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Entropy - Distance Measures Based on Pythagorean Fuzzy Sets using VIKOR Method

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***Abstract:* -** While making important decisions in life we often come across situations where we face doubt in choosing best options according to our needs. This becomes more difficult when a lot of options are available and lots of attributes are to be looked. Decision making is one such technique which makes it easier to choose best options among the available ones. In this editorial we aim to provide a method which will serve as an aid in decision making. New distance measures have been proposed for the PFS. Entropy measure is also proposed to calculate the weights needed. VIKOR method is applied to rank the attributes to get the best alternatives from pool of options. Numerical illustrations have been taken along with some real-life examples.

Keywords: Decision making, Distance measures, Entropy, Pythagorean fuzzy sets, VIKOR.

MSC: 94D05, 90B50, 03B52, 03E72.

I. Introduction

Decision making is a part of our day to day life with some decisions being easy and some being complex. When there are more than one attributes to be considered decision making becomes quite a tedious task, to ease the process there are many techniques which help in taking best decision according to the need. Decision making can be done both qualitatively and quantitatively. Multi Criteria Decision Making (MCDM) is a branch of Operation Research (OR) that deals with different methods of decision making involving more than one criterion. There are numerous methods available under this category like Analytical Hierarchical Process (AHP), Case-based Reasoning, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Simple Additive Weighting (SAW), Vlsekriterijumska Optimizacija I Kompromisno Resenje (VIKOR) etc. Since taking decision in real life is not a simple task due to uncertainty involved in it became necessary to extended the classical set theory to fuzzy set theory. Fuzzy set theory was introduced in by Zadeh [1] and it involved the degree of randomness between the variables. But by the time it was irrelevant in some cases due to inclusion of only membership function. In 1986 Atanassov [2-3] generalized FS to IFS which include non-membership function and hesitancy function such that $\rho(\mathfrak{y}) + \xi(\mathfrak{y}) \leq 1$ and $\rho(\mathfrak{y}) + \xi(\mathfrak{y}) + \theta(\mathfrak{y}) = 1$ where '$\rho(\mathfrak{y})$' is the degree of membership '$\xi(\mathfrak{y})$' is the degree of non- membership and '$\theta(\mathfrak{y})$' is the hesitancy function. Later on it was realized that there exists some cases which do not satisfy Atanassov's condition for instance consider the set A = < 0.6, 0.5 > here $\rho(\mathfrak{y})$ = 0.6, $\xi(\mathfrak{y})$ = 0.5 then $\rho(\mathfrak{y}) + \xi(\mathfrak{y})$ = 0.6 + 0.5 > 1. Yager [4-5] in 2013 extended the theory to Pythagorean Fuzzy sets such that $\rho^2(\mathfrak{y})$ + $\xi^2(\mathfrak{y})$ + $\theta^2(\mathfrak{y})$ = 1 where '$\rho(\mathfrak{y})$', '$\xi(\mathfrak{y})$' and '$\theta(\mathfrak{y})$' denotes membership function, non- membership function and hesitancy function respectively. MCDM problems are solved with the help of distance measure using methods like TOPSIS, AHP, VIKOR etc. Abdullah et al. [6] discussed VIKOR method with PFSs and their applications. Chen and Zhou [7] extended PF VIKOR method with generalized distance measure using MCDM. Xingli et al. [8] discussed linguistic method for VIKOR method to solve MCDM problems. Samal and Dash [9] proposed a comparison between TOPSIS and VIKOR for ranking decision making models.

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Entropy is defined as the measure of uncertainty of a function. It was developed by Shannon in [10]. De Luca and Termini [11] developed the axiomatic properties for fuzzy entropy based on Shannon entropy measure. It is used to obtain weights for MCDM method. Yang and Hussain [12] discussed the concept of fuzzy entropy for PFSs with application to MCDM. Gandotra et al. [13] proposed new entropy measure with application to MCDM. Chaurasiya and Jain [14] suggested PF entropy measure based for MCDM health problem. Entropy and Distance measure shows the characteristics in PFSs. Similarity and distance measure are applied in variety of real life MCDM problems along with fuzzy sets. Similarity measure helps in finding closeness between objects distance measure does the opposite that is finding the distance between the objects. Utilizing the usefulness of PFSs . Peng [15] suggested some results based on PFSs. Li et al. [16-17] also discussed about the distance measure of PFSs. Ejegwa [18] extended the distance measures for IFSs to PFSs and applied them to MCDM problems. Onyeke and Ejegwa [19] further suggested a weighted distance measure and its applications in decision making using PFSs. Arora and Taruna [20] proposed a sine distance measure. Further, Bhatia et al. [21] proposed a distance measure for MCDM using TOPSIS. Li et al. discussed image stenographic policy with constrained distance measure [22]. Raj et al. [23] suggested cosine distance and similarity entropy measures for fuzzy soft sets. Taherdoost et al. [24] proposed VIKOR method for effective management policies. Further, Yu et al. [25] suggested safety evaluation method for quayside container cranes based on VIKOR method. Prawar et al. [26] suggested how cold standby helps in enhancing system which helps in better decision making for the system.
The following paper is organized as follows:
Section 2 discusses the definition of concepts applied in the article and existing measures in the concerned field followed by proposed measures in Section 3. In section 4 numerical example has been given to check the validity of the measure. Section 5 is dedicated to proposed entropy and its properties. In section 6 algorithm of VIKOR method is elaborated as a ranking tool and a case study is discussed using proposed measure. In the end the work is concluded with possibilities for future and references that serve as an aid in completion of the work.

II. Preliminaries

In this section we will discuss some basic definitions and existing measures.

Fuzzy Sets

Definition 2.1. *Zadeh (1965):* Let $\mathfrak{Y}$ be a finite universe of discourse. A fuzzy set $\dot{\mathrm{M}}$ in $\mathfrak{N}$ is defined as :
$\dot{\mathrm{M}} = < \mathfrak{y} , \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ where $\rho_{\mathcal{M}}(\mathfrak{y})$ represents the degree of inclusion of element $\mathfrak{y}$ in $\mathfrak{Y}$ and $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}) : \mathfrak{Y} \rightarrow [0,1]$.

Intuitionistic Fuzzy Sets

Definition 2.2. *Atanassov (1986):* Let $\dot{\mathrm{M}}$ be an IFS in $\mathfrak{Y}$ then it can be defined as:
$\dot{\mathrm{M}} = < \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) , \xi_{\dot{\mathrm{M}}}(\mathfrak{y}) \mid \forall\, \mathfrak{y} \in \mathfrak{Y}\}$ where $\rho_{\dot{\mathrm{M}}}(\mathfrak{y})+\xi_{\dot{\mathrm{M}}}(\mathfrak{y}) \leq 1$.
$\rho_{\dot{\mathrm{M}}}(\mathfrak{y}) : \mathfrak{Y} \rightarrow [0,1]$ is the degree of inclusion of element $\mathfrak{y}$ in $\mathfrak{Y}$ and $\xi_{\dot{\mathrm{M}}}(\mathfrak{y}) : \mathfrak{Y} \rightarrow [0,1]$ is the degree of exclusion of element $\mathfrak{y}$ in $\mathfrak{Y}$.
The degree of hesitancy can be represented as $\theta_{\dot{\mathrm{M}}}(\mathfrak{y}) = 1 - \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) - \xi_{\dot{\mathrm{M}}}(\mathfrak{y})$.

Pythagorean Fuzzy Sets

Definition 2.3. *Yager (2013):* Let $\mathfrak{Y}$ be a finite universe of discourse. A Pythagorean fuzzy set $\dot{\mathrm{M}}$ in $\mathfrak{Y}$ is defined as:
$\dot{\mathrm{M}} = < \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) , \xi_{\dot{\mathrm{M}}}(\mathfrak{y}) \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ where $\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y})+ \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}) \leq 1$.
$\rho_{\dot{\mathrm{M}}}(\mathfrak{y}) : \mathfrak{Y} \rightarrow [0,1]$ is the degree of inclusion of element $\mathfrak{y}$ in $\mathfrak{Y}$ and $\xi_{\dot{\mathrm{M}}}(\mathfrak{y}) : \mathfrak{Y} \rightarrow [0,1]$ is the degree of exclusion of element $\mathfrak{y}$ in $\mathfrak{Y}$.

The degree of hesitancy can be represented as $\theta_{\dot{\mathrm{M}}}(\mathfrak{y})= \sqrt{1 - (\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}) + \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}))}$

Existing Distance measures for PFSs

Before discussing proposed measure we will discuss some existing distance measure.

Definition 2.4. *Arora and Taruna (2021).* Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be two PFSs in $\mathfrak{Y}$ such that $\mathfrak{Y} = \{ \mathfrak{y}_1 , \mathfrak{y}_2 , \ldots\ldots\ldots , \mathfrak{y}_{n-1} , \mathfrak{y}_n \}$ then distance measure between $\dot{\mathrm{M}}$ and $\mathfrak{N}$ can be defined as

$$\mathcal{DM}^1(\dot{\mathrm{M}},\mathfrak{N}) = \frac{2}{n}\sum_{i=1}^{n}\frac{\sin\left\{\frac{\pi}{2}\left|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)\right|\right\}}{1+\sin\left\{\frac{\pi}{2}\left|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)\right|\right\}} \quad (1)$$

Definition 2.5. *Ejegwa and Awolola (2021).* Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be two PFSs in $\mathfrak{Y}$ such that $\mathfrak{Y} = \{ \mathfrak{y}_1 , \mathfrak{y}_2 , \ldots\ldots\ldots , \mathfrak{y}_{n-1} , \mathfrak{y}_n \}$ then distance measure between $\dot{\mathrm{M}}$ and $\mathfrak{N}$ can be defined as

$$\mathcal{DM}^2(\dot{\mathrm{M}},\mathfrak{N})=\frac{1}{4n}\sum_{i=1}^{n}[|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)| + ||\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi_{\mathcal{M}}(\mathfrak{y}_i)| - |\rho_{\mathfrak{N}}(\mathfrak{y}_i) - \xi_{\mathfrak{N}}(\mathfrak{y}_i)|| + ||\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta_{\dot{\mathrm{M}}}(\mathfrak{y}_i)| - |\rho_{\mathfrak{N}}(\mathfrak{y}_i) - \theta_{\mathfrak{N}}(\mathfrak{y}_i)||] \quad (2)$$

$$\mathcal{DM}^3(\dot{\mathrm{M}},\mathfrak{N})=\frac{1}{4n}\sum_{i=1}^{n}[|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)| + |\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi_{\mathfrak{N}}(\mathfrak{y}_i)| + |\theta_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta_{\mathfrak{N}}(\mathfrak{y}_i)| + 2\max\{\sum_{i=1}^{n}[|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)|, |\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi_{\mathfrak{N}}(\mathfrak{y}_i)|, |\theta_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta_{\mathfrak{N}}(\mathfrak{y}_i)| \} \quad (3)$$

$$\mathcal{DM}^4(\dot{\mathrm{M}},\mathfrak{N})=(\frac{1}{4n}\sum_{i=1}^{n}[|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)|^2 + |\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi_{\mathfrak{N}}(\mathfrak{y}_i)|^2 + |\theta_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta_{\mathfrak{N}}(\mathfrak{y}_i)|^2 + 2\max\{\sum_{i=1}^{n}[|\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho_{\mathfrak{N}}(\mathfrak{y}_i)|^2, |\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi_{\mathfrak{N}}(\mathfrak{y}_i)|^2, |\theta_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta_{\mathfrak{N}}(\mathfrak{y}_i)|^2 \}])^{1/2} \quad (4)$$

Definition 2.6. *Bhatia et al. (2022).* Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be two PFSs in $\mathfrak{Y}$ such that $\mathfrak{Y} = \{ \mathfrak{y}_1 , \mathfrak{y}_2 , \ldots\ldots\ldots , \mathfrak{y}_{n-1} , \mathfrak{y}_n \}$ then distance measure between $\dot{\mathrm{M}}$ and $\mathfrak{N}$ can be defined as

$$\mathcal{DM}^5(\dot{\mathrm{M}},\mathfrak{N})=1 - \frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\pi}{2} - \frac{\pi}{6}\left(\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho^2_{\mathfrak{N}}(\mathfrak{y}_i)\right| + \left|\xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\mathfrak{N}}(\mathfrak{y}_i)\right| + |\theta^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \theta^2_{\mathfrak{N}}(\mathfrak{y}_i)|\right) \quad (5)$$

Definition 2.7. *Hussain et al. (2023).* Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be two PFSs in $\mathfrak{Y}$ such that $\mathfrak{Y} = \{ \mathfrak{y}_1 , \mathfrak{y}_2 , \ldots\ldots\ldots , \mathfrak{y}_{n-1} , \mathfrak{y}_n \}$ then distance measure between $\dot{\mathrm{M}}$ and $\mathfrak{N}$ can be defined as

$$\mathcal{DM}^6(\dot{\mathrm{M}},\mathfrak{N})=\frac{1}{3n}\sum_{i=1}^{n}\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho^2_{\mathfrak{N}}(\mathfrak{y}_i)\right| + \left|\xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\mathfrak{N}}(\mathfrak{y}_i)\right| + \frac{1}{2}\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho^2_{\mathfrak{N}}(\mathfrak{y}_i) + \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\mathfrak{N}}(\mathfrak{y}_i)\right| \quad (6)$$

$$\mathcal{DM}^7(\dot{\mathrm{M}},\mathfrak{N})=\frac{1}{3}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho^2_{\mathfrak{N}}(\mathfrak{y}_i)\right|^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left|\xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\mathfrak{N}}(\mathfrak{y}_i)\right|^2} + \sqrt{\frac{1}{4n}\sum_{i=1}^{n}\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \rho^2_{\mathfrak{N}}(\mathfrak{y}_i) + \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\mathfrak{N}}(\mathfrak{y}_i)\right|^2} \quad (7)$$

Definition 2.8. *Kumar et al. (2023).* Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be two PFSs in $\mathfrak{Y}$ such that $\mathfrak{Y} = \{ \mathfrak{y}_1 , \mathfrak{y}_2 , \ldots\ldots\ldots , \mathfrak{y}_{n-1} , \mathfrak{y}_n \}$ then distance measure between $\dot{\mathrm{M}}$ and $\mathfrak{N}$ can be defined as

$$\mathcal{DM}^8(\dot{\mathrm{M}},\mathfrak{N}) = 1 - \frac{\sum_{i=1}^{n}\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i).\rho^2_{\mathfrak{N}}(\mathfrak{y}_i) + \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i).\xi^2_{\mathfrak{N}}(\mathfrak{y}_i)}{\sum_{i=1}^{n}\left[\left(\rho^4_{\dot{\mathrm{M}}}(\mathfrak{y}_i) \vee \rho^4_{\mathfrak{N}}(\mathfrak{y}_i)\right) + \left(\xi^4_{\dot{\mathrm{M}}}(\mathfrak{y}_i) \vee \xi^4_{\mathfrak{N}}(\mathfrak{y}_i)\right)\right]} \quad (8)$$

Entropy

Some existing entropy measures are as follows:

Definition 2.8. *Gandotra et al. (2021).* Let $\dot{\mathrm{M}}$ be an PFS in $\mathfrak{Y}$ such that $\dot{\mathrm{M}} = < \mathfrak{y} , \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) , \xi_{\dot{\mathrm{M}}}(\mathfrak{y}) \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ then the Pythagorean fuzzy entropy is defined as :

$$\mathrm{E}(\dot{\mathrm{M}}) = \frac{1}{n}\sum_{i=1}^{n}\left[\sec\left(\frac{\pi}{3} - \frac{\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i)\right|}{3}\pi\right) - 1\right] \quad (9)$$

Definition 2.9. *Chaurasiya and Jain (2022).* Let $\dot{\mathrm{M}}$ be an PFS in $\mathfrak{Y}$ such that $\dot{\mathrm{M}} = < \mathfrak{y} , \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) , \xi_{\dot{\mathrm{M}}}(\mathfrak{y}) \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ then the Pythagorean fuzzy entropy is defined as :

$$\mathrm{E}(\dot{\mathrm{M}}) = \frac{1}{n}\sum_{i=1}^{n}\left[\left\{\sqrt{2}\cos\left(\pi\left(\frac{\left|\rho^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i) - \xi^2_{\dot{\mathrm{M}}}(\mathfrak{y}_i)\right|}{4}\right)\right) - 1\right\}\frac{1}{\sqrt{2}-1}\right] \quad (10)$$

III. Proposed Distance measure

In this section we will propose distance measure between two PFSs $\dot{\mathrm{M}}$ and $\mathfrak{N}$ as follows and also prove the properties satisfied by them.

Definition 3.1. Let $\dot{\mathrm{M}}$ and $\mathfrak{N}$ be the two PFSs in $\mathfrak{Y}$ such that $\dot{\mathrm{M}} = \{< \rho_{\dot{\mathrm{M}}}(\mathfrak{y}) , \xi_{\dot{\mathrm{M}}}(\mathfrak{y}) > \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ and $\mathfrak{N} = \{< \rho_{\mathfrak{N}}(\mathfrak{y}) , \xi_{\mathfrak{N}}(\mathfrak{y}) > \mid \forall\, \mathfrak{y} \in \mathfrak{Y} \}$ then the distance measure between the two sets can be formulated as:

$$\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})=1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}\left(\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\right)\right)\right) \quad (11)$$

$$\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})=1-\frac{1}{n}\sum_{i=1}^{n}w_i\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}\left(\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\right|\right)\right)\right) \quad (12)$$

Where $\theta_{\dot{\text{M}}}(\mathfrak{y})=\sqrt{1-\rho_{\dot{\text{M}}}^2(\mathfrak{y})-\xi_{\dot{\text{M}}}^2(\mathfrak{y})}$ and $\theta_{\mathfrak{N}}(\mathfrak{y})=\sqrt{1-\rho_{\mathfrak{N}}^2(\mathfrak{y})-\xi_{\mathfrak{N}}^2(\mathfrak{y})}$

$w=(w_1,w_2,\dots,w_n)^T$ be the vector denoting weight corresponding to $\mathfrak{y}_i(i=1,2,\dots,n)$, where $w_k\in[0,1]$, $k=1,2,\dots,n$, $\sum_{k=1}^{n}w_k=1$. If $\omega=\left(\frac{1}{n},\frac{1}{n},\dots\frac{1}{n}\right)^T$, then the weighted logarithmic divergence measure becomes proposed measure. If $w_k=1$, $k=1,2,\dots,n$, then $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})=\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})$.

Theorem and Proof

$\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})$ *and* $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})$ *,* $\dot{\text{M}}$ *and* $\mathfrak{N}$ *in* $\mathfrak{Y}$ *the then divergence measure fulfils following properties:*

1. $0\le\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})$, $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})\le 1$
2. $\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})$, $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})=0\Leftrightarrow\dot{\text{M}}=\mathfrak{N}$
3. $\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})=\mathcal{DM}^{10}(\mathfrak{N},\dot{\text{M}})$ and $\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})=\mathcal{DM}^{10}(\mathfrak{N},\dot{\text{M}})$
4. If $\mathcal{O}$ is a PFS in $\mathfrak{Y}$ and $\dot{\text{M}}\subseteq\mathfrak{N}\subseteq\mathcal{O}$, then

(i) $\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})\le\mathcal{DM}^9(\dot{\text{M}},\mathcal{O})$ and $\mathcal{DM}^9(\mathfrak{N},\mathcal{O})\le\mathcal{DM}^9(\dot{\text{M}},\mathcal{O})$

(ii) $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})\le\mathcal{DM}^{10}(\dot{\text{M}},\mathcal{O})$ and $\mathcal{DM}^{10}(\mathfrak{N},\mathcal{O})\le\mathcal{DM}^{10}(\dot{\text{M}},\mathcal{O})$

Proof:

1. $0\le\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})\le 1$

$\because 0\le\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\le 1,\ 0\le\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\le 1,\ 0\le\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\le 1$

$\Rightarrow 0\le\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\le 3$

$\Rightarrow 0\le\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\le\frac{\pi}{4}$

$$\Rightarrow 0\le\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}\left(\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\right)\right)\right)\le 1$$

$\Rightarrow 0\le\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})\le 1$

Similarly, we can prove for $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})$

2. $\mathcal{DM}^9(\dot{\text{M}},\mathfrak{N})=0\Leftrightarrow\dot{\text{M}}=\mathfrak{N}$

$$\Leftrightarrow 1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}\left(\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\right)\right)\right)=0$$

$$\Leftrightarrow\sum_{i=1}^{n}\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}\left(\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|+\left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|\right)\right)\right)=\text{n}$$

$\Leftrightarrow\left|\rho_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|=0,\ \left|\xi_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|=0,\ \left|\theta_{\dot{\text{M}}}^2(\mathfrak{y}_i)-\theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|=0$

$\Leftrightarrow\rho_{\dot{\text{M}}}(\mathfrak{y}_i)=\rho_{\mathfrak{N}}(\mathfrak{y}_i),\ \xi_{\dot{\text{M}}}(\mathfrak{y}_i)=\xi_{\mathfrak{N}}(\mathfrak{y}_i),\ \theta_{\dot{\text{M}}}(\mathfrak{y}_i)=\theta_{\mathfrak{N}}(\mathfrak{y}_i)$

$\Leftrightarrow\dot{\text{M}}=\mathfrak{N}$

Similarly, we can prove for $\mathcal{DM}^{10}(\dot{\text{M}},\mathfrak{N})$

3. $\mathcal{DM}^9(\dot{M},\mathfrak{N}) = \mathcal{DM}^9(\mathfrak{N},\dot{M})$

Proof is obvious

Similarly for $\mathcal{DM}^{10}(\dot{M},\mathfrak{N})$.

4. If $\mathcal{O}$ is a PFS in $\mathfrak{Y}$ and $\dot{M}\subseteq \mathfrak{N} \subseteq\mathcal{O}$, then $\mathcal{DM}^9(\dot{M} , \mathfrak{N}) \le \mathcal{DM}^9(\dot{M} , \mathcal{O})$ and $\mathcal{DM}^{10}(\mathfrak{N} , \mathcal{O}) \le \mathcal{DM}^{10}(\dot{M} , \mathcal{O})$.

Proof. For $\mathcal{DM}^9(\dot{M},\mathfrak{N})$, If $\dot{M}\subseteq \mathfrak{N} \subseteq\mathcal{O}$, then for $\mathfrak{y}_i\in \mathfrak{Y}$, we have $0\le \rho_{\dot{M}}(\mathfrak{y}_i) \le \rho_{\mathfrak{N}}(\mathfrak{y}_i) \le \rho_{\mathcal{O}}(\mathfrak{y}_i) \le 1$ and $1\ge \xi_{\dot{M}}(\mathfrak{y}_i) \ge \xi_{\mathfrak{N}}(\mathfrak{y}_i)\ge \xi_{\mathcal{O}}(\mathfrak{y}_i) \ge 0$.

$\Rightarrow \le\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| \le \left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right|,\ \left|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| \le \left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right|$

$\Rightarrow \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| \le \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right|,\ \left|\xi_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| \le \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right|$

$\Rightarrow \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| \le \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|,\ \left|\theta_{\mathfrak{N}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right| \le \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|$

Adding above equations

$\Rightarrow \left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{N}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| \le \left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|$

And $\left|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\mathfrak{N}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right| \le \left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|$

$\Rightarrow \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathfrak{N}}^2(\mathfrak{y}_i)\right|)) \ge \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|))$

And $\tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\mathfrak{N}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|)) \ge \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|))$

$\Rightarrow 1 - \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4} - \left(\frac{\pi}{12}\left(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\dot{N}}^2(\mathfrak{y}_i)\right|\right)\right)\right) \le 1 - \frac{1}{n}\sum_{i=1}^{n} \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|))$

and $1 - \frac{1}{n}\sum_{i=1}^{n} \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\mathfrak{N}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right| \le 1 - \frac{1}{n}\sum_{i=1}^{n} \tan(\frac{\pi}{4} - \frac{\pi}{12}(\left|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\mathcal{O}}^2(\mathfrak{y}_i)\right| + \left|\theta_{\dot{M}}^2(\mathfrak{y}_i) - \theta_{\mathcal{O}}^2(\mathfrak{y}_i)\right|))$

$\Rightarrow \mathcal{DM}^9(\dot{M} , \mathfrak{N}) \le \mathcal{DM}^9(\dot{M} , \mathcal{O})$ and $\mathcal{DM}^9(\mathfrak{N} , \mathcal{O}) \le \mathcal{DM}^9(\dot{M} , \mathcal{O})$

IV. Numerical Example

In this section, we will verify the distance measure proposed using an example:

Let $\dot{M} = \{ \mathfrak{y}_1,0.6,0.2 \} , \{ \mathfrak{y}_2 ,0.4 , 0.6 \} , \{ \mathfrak{y}_3 , 0.5,0.3\}$

$\mathfrak{N} = \{\mathfrak{y}_1, 0.8 , 0.1\} , \{\mathfrak{y}_2 , 0.7,0.3\} , \{\mathfrak{y}_3 , 0.6,0.3\}$

$\mathcal{O}= \{\mathfrak{y}_1 , 0.9 , 0.1\} , \{\mathfrak{y}_2 , 0.8,0.2\} , \{\mathfrak{y}_3 , 0.7, 0.1\}$

$$\mathcal{DM}^9(\dot{M}, \mathfrak{N})=1 - \frac{1}{3}\left[\tan\left(\frac{\pi}{4} - \left(\frac{\pi}{12}(|0.6^2 - 0.8^2| + |0.2^2 - 0.1^2| + |0.77^2 - 0.59^2|)\right)\right) + \tan\left(\frac{\pi}{4} - \frac{\pi}{12}(|0.4^2 - 0.7^2| + |0.6^2 - 0.3^2| + |0.69^2 - 0.64^2|)\right) + tan\left(\frac{\pi}{4} - \frac{\pi}{12}(|0.5^2 - 0.6^2| + |0.3^2 - 0.3^2| + |0.81^2 - 0.74^2|)\right)\right]$$

$\Rightarrow \mathcal{DM}^9(\dot{M}, \mathfrak{N}) = 0.22$

$\mathcal{DM}^9(\mathfrak{N},\mathcal{O})=1-\frac{1}{3}\left[\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}(|0.8^2-0.9^2|+|0.1^2-0.1^2|+|0.59^2-0.42^2|)\right)\right)+\tan\left(\frac{\pi}{4}-\frac{\pi}{12}(|0.7^2-0.8^2|+|0.3^2-0.2^2|+|0.64^2-0.56^2|)\right)+tan\left(\frac{\pi}{4}-\frac{\pi}{12}(|0.6^2-0.7^2|+|0.3^2-0.1^2|+|0.74^2-0.70^2|)\right)\right]$

$\Rightarrow \mathcal{DM}^9(\mathfrak{N},\mathcal{O})=0.14$

$\mathcal{DM}^9(\dot{\mathrm{M}},\mathcal{O})=1-\frac{1}{3}\left[\tan\left(\frac{\pi}{4}-\left(\frac{\pi}{12}(|0.6^2-0.9^2|+|0.2^2-0.1^2|+|0.77^2-0.42^2|)\right)\right)+\tan\left(\frac{\pi}{4}-\frac{\pi}{12}(|0.4^2-0.8^2|+|0.6^2-0.2^2|+|0.69^2-0.56^2|)\right)+tan\left(\frac{\pi}{4}-\frac{\pi}{12}(|0.6^2-0.7^2|+|0.3^2-0.1^2|+|0.74^2-0.70^2|)\right)\right]$

$\Rightarrow \mathcal{DM}^9(\dot{\mathrm{M}},\mathcal{O})=0.34$

In the same way we can prove for equation (12).

V. PROPOSED ENTROPY

In this part we will propose the entropy measure as follows:

$$E(\dot{\mathrm{M}})=\frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4}-\frac{|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)|}{4}\pi\right\}\right] \quad (13)$$

The proposed entropy measure needs to satisfy the following properties:

Let $\dot{\mathrm{M}}=\{\mathfrak{y}_i,\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i),\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)|\mathfrak{y}_i\in\mathfrak{Y}\}$ and $\mathfrak{N}=\{\mathfrak{y}_i,\rho_{\mathfrak{N}}(\mathfrak{y}_i),\xi_{\mathfrak{N}}(\mathfrak{y}_i)|\mathfrak{y}_i\in\mathfrak{Y}\}$ be two PFS($\mathfrak{Y}$), the entropy for $E: PFSs\rightarrow[0,1]$ which is a crisp function should match the following criteria:

(1) $0\le E(\dot{\mathrm{M}}),E(\mathfrak{N})\le 1$

(2) If $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=0,\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=1$ or $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=1,\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=0$, then $E(\dot{\mathrm{M}})=0$.

(3) $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i), then\ E(\dot{\mathrm{M}})=1$.

(4) $E(\dot{\mathrm{M}}^c)=E(\dot{\mathrm{M}})$.

(5) $E(\dot{\mathrm{M}})\le E(\mathfrak{N})$, if $\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\le\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)$, $\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\le\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$ or $\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\ge\rho_{\mathfrak{N}}^2(\mathfrak{y}_i)$, $\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\ge\xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$ for all $\mathfrak{y}_i\in\mathfrak{Y}$.

Proof :

(1) $0\le E(\dot{\mathrm{M}}),E(\mathfrak{N})\le 1$

$0\le\left|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\right|\le 1$

$\Rightarrow 0\le\frac{\pi}{4}\left|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\right|\le\frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{4}\le-\frac{\pi}{4}\left|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\right|\le 0$

$\Rightarrow 0\le\frac{\pi}{4}-\frac{\pi}{4}\left|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\right|\le\frac{\pi}{4}$

$\Rightarrow 0\le\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\left|\rho_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)-\xi_{\dot{\mathrm{M}}}^2(\mathfrak{y}_i)\right|\right)\le 1$

(2) If $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=0,\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=1$ or $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=1,\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=0$, then $E(\dot{\mathrm{M}})=0$.

$\Rightarrow$If $\rho_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=0,\xi_{\dot{\mathrm{M}}}(\mathfrak{y}_i)=1$ then

$\Rightarrow E(\dot{\mathrm{M}})=\frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4}-\frac{|-1|}{4}\pi\right\}\right]$

$\Rightarrow E(\dot{\mathrm{M}})=0$

Similarly for other case.

(3) $\rho_{\dot{M}}(\mathfrak{y}_i) = \xi_{\dot{M}}(\mathfrak{y}_i), then\ E(\dot{M}) = 1$

⇒If $\rho_{\dot{M}}(\mathfrak{y}_i) = \xi_{\dot{M}}(\mathfrak{y}_i)$

$\Rightarrow E(\dot{M}) = \frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4} - \frac{0}{4}\pi\right\}\right]$

$\Rightarrow E(\dot{M}) = 1$

(4) $E(\dot{M}^c) = E(\dot{M})$

$$E(\dot{M}^c) = \frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4} - \frac{|\xi_{\dot{M}}^2(\mathfrak{y}_i) - \rho_{\dot{M}}^2(\mathfrak{y}_i)|}{4}\pi\right\}\right] = \frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4} - \frac{|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\dot{M}}^2(\mathfrak{y}_i)|}{4}\pi\right\}\right] = E(\dot{M})$$

(5) $E(\dot{M}) \leq E(\mathfrak{N})$, if $\rho_{\dot{M}}^2(\mathfrak{y}_i) \leq \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)$, $\xi_{\dot{M}}^2(\mathfrak{y}_i) \leq \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$ or $\rho_{\dot{M}}^2(\mathfrak{y}_i) \geq \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)$, $\xi_{\dot{M}}^2(\mathfrak{y}_i) \geq \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$ for all $\mathfrak{y}_i \in \mathfrak{Y}$.

If $\rho_{\dot{M}}^2(\mathfrak{y}_i) \geq \rho_{\mathfrak{N}}^2(\mathfrak{y}_i)$, $\xi_{\dot{M}}^2(\mathfrak{y}_i) \geq \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$

⇒ Subtracting the two

$\Rightarrow \rho_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\dot{M}}^2(\mathfrak{y}_i) \geq \rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)$

$$\Rightarrow \tan\left(\frac{\pi}{4} - \frac{|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\dot{M}}^2(\mathfrak{y}_i)|}{4}\pi\right) \leq \tan\left(\frac{\pi}{4} - \frac{|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)|}{4}\pi\right)$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4} - \frac{|\rho_{\dot{M}}^2(\mathfrak{y}_i) - \xi_{\dot{M}}^2(\mathfrak{y}_i)|}{4}\pi\right\}\right] \leq \frac{1}{n}\sum_{i=1}^{n}\left[tan\left\{\frac{\pi}{4} - \frac{|\rho_{\mathfrak{N}}^2(\mathfrak{y}_i) - \xi_{\mathfrak{N}}^2(\mathfrak{y}_i)|}{4}\pi\right\}\right]$$

$\Rightarrow E(\dot{M}) \leq E(\mathfrak{N})$

Hence $E(\dot{M})$ satisfies all properties of entropy measure.

VI. Application Of Distance Measure Using VIKOR Method

Briefing of VIKOR method

Step 1: Constructing a matrix of criteria and different alternatives

Let $\mathcal{M}$ = {$\mathcal{M}_1$, $\mathcal{M}_2$,…….. , $\mathcal{M}_n$} be the set of n alternatives and the DM will choose from Ĉ = { $\hat{C}_1$, $\hat{C}_2$, …….. , $\hat{C}_m$ } the set of m criteria. Let $\mathcal{L} = [\ell_{ij}^k]_{m\times n}$ be the linguistic matrix by k DM, here ℓ_{ij}^k is the value of alternative $\mathcal{M}_i$ vs $\hat{C}_i$ criteria in terms of linguistic variables for k^{th} DM.

Step 2: Normalizing the decision matrix.

The fuzzy decision matrix is calculated by interchanging the values of membership and non-membership value in case of worst criteria and no interchange in case of best criteria.

Step 3: Determination of weights by Proposed Entropy measure.

The weights of the decision maker are calculated using equation (13).

Step 4: Calculate the fuzzy PIS and fuzzy NIS.

In VIKOR method, the criteria for assessment are divided into two categories the best and the worst criteria. The PFPIS and PFNIS can be specified as:

$\mathcal{M}^{k+}$ = { r_1^{k+} , r_2^{k+} , …… , r_n^{k+} } and $\mathcal{M}^{k-}$ = { r_1^{k-} , r_2^{k-} , …… , r_n^{k-} } then

$\mathcal{M}^{k+}$ = {(max$_i$ (r_i^k), min$_i$ (r_i^k))} for $\hat{C}_i$

$\mathcal{M}^{k-}$ = {(min$_i$ (r_i^k), max$_i$ (r_i^k))} for $\hat{C}_i$

Step 5: Calculate divergence measure for PIS and NIS

$$\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i) = 1 - \frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{4} - \left(\frac{\pi}{12}\left(\left|\rho_{\mathcal{A}_i^+}^2(\mathfrak{y}_i) - \rho_{\mathcal{A}_i}^2(\mathfrak{y}_i)\right| + \left|\xi_{\mathcal{A}_i^+}^2(\mathfrak{y}_i) - \xi_{\mathcal{A}_i}^2(\mathfrak{y}_i)\right| + \left|\theta_{\mathcal{A}_i^+}^2(\mathfrak{y}_i) - \theta_{\mathcal{A}_i}^2(\mathfrak{y}_i)\right|\right)\right)\right) \quad (14)$$

$\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i^-)$=1 $- \frac{1}{n}\sum_{i=1}^{n} \tan\left(\frac{\pi}{4} - \left(\frac{\pi}{12}\left(\left|\rho^2_{\mathcal{A}_i^+}(\mathfrak{y}_i) - \rho^2_{\mathcal{A}_i^-}(\mathfrak{y}_i)\right| + \left|\xi^2_{\mathcal{A}_i^+}(\mathfrak{y}_i) - \xi^2_{\mathcal{A}_i^-}(\mathfrak{y}_i)\right| + \left|\theta^2_{\mathcal{A}_i^+}(\mathfrak{y}_i) - \theta^2_{\mathcal{A}_i^-}(\mathfrak{y}_i)\right|\right)\right)\right)$
(15)

$$\mathcal{S}_i = \sum_{i=1}^{n}\left(\mathcal{w}_j * \frac{\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i)}{\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i^-)}\right) \quad (16)$$

$$\mathcal{R}_i = max_i\left(\mathcal{w}_j * \frac{\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i)}{\mathcal{DM}(\mathcal{A}_i^+, \mathcal{A}_i^-)}\right) \quad (17)$$

$$\mathcal{Q}_i = \kappa\left(\frac{\mathcal{S}_i - \mathcal{S}^+}{\mathcal{S}^- - \mathcal{S}^+}\right) + (1 - \kappa)\left(\frac{\mathcal{R}_i - \mathcal{R}^+}{\mathcal{R}^- - \mathcal{R}^+}\right) \quad (18)$$

Here κ and $1 - \kappa$ denotes the weights of best and worst criteria and $\mathcal{S}^+ = min_i\ \mathcal{S}_i$, $\mathcal{S}^- = max_i\ \mathcal{S}_i$, $\mathcal{R}^+ = min_i\ \mathcal{R}_i$, $\mathcal{R}^- = max_i\ \mathcal{R}_i$.
Step 6 : Determine the compromise solution
The values of $\mathcal{S}_i$, $\mathcal{R}_i$ and $\mathcal{Q}_i$ are ranked in decreasing order if the following two conditions are satisfied:
Case 1 : Acceptable advantage

$$\mathcal{Q}(\mathcal{A}^2) - \mathcal{Q}(\mathcal{A}^1) \geq \frac{1}{n-1} \quad (19)$$

where $\mathcal{Q}(\mathcal{A}^1)$ is the first and $\mathcal{Q}(\mathcal{A}^2)$ is second ranked and n is the number of alternatives.
Case 2: Adequate stability

$\mathcal{Q}(\mathcal{A}^1)$ is ranked by $\mathcal{S}_i$ and $\mathcal{R}_i$. In case 1 is not satisfied then $\mathcal{Q}(\mathcal{A}^2) - \mathcal{Q}(\mathcal{A}^1) < \frac{1}{n-1}$ (20)

Step 7: Rank the alternatives
The alternatives are ranked according to decreasing values of $\mathcal{S}_i$, $\mathcal{R}_i$ and $\mathcal{Q}_i$. The lowest value of $\mathcal{Q}_i$ being the best alternative.
The VIKOR method can be explained with the help of flowing flow chart (Figure 1).

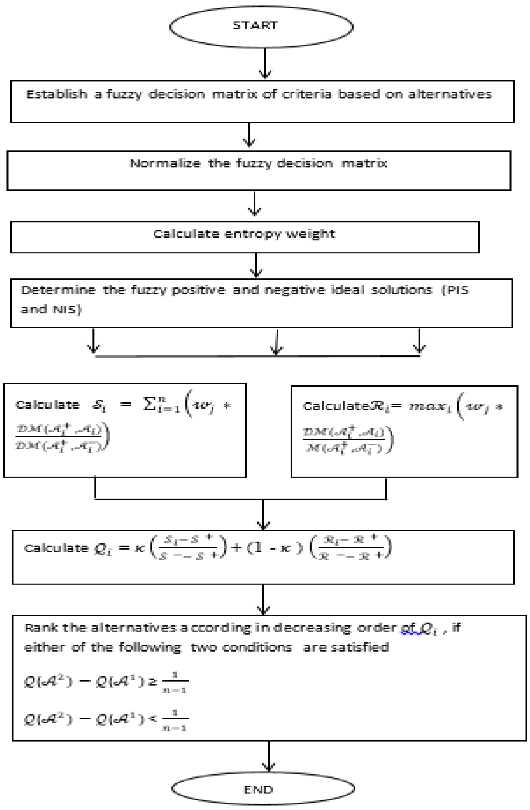

Figure 1: VIKOR method

6.1. Case study

When it comes to making an appropriate decision there are many options which are prioritized by a person. In this section we will talk about two such cases where decision has to be taken by ranking different options available so as to make best decision out of available options.

6.1.1. Career Selection

Choosing a best career option is also a tedious task. Sometimes a person is very clear with the option that has to be chosen but it is not always the case. Many students after graduating from school face these issues. Here we have listed five career options career 1(J_1), career 2(J_2), career 3(J_3) , career 4(J_4) and career 5(J_5) that are considered by most of the students who are generally confused and rank career on the basis of benefits like minimum number of years spent ($\mathfrak{C}_1$), difficulty level($\mathfrak{C}_2$), fees($\mathfrak{C}_3$), average salary($\mathfrak{C}_4$), growth perspectives($\mathfrak{C}_5$) provided after doing them.

Step 1: Decision matrix in the form of PFN is given as:

Table 1: Data set in the form of decision matrix

$\mathcal{DM}$	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$
J_1	<0.7,0.2>	<0.85,0.1>	<0.9,0.3>	<0.8,0.1>	<0.8,0.2>
J_2	<0.0,1.0>	<0.6,0.2>	<0.6,0.1>	<0.6,0.2>	<0.5,0.3>
J_3	<0.6,0.2>	<0.4,0.3>	<0.2,0.6>	<0.4,0.3>	<0.5,0.4>
J_4	<0.5,0.2>	<0.7,0.4>	<0.6,0.2>	<0.7,0.2>	<0.7,0.1>
J_5	<0.6,0.1>	<0.5,0.6>	<0.4,0.15>	<0.5,0.4>	<0.6,0.3>

Step 2: Normalizing data

On the basis of the data, these options are to be ranked and the best option is to be determined. In MCDM there are two types of criteria: benefit and cost criteria. Benefit criteria are criteria which desire higher value and cost criteria are those with low values. In the following case $\mathfrak{C}_1$, $\mathfrak{C}_2$, $\mathfrak{C}_3$ are cost criteria and $\mathfrak{C}_4$ and $\mathfrak{C}_5$ are benefit criteria. The normalized value can be found by interchanging membership and non- membership value for cost criteria and no interchange for benefit criteria.

Table 2: Normalization of the decision matrix

$\mathcal{DM}$	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$
J_1	<0.2,0.7>	<0.1,0.85>	<0.3,0.9>	<0.8,0.1>	<0.8,0.2>
J_2	<1.0,0.0>	<0.2,0.6>	<0.1,0.6>	<0.6,0.2>	<0.5,0.3>
J_3	<0.2,0.6>	<0.3,0.4>	<0.6,0.2>	<0.4,0.3>	<0.5,0.4>
J_4	<0.2,0.5>	<0.4,0.7>	<0.2,0.6>	<0.7,0.2>	<0.7,0.1>
J_5	<0.1,0.6>	<0.6,0.5>	<0.15,0.4>	<0.5,0.4>	<0.6,0.3>

Step 3: Calculate entropy weight

Entropy weights are calculated by using the equation (13).

The weights calculated are w_1 = 0.3076, w_2 = 0.5037, w_3 = 0.7683, w_4 = 0.5561, w_5= 0.7436.

Step 4: Determine the FPIS and FNIS

Table 3: FPIS and FNIS for each criteria

$\mathcal{DM}$	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$
A^+	<0.1.0.7>	<0.1,0.85>	<0.1,0.9>	<0.8,0.1>	<0.8,0.1>
A^-	<0.2,0.0>	<0.6,0.5>	<0.6,0.2>	<0.4,0.4>	<0.5,0.4>

Step 5: Compute distance measure values by using equations (14) and (15)

Table 4: Separation measure for FPIS

	$\mathcal{DM}$	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$
FOR FPIS	J_1	0.1156	0.0871	0.0756	0.1561	0.1546
	J_2	0.1140	0.1201	0.1054	0.1510	0.1401
	J_3	0.1337	0.1137	0.1248	0.1277	0.1457
	J_4	0.1497	0.1104	0.1080	0.1503	0.1526
	J_5	0.1364	0.1328	0.0945	0.1457	0.1485

Table 5: Separation measure for FNIS

	$\mathcal{DM}$	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$
FOR FNIS	J_1	0.1196	0.1351	0.1212	0.1177	0.1294
	J_2	0.0535	0.1527	0.1556	0.1527	0.1539
	J_3	0.1381	0.1337	0.1383	0.1616	0.1482
	J_4	0.1551	0.1443	0.1527	0.1376	0.1526
	J_5	0.1411	0.1408	0.1583	0.1482	0.1485

Step 6: Compute $\mathcal{S}^+$, $\mathcal{R}^+$ and $\mathcal{Q}_i$ using equations (16), (17) and (18)

Table 6: Compute $\mathcal{S}_i$, $\mathcal{R}_i$ and $\mathcal{Q}_i$

$\mathcal{DM}$	$\mathcal{S}_i$	$\mathcal{R}_i$	$\mathcal{Q}_i$
J_1	3.5784	1.0501	0
J_2	3.9452	1.2095	0.5461
J_3	4.1046	1.4314	0.9838
J_4	4.1223	1.2391	0.7478
J_5	3.9771	1.0841	0.4111

Step 7: Determine the ranking for criteria

Table 7: Ranking result obtained from VIKOR method

$\mathcal{DM}$	Ranking
J_1	1
J_2	3
J_3	5
J_4	4
J_5	2

Table 7 shows J_1 is the best ranked as it has minimum value of $\mathcal{Q}_i$. Then it has to check two conditions using DQ = $\frac{1}{n-1}$ where n = 5 which gives DQ = 0.25.

Check $\mathcal{Q}(\mathcal{A}^2) - \mathcal{Q}(\mathcal{A}^1)$ = 1 ≥ 0.25. Since the condition 1 is satisfied J_1 and J_5 are the best alternative.

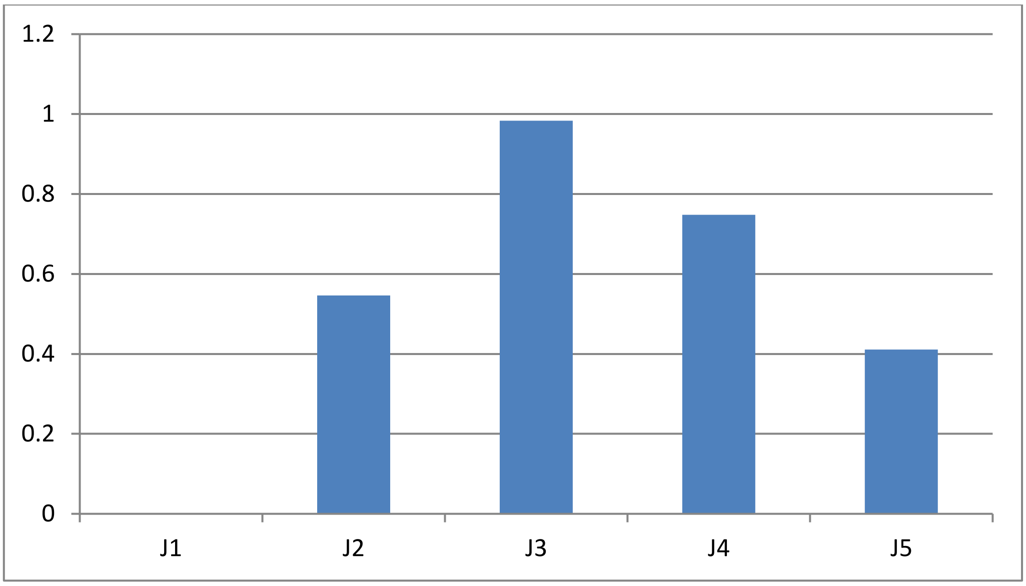

Figure 2: Ranking of alternatives

Result Discussion

From the graph we can see J_1 corresponds to the least value of Q_i and hence after checking the conditions by equations (17) and (18) we can conclude J_1 is the best alternative.

VII. Conclusion

VIKOR method is used as a ranking tool for solving complex decision making problems. It is easy to understand which makes widely usable. In this paper we have presented distance measures under PFS which are easily applicable with MCDM techniques making it an easy tool which can be used in variety of situations. An entropy measure is also proposed to evaluate the weights given to each criteria making it more reliable as the weights are not hypothetically assumed but calculated. Axioms for both the distance measure and entropy have been proposed and numerical illustrations are shown to ensure the validity of this measure. A real life application has been presented to show the applicability of the measure in real world and express that this entropy distance measure can be applied to variety of other decision making problems. Flow charts and graphs are given to discuss the result in an efficient manner. The proposed work is not limited to VIKOR method, it can be applied to other MCDM techniques like TOPSIS, CORPAS, AHP, SAW etc. also along with it can be applied it in clustering , image processing , supplier selection and many more.

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Data Availability

All the tables and images have been created by the authors.

Conflicts of Interest

Authors affirm that there is no conflict of Interest.

Statements and Declarations

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