is not able to quantitatively satisfy the demanded norms for optimality to the quality indicators of the control system. The second multitude $R_{PID}^{\checkmark}$ (19.c) fully satisfies the requirements (17) and defines ***the efficiency domain*** of $R_{PID}^{\checkmark}$ (19.d)

For comparison and evaluation, the considered multitudes $\mathcal{G}$ (14), $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ (18.c), $R_{PID}^{\checkmark}$ (19.c) are illustrated graphically by figure5 in proportion to the numerical axis of the frequency ω in the parameterized frequency characteristic of the controllers.

It is obvious (figure5) that $R_{PID}^{\checkmark} \subset \mathcal{R}_{I\mathcal{D}}^{\checkmark}$ ($R_{PID}^{\checkmark}$ is a sub-multitude of the multitude $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$). An addition to the multitude $R_{PID}^{\checkmark}$ regarding the multitude $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ is a multitude $\overline{R}$ (20) containing all elements $x_{\mathcal{R}_{I\mathcal{D}}^{\checkmark}}$ of the multitude $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ that do not belong to the multitude $R_{PID}^{\checkmark}$ with elements $x_{R_{PID}^{\checkmark}}$. The multitude obtained from the difference $\mathcal{R}_{I\mathcal{D}}^{\checkmark} / R_{PID}^{\checkmark}$ of the multitudes $R_{PID}^{\checkmark}$ and $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ is called ***an addition*** of $R_{PID}^{\checkmark}$ to $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$. The addition is marked by $\overline{R}$ and is the analytical proof of the unambiguous advantage of the efficiency of $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ (18.d) over the efficiency of $R_{PID}^{\checkmark}$ (19.d), i.e. ***the domain*** $\mathcal{R}_{I\mathcal{D}}^{\checkmark}$ (18.d) ***is wider than*** $R_{PID}^{\checkmark}$ (19.d).

(16.a)
$$\begin{gathered}\boldsymbol{\alpha} = \mathcal{F}_{\alpha}\left(k_G, T_G, \tau_G, \boldsymbol{\Omega}_{\mathcal{G}}, \sigma\right), \boldsymbol{\beta} = \mathcal{F}_{\beta}\left(k_G, T_G, \tau_G, \boldsymbol{\Omega}_{\mathcal{G}}, \sigma\right) \\ \boldsymbol{\omega}_{I\text{б}} = \boldsymbol{\omega}_{I\text{б}}(\boldsymbol{\alpha});\ \boldsymbol{\omega}_{\mathcal{D}\text{ћ}} = \boldsymbol{\omega}_{\mathcal{D}\text{б}}(\boldsymbol{\beta});\ \boldsymbol{\Delta\omega}_{I\mathcal{D}} = \boldsymbol{\Delta\omega}_{I\mathcal{D}}\left(\boldsymbol{\omega}_{I\text{б}}, \boldsymbol{\omega}_{\mathcal{D}\text{ћ}}\right),\end{gathered}$$

(16.b)
$$\begin{gathered}T_I = \mathcal{F}_{T_I}\left(k_G, T_G, \tau_G, \boldsymbol{\Omega}_{\mathcal{G}}, \sigma\right), T_D = \mathcal{F}_{T_D}\left(k_G, T_G, \tau_G, \boldsymbol{\Omega}_{\mathcal{G}}, \sigma\right) \\ \omega_I = \omega_I(T_I), \omega_D = \omega_D(T_D); \Delta\omega_R = \Delta\omega_R(T_I, T_D),\end{gathered}$$

(17)
$$\sigma = \sigma\left(GM, PM, t_{reg}, RS, RP, \ldots\right),$$

(18.a)
$$\mathcal{R}_{I\mathcal{D}} = \left\{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\omega}_{I\text{б}}, \boldsymbol{\omega}_{\mathcal{D}\text{ћ}}, \boldsymbol{\Delta\omega}_{I\mathcal{D}}\right\},$$

(18.b)
$$\mathcal{R}_{I\mathcal{D}}^{\blacksquare} = \left\{\boldsymbol{\alpha}^{\blacksquare}, \boldsymbol{\beta}^{\blacksquare}, \boldsymbol{\omega}_{I\text{б}}^{\blacksquare}, \boldsymbol{\omega}_{\mathcal{D}\text{ћ}}^{\blacksquare}, \boldsymbol{\Delta\omega}_{I\mathcal{D}}^{\blacksquare}\right\}, \left(\mathcal{R}_{I\mathcal{D}}^{\blacksquare} \subset \mathcal{R}_{I\mathcal{D}}\right),$$

(18.c)
$$\mathcal{R}_{I\mathcal{D}}^{\checkmark} = \left\{\boldsymbol{\alpha}^{\checkmark}, \boldsymbol{\beta}^{\checkmark}, \boldsymbol{\omega}_{I\text{б}}^{\checkmark}, \boldsymbol{\omega}_{\mathcal{D}\text{ћ}}^{\checkmark}, \boldsymbol{\Delta\omega}_{I\mathcal{D}}^{\checkmark}\right\}, \left(\mathcal{R}_{I\mathcal{D}}^{\checkmark} \subset \mathcal{R}_{I\mathcal{D}}\right),$$

(18.d)
$$\mathcal{R}_{I\mathcal{D}}^{\checkmark} \underset{\{\sigma\}}{\Leftrightarrow} \mathcal{G},$$

(19.a)
$$R_{PID} = \left\{T_I, T_D, \omega_I, \omega_D, \Delta\omega_R\right\},$$

(19.b)
$$R_{PID}^{\blacksquare} = \left\{T_I^{\blacksquare}, T_D^{\blacksquare}, \omega_I^{\blacksquare}, \omega_D^{\blacksquare}, \Delta\omega_R^{\blacksquare}\right\}, \left(R_{PID}^{\blacksquare} \subset R_{PID}\right),$$

(19.c)
$$R_{PID}^{\checkmark} = \left\{T_I^{\checkmark}, T_D^{\checkmark}, \omega_I^{\checkmark}, \omega_D^{\checkmark}, \Delta\omega_R^{\checkmark}\right\}, \left(R_{PID}^{\checkmark} \subset R_{PID}\right),$$

(19.d)
$$R_{PID}^{\checkmark} \underset{\{\sigma\}}{\Leftrightarrow} \mathcal{G},$$

(20)
$$\overline{R} \overset{def}{=} \mathcal{R}_{I\mathcal{D}}^{\checkmark} / R_{PID}^{\checkmark} = \left\{x_{\mathcal{R}_{I\mathcal{D}}^{\checkmark}} \mid x_{R_{PID}^{\checkmark}} \in \mathcal{R}_{I\mathcal{D}}^{\checkmark} \wedge x_{\mathcal{R}_{I\mathcal{D}}^{\checkmark}} \notin R_{PID}^{\checkmark}\right\}.$$

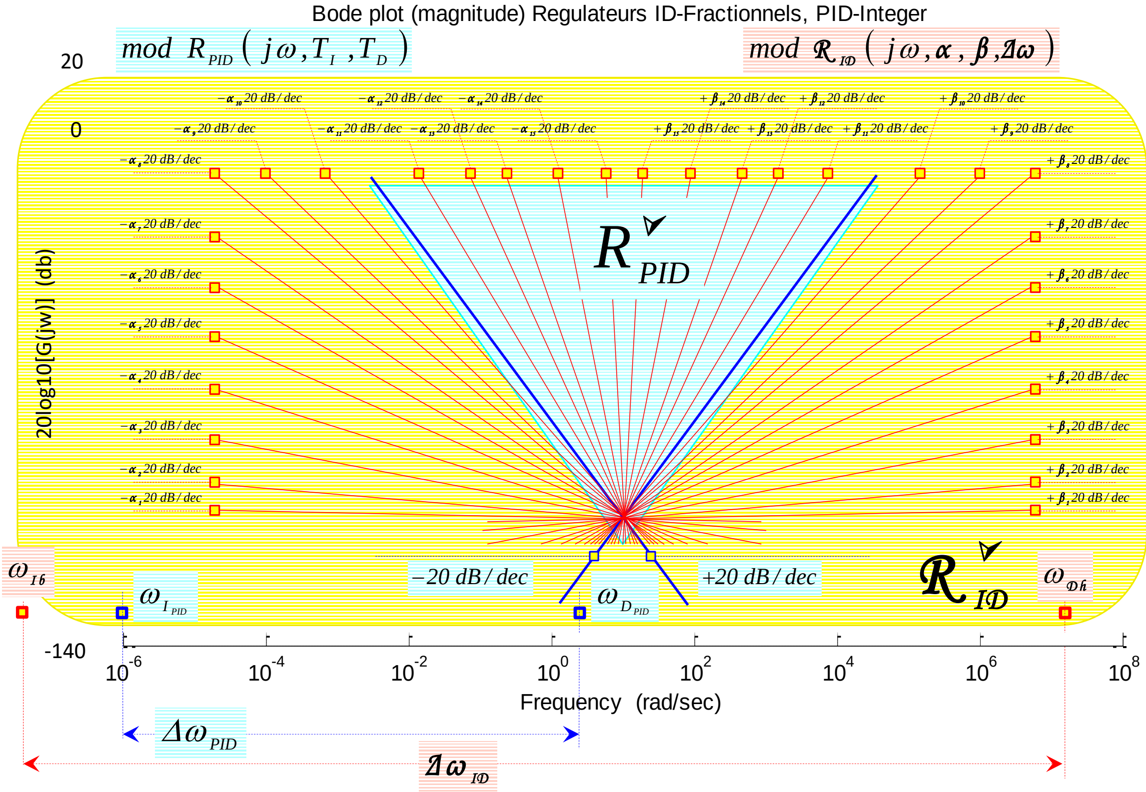

Figure5.a

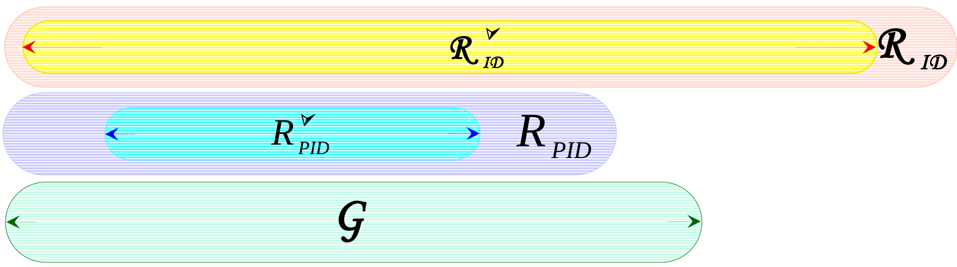

Figure5.b

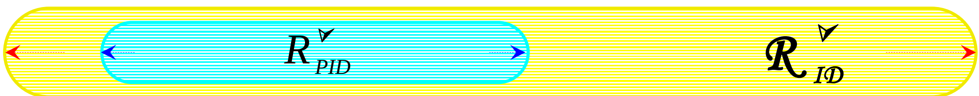

Figure5.c

V. Conclusion

The results of the comparative analysis of the functional capabilities in (10) of the characteristics of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ -controller and R_{PID} -controller (figure1, figure2, Table 3, Table 4), based on the indicators ***dephasing, advancement, frequency range, efficiency domain***, unequivocally confirm the advantages of $\mathcal{R}_{I^{\alpha}D^{\beta}}$ -controllers over R_{PID} -controllers. Ratios (11)$\div$(13), (20) prove that the functional capabilities of the fractional $\mathcal{R}_{I^{\alpha}D^{\beta}}$ -controllers are many times wider (figure1.a, figure3, figure5) from those of integer order R_{PID} -controllers.

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