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## Synergetic Control of Spatial Motion for Spacecraft



**Abstract:** - This article demonstrates the implementation of new principles and nonlinear methods for synergetic control theory to develop rules that govern the spatial movement of a spacecraft. The text introduces a model of control systems that incorporates the principles of "expansion-contraction" of phase space and the notion of equivalence (conservation) of controls. The proposed approach employs a coordinated control strategy that encompasses all state variables to effectively transition the item to the target state. In certain control algorithms, the interconnections between control channels are established directly within the regulator, rather than being mediated through the control object. Subsequently, the implemented techniques generated control rules that guarantee the spacecraft's transition from any initial condition within a specific acceptable range to the target state. The findings obtained from the nonlinear model demonstrate the overall asymptotic stability, robustness to parameter variations, and resistance to external disturbances of the closed system "SC-autopilot".

**Keywords:** Angular motion, invariant manifolds, spacecraft, synergetic synthesis.

### I. INTRODUCTION

A crucial criterion in the design of control systems for contemporary aircraft is the imperative precision in determining and regulating the parameters of their motion. Consequently, it is necessary to consider multiple unregulated variables when formulating suitable control algorithms. Initially, we are discussing unpredictable variables that influence the airplane while it is in motion. These factors encompass air disruptions such as variances in density and wind gusts, inaccuracies in determining control actions, discrepancies in geometric, aerodynamic, and other properties of the controlled item from the calculated values, and several other factors [4]-[8]. It is important to acknowledge that the majority of aviation accidents are caused by adverse meteorological conditions. A microburst, which is a localized disturbance of the atmospheric conditions, is the most hazardous meteorological occurrence for aircraft missions. It manifests as a vortex ring of powerful winds. The occurrence poses a significant risk to aircraft throughout the process of landing and departure [9]-[13]. The literature study in this topic provides a wealth of control strategies for aircraft control, with a focus on control algorithms [14]-[18].

The study [14] offers a thorough examination of aircraft transformation. This paper examines both the general and specific challenges involved in the creation of intelligent morphing aircraft. The paper [15] introduces a controller called MnHR-NEPID, which is designed for regulating hormones with multiple nodes using neuroendocrine proportional integral differential regulation. The controller is based on the dynamics of adaptive safe experiments (ASED) and is specifically designed for nonlinear systems with multiple inputs and multiple outputs (MIMO). The article [16] examined the utilization of a data-driven sigmoid secretion rate of neuroendocrine PID (SbSRNEPID) in a two-rotor MIMO system (TRMS) using the ASED algorithm. The paper [17] discusses the findings of a study on the flight dynamics of a theoretical aircraft capable of maneuvering. This aircraft has inherent drawbacks in terms of stability and controllability. However, these drawbacks are mitigated by algorithms implemented in a stability and controllability enhancement system. This system introduces a sophisticated boundary for the permissible angle of attack, which varies depending on the flight mode. The article [18] examines different vertical takeoff and landing (VTOL) configurations and subsequently discusses the dynamics, modeling tools, and control strategies for a quadcopter. A technique was created in [19] to construct a robust control system for  $H_\infty$  synthesis

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tools. This methodology enables more effective handling of parameter perturbation and load perturbation. The study in [20] investigates the application of  $H_2$  and  $H_\infty$  synthesis methods to regulate the vertical plane flight of an airplane during landing, specifically in the presence of uncertain disturbances. The study in [21] examines the development of a resilient mixed  $H_2/H_\infty$  type controller for spacecraft flight control in the presence of disturbances. In practice, the synthesis of autopilots typically relies on control theory methods and approaches. These methods are based on linearized mathematical models of moving objects, which are represented as input-output ratios. During specific driving modes that require exceptional maneuverability, it is crucial to have an autopilot system that precisely considers the characteristics of the controlled vehicle. The resolution to this issue entails creating motion control regulations by utilizing a comprehensive nonlinear mathematical model of the item. This model provides the most precise representation of the system's dynamics from a physical perspective.

This article suggests applying the principles and methods of synergetic control theory to create the laws of control for the spatial motion of spacecraft. This will ensure that the desired modes of motion are achieved while considering the inherent nonlinear properties of their mathematical models. Instead of using individual stabilizing controls for each control channel (contour), this strategy employs joint (coordinated) control over all variables to achieve the desired state of the object. In certain control algorithms, the interconnections between control channels are established directly within the regulator, instead of being transmitted through the control object.

The article is structured in the following manner. Section 2 introduces a model that combines control systems in a synergistic manner, and focuses on mathematically describing the angular motion of the spacecraft. Section 3 showcases the outcomes of the combined synthesis of the laws that govern the spatial movement of the spacecraft. These laws guarantee the long-term stability of a closed nonlinear system. The efficacy of the proposed control is validated by the outcomes of modeling the synthesized system. Section 4 provides a concise summary of the primary findings of this work and suggests potential areas for future investigation.

## II. RESEARCH METHODS

### 2.1. A model of synergetic synthesis of control systems

The synergetic synthesis problem is formulated by describing the system using differential equations. These equations, denoted as

$$\dot{x}(t) = F(x, u, q, J, M), \quad (1)$$

represent the object's coordinates of the state  $x(t)$  and include external forces such as desired controls  $u(t)$ , specified  $q(t)$ , external  $M(t)$ , and parametric  $J(t)$  influences [22]. The original system, which includes a dynamic object and external forces and disturbances acting on it, is converted into an enhanced model of system synthesis through direct and feedback closure. More precisely, the issue at hand is the creation of the appropriate control laws  $u(x_1, \dots, x_n, z_1, \dots, z_r)$  as a function of the state coordinates of the expanded system. The variables  $z_1, \dots, z_r$  represent the coordinates of the information models for the driving, external, and parametric perturbations. These models are expressed as extra differential equations:

$$\dot{z}(t) = \varphi(x, z), \quad (2)$$

where  $z$  is the estimate of the corresponding disturbing effects.

Equations (1) and (2) jointly constitute a model for the synergistic synthesis of control systems. In the synergetic theory of control of nonlinear dynamic objects, it is recommended to recognize two crucial overarching principles: the "expansion-compression" concept of phase space and the equivalence (conservation) principle of controls. These principles serve as the foundation for a synergistic approach to synthesizing nonlinear control systems.

A novel approach called Analytical Construction of Aggregated Regulators (ACAR) has been developed, utilizing the concept of "expansion-compression" of the phase space. This method involves the introduction of attractive invariant manifolds  $\psi_s(x_1, \dots, x_n) = 0$ , as shown in Fig. 1. These manifolds effectively coordinate the natural properties (such as energy, mechanical, thermal, etc.) of the object with the control task requirements. The mathematical formulation of the problem of synergetic synthesis involves generating a set of feedback control laws  $u(\psi) = u(x)$ , that initially moves the system from any starting state to the vicinity of manifolds  $\psi_s(x) = 0$ . Subsequently, these control laws ensure that the system moves stably along these manifolds until it reaches the target attractors. These attractors ensure the fulfillment of specific invariants, such as technological, mechanical, energy, and others.

The application of the fundamental principles of the ACAR technique in managing nonlinear systems (1) results in the following phases. The first step involves writing the initial differential equations of the object (1), which may be expressed as

$$\begin{aligned} \dot{x}_k(t) &= f_k(x_1, \dots, x_n) + M_k(t), k = 1, 2, \dots, m - 1, m \leq n, \\ \dot{x}_{k+1}(t) &= f_{k+1}(x_1, \dots, x_n) + u_{k+1} + M_{k+1}(t); \\ \dot{x}_n(t) &= f_n(x_1, \dots, x_n) + u_n + M_r(t), \end{aligned} \tag{3}$$

where  $x_1, \dots, x_n$  are the coordinates of the object's state;  $M_1(t), \dots, M_r(t)$  are external disturbing influences;  $u_{k+1}, \dots, u_k$  are controls.

In the subsequent phase, the equations pertaining to the issue of forecasting and mitigating disruptions are incorporated into the system (3):

$$\dot{z}_j(t) = g_j(z_1, \dots, z_r, x_1, \dots, x_n), j = 1, \dots, r. \tag{4}$$

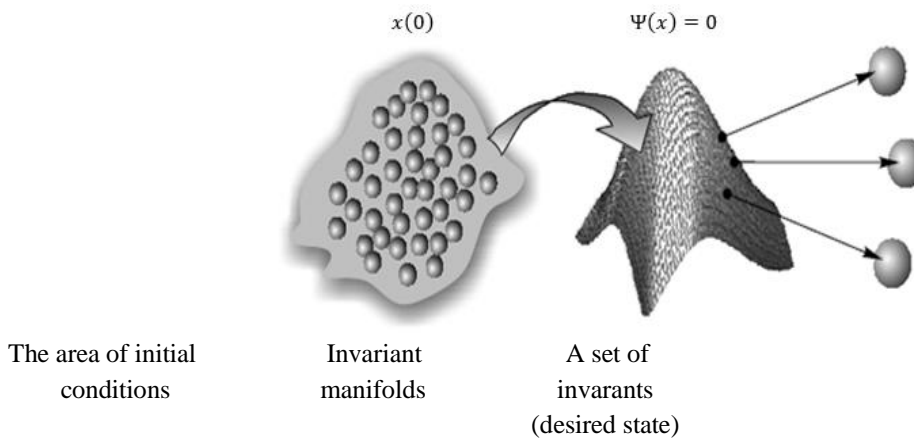


Fig. 1 Problem statement for the synergetic synthesis of systems

When formulating equations (4), there are two distinct tasks: first, describing real perturbations  $M_1(t), \dots, M_r(t)$  as partial solutions of additional differential equations, and second, establishing connections between the equations of the original object and the perturbation equations. Once the coupling equations are chosen, we derive an expanded system of differential equations:

$$\begin{aligned} \dot{z}_j(t) &= g_j(z_1, \dots, z_r, x_1, \dots, x_n), j = 1, \dots, r; \\ \dot{x}_i(t) &= f_i(x_1, \dots, x_n) + z_j, i = r + 1, \dots, m - 1; \\ \dot{x}_{i+1}(t) &= f_{i+1}(x_1, \dots, x_n) + u_{i+1} + z_{j+1}; \\ \dot{x}_n(t) &= f_n(x_1, \dots, x_n) + u_n + z_r. \end{aligned} \tag{5}$$

The extended model (5) enables us to address the task of synergetic synthesis of laws. This task involves finding a control vector  $u(u_1, \dots, u_m)$  that will move the image point (IP) of the extended system (5) from any initial state within an acceptable domain to the target invariant manifolds (IM)

$$\psi_s(x_1, \dots, x_n, z_1, \dots, z_r) = 0, s = 1, 2, \dots, m \tag{6}$$

Consequently, due to the movement along  $\psi_s = 0$  (1.70), IP reaches the designated ultimate state. In this scenario, the external disturbances  $M_1(t), \dots, M_r(t)$  are effectively reduced and the long-term stability of motion is ensured. Additionally, the trajectories of the closed system attain a minimum value of the accompanying optimizing functional (AOF).

$$J_\Sigma = \int_0^\infty [\sum_{s=1}^m \varphi_s^2(\psi_s) + \sum_{s=1}^m T_s^2 \psi_s^2(t)] dt \tag{7}$$

The movement of the image point (IP) of the synthesized system, as stated in equation (7), must adhere to a set of functional equations including macro variables

$$T_s \psi_s(t) + \varphi_s(\psi_s) = 0, s = 1, 2, \dots, m \tag{8}$$

The equations (8) are the Euler-Lagrange equations, which determine the minimum value of AOF (7). This value reflects the integral features of the synthesized systems. Based on reference (7), it is possible to develop different criteria to assess the quality of systems [22]-[24]. Equations (8) are *invariant relations* that are extensively employed in the field of analytical mechanics.

The ACAR approach is employed to solve the aforementioned extended synthesis problem. According to this method, when "external" controls  $u_{i+1}, \dots, u_n$  are applied, the IP of the extended system (4) will be located near the intersection of IM  $\psi_s = 0$ . The movement along this intersection is characterized by decomposed equations of "internal" dynamics.

$$\begin{aligned} \dot{z}_{j\psi}(t) &= g_i(z_{1\psi}, \dots, z_{r\psi}, u_{i+1}, \dots, u_m, x_{1\psi}, \dots, x_{m-1\psi}); \\ \dot{x}_{i\psi}(t) &= f_i(x_{1\psi}, \dots, x_{m-1\psi}, u_{i+1}, \dots, u_m); \\ j &= 1, \dots, r; \quad i = r + 1, \dots, m - 1, \end{aligned} \tag{9}$$

where  $u_{i+1}, \dots, u_m$  are the "internal" controls.

The proposed technique yields a set of interrelated "internal" controls that can be used to construct the appropriate macro variables. For instance, these variables can be represented by the formula

$$\begin{aligned} \psi_s &= \gamma_{s1}(x_{i+1} - u_1) + \dots + \gamma_{sm}(x_n - u_n), \\ s &= 1, \dots, m. \end{aligned} \tag{10}$$

Based on functional equations (8) and macro-variables  $\psi_s$  (10), the "external" control laws are determined using the equations of the system (5).

$$\begin{aligned} u_{i+1} &= -f_{i+1}(x_1, \dots, x_n) - z_{j+1} - \frac{D_1}{D}; \dots \dots \dots \tag{11} \\ u_n &= -f_n(x_1, \dots, x_n) - z_n - \frac{D_n}{D}, \end{aligned}$$

where

$$\begin{aligned} D &= \begin{vmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2m} \\ \dots & \dots & \dots & \dots \\ \gamma_{m1} & \gamma_{m2} & \dots & \gamma_{mm} \end{vmatrix} \neq 0; \\ D_1 &= \begin{vmatrix} \Phi_1 & \gamma_{12} & \dots & \gamma_{1m} \\ \Phi_2 & \gamma_{22} & \dots & \gamma_{2m} \\ \dots & \dots & \dots & \dots \\ \Phi_m & \gamma_{m2} & \dots & \gamma_{mm} \end{vmatrix} \neq 0 \text{ when } \Phi_s = 0; \\ D_n &= \begin{vmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1,m-1} & \Phi_1 \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2m} & \Phi_2 \\ \dots & \dots & \dots & \dots & \dots \\ \gamma_{m1} & \gamma_{m2} & \dots & \gamma_{mm} & \Phi_m \end{vmatrix} \neq 0 \text{ when } \Phi_s \neq 0; \end{aligned}$$

$$\Phi_s = \gamma_{s1}\dot{u}_1(t) + \gamma_{s2}\dot{u}_2(t) + \dots + \gamma_{sn}\dot{u}_n(t) - \frac{1}{T_s} \varphi_s(\psi_s).$$

### 2.2 Formulation of problems of synthesis

The angular motion of the spacecraft relative to the center of mass is described by Euler's dynamic equations [22]:

$$\begin{aligned} A\dot{\omega}_1(t) &= (B - C)\omega_2\omega_3 + M_1; \\ B\dot{\omega}_2(t) &= (C - A)\omega_1\omega_3 + M_2; \\ C\dot{\omega}_3(t) &= (A - B)\omega_1\omega_2 + M_3; \end{aligned} \tag{12}$$

where  $M_1, M_2, M_3$  are the control moments developed by the corresponding engines;  $A, B, C$  are the main central moments of inertia of the spacecraft;  $\omega_1, \omega_2, \omega_3$  are the projection of the angular velocity vector of the spacecraft onto its main central axes of inertia ( $i_1, i_2, i_3$ ). Let us incorporate these into the kinematic Poisson equations:

$$\begin{aligned}\dot{\lambda}_1(t) &= \omega_3\lambda_2 - \omega_2\lambda_3; \\ \dot{\lambda}_2(t) &= \omega_1\lambda_3 - \omega_3\lambda_1; \\ \dot{\lambda}_3(t) &= \omega_2\lambda_1 - \omega_1\lambda_2,\end{aligned}\tag{13}$$

which is used to determine the orientation of a spacecraft. The variables  $\lambda_1, \lambda_2$  and  $\lambda_3$  represent the spacecraft's divergence from the fixed vertical axis. Assume that a link is formed based on the orientation coordinates using the following initial integral:

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1.\tag{14}$$

Then the Euler (12) and Poisson (13) equations, taking into account the invariant (14), collectively describe a system of uniaxial stabilization of the spacecraft equilibrium by means of engines. For  $M_1 = M_2 = M_3 = 0$ , these equations have a particular solution

$$\omega_1 = \omega_2 = \omega_3 = 0;$$

$$\lambda_1 = \lambda_3 = 0;$$

$$\lambda_2 = 1,$$

Corresponding to the equilibrium position of the spacecraft, in which the direction of one of the main central axes of inertia coincides with the direction of the fixed vertical axis. Let's formulate the problem of stabilizing the equilibrium position of the spacecraft using controls. To do this, by introducing new variables  $x_1 = \omega_2, x_2 = \lambda_1, x_3 = \lambda_3, x_4 = \omega_1, x_5 = \omega_3, x_6 = \lambda_2 - 1, u_1 = M_1/B, u_4 = M_2/A, u_5 = M_3/C$  and excluding  $x_6$  by means of the first integral  $\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$ , we will derive the following differential equations of perturbed motion based on the Euler and Poisson equations [25]:

$$\begin{aligned}\dot{x}_1(t) &= \left(\frac{C-A}{B}\right)x_4x_5 + u_1, \\ \dot{x}_2(t) &= -x_1x_3 + x_5\sqrt{1-x_2^2-x_3^2}, \\ \dot{x}_3(t) &= x_1x_2 - x_4\sqrt{1-x_2^2-x_3^2}, \\ \dot{x}_4(t) &= \left(\frac{B-C}{A}\right)x_1x_5 + u_4, \\ \dot{x}_5(t) &= \left(\frac{A-B}{C}\right)x_1x_4 + u_5.\end{aligned}\tag{15}$$

The set of equations (15) represents the rotational movement of the spacecraft with respect to the center of mass near the target position, specifically when  $x_k = 0, k = 1, \dots, 5$ , and the device is oriented along a single axis in inertial space.

The task at hand is to develop control laws, denoted as  $u_r(x_1, \dots, x_5), r = 1, 4, 5$ . These control laws are aimed at guiding the spacecraft (15) from any initial condition within a specified permitted range to a desirable state. Put simply, it is essential to address the issue of analytically designing a comprehensive three-channel autopilot for the spaceship by utilizing its complete nonlinear dynamic model. This problem is a crucial aspect of the practical theory of controlling aircraft for a wide range of uses.

### III. RESULTS AND DISCUSSION

The ACAR method [22]-[24] suggests that the synthesized system can be decomposed sequentially based on the ratio

$$\dim z = n - \lambda m,\tag{16}$$

where  $z$  represents the dimensions of the decomposed system,  $n$  represents the dimensions of the initial system,  $m$  represents the number of control channels, and  $\lambda$  represents the number of consistently introduced invariant manifolds.

Based on (16), it is feasible to promptly break down the original problem (15) with a dimension of  $n = 5$ , without first addressing the decomposed problem of regulating a second-order system ( $\lambda = 1, m = 3$ ):

$$\begin{aligned} \dot{x}_{2\psi}(t) &= -x_{1\psi}x_{3\psi} + x_{5\psi}\sqrt{1 - x_{2\psi}^2 - x_{3\psi}^2}, \\ \dot{x}_{3\psi}(t) &= x_{1\psi}x_{2\psi} - x_{4\psi}\sqrt{1 - x_{2\psi}^2 - x_{3\psi}^2}. \end{aligned} \tag{17}$$

Equations (17) represent the motion of IP along the manifold  $\psi_{1,4,5} = 0$ . This manifold is formed by the intersection of the manifolds  $\psi_1 = 0, \psi_4 = 0$ , and  $\psi_5 = 0$ . IP reaches this manifold due to the influence of controls  $u_1, u_4$  and  $u_5$ . In subsystem (17), the variables  $v_1 = x_{1\psi}, v_4 = x_{4\psi}$ , and  $v_5 = x_{5\psi}$  should be selected as the "internal" controls of  $v_k$ . This is because the first derivatives  $\dot{x}_1(t), \dot{x}_4(t)$  and  $\dot{x}_5(t)$  of these variables appear on the left side of the equations (15), where the desired controls  $u_1, u_4$ , and  $u_5$  are present on the right side.

Let's write equations (17) in the form

$$\begin{aligned} \dot{x}_{2\psi}(t) &= -v_1x_{3\psi} + v_5\sqrt{1 - x_{2\psi}^2 - x_{3\psi}^2}, \\ \dot{x}_{3\psi}(t) &= v_1x_{2\psi} - v_4\sqrt{1 - x_{2\psi}^2 - x_{3\psi}^2}. \end{aligned} \tag{18}$$

The objective is to design internal controls  $v_1, v_4, v_5$  that will move the IT system, as described in equation (18), along the intersection of manifolds  $\psi_{1,4,5} = 0$ , from any beginning state to the origin of the subspace of states  $x_{2\psi} = x_{3\psi} = 0$ .

Let's choose the macro variable  $\psi_1 = x_1$  and based on the functional equation [22]-[24]

$$T_1\dot{\psi}_1(t) + \psi_1 = 0$$

and the first equation of the system (15), we can obtain the control  $u_1$  in the form of

$$u_1 = \left(\frac{A-C}{B}\right)x_4x_5 - \frac{1}{T_1}(x_1 - x_{1s}). \tag{19}$$

Then the first equation of the initial system (15), taking into account (19), will have the form of

$$T_1\dot{x}_1(t) + x_1 = x_{1s}, \tag{20}$$

which implies

$$x_1(t) = v_1 = (x_{10} - x_{1s})e^{\frac{-t}{T_1}} + x_{1s}. \tag{21}$$

Let's select the remaining linear "internal" controls in the form

$$v_4 = \rho x_{3\psi} \text{ и } v_5 = -\gamma x_{2\psi}, \tag{22}$$

To perform the relations (22), we introduce the following macro variables:

$$\begin{aligned} \psi_4 &= \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + x_5; \\ \psi_5 &= \alpha_2x_2 + \alpha_3x_3 + \alpha_4x_4 + x_5. \end{aligned} \tag{23}$$

As a result of the synthesis, we will find the values

$$\gamma = \alpha_2 > 0; \rho = \frac{\beta_3 - \alpha_3}{\alpha_4 - \beta_4} > 0, \tag{24}$$

where  $\alpha_2 = \beta_2 > 0, \alpha_3\beta_4 = \beta_3\alpha_4, \alpha_3 < 0, \beta_4 < 0$ .

To synthesize the external controls  $u_4$  and  $u_5$ , we introduce functional equations in accordance with the ACAR method [22]-[24]

$$\begin{aligned} T_4\dot{\psi}_4(t) + \psi_4 &= 0, T_4 > 0, \\ T_5\dot{\psi}_5(t) + \psi_5 &= 0, T_5 > 0. \end{aligned} \tag{25}$$

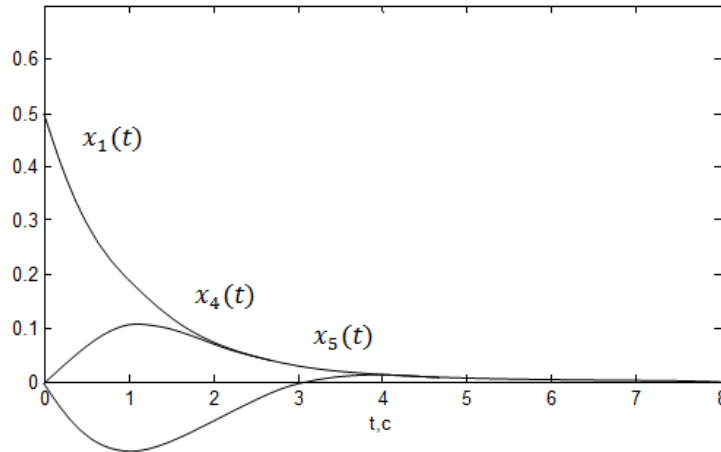
Based on equations (25), taking into account (23) and (15), we obtain the following vector control laws:

$$\begin{aligned}
 (\beta_4 - \alpha_4)u_4 &= -(\beta_3 - \alpha_3)x_1x_2 + (\beta_3 - \alpha_3)x_4\sqrt{1 - x_2^2 - x_3^2} \\
 &+ (\beta_4 - \alpha_4)\left(\frac{C-B}{A}\right)x_1x_5 - \frac{1}{T_4}\psi_4 + \frac{1}{T_5}\psi_5;
 \end{aligned}
 \tag{26}$$

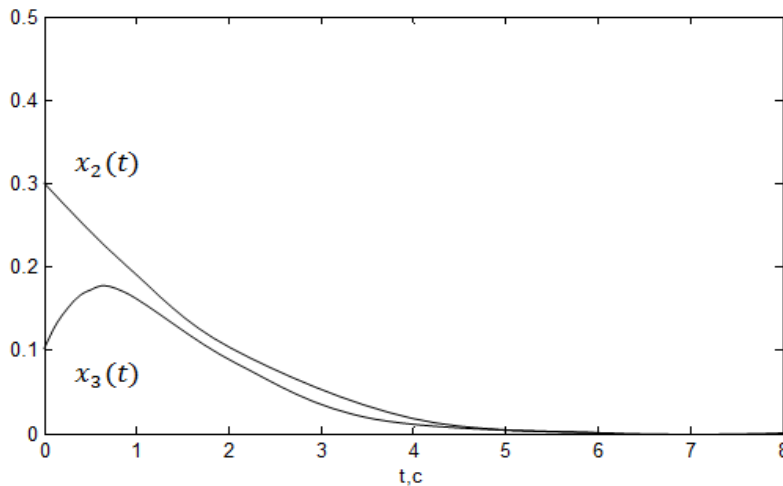
$$\begin{aligned}
 (\beta_4 - \alpha_4)u_5 &= \alpha_2(\beta_4 - \alpha_4)x_1x_3 - \alpha_2(\beta_4 - \alpha_4)x_5\sqrt{1 - x_2^2 - x_3^2} \\
 &+ (\beta_4 - \alpha_4)\left(\frac{B-A}{C}\right)x_1x_4 - \frac{\alpha_4}{T_4}\psi_4 - \frac{\beta_4}{T_5}\psi_5.
 \end{aligned}
 \tag{27}$$

The control rules  $u_4$  (26) and  $u_5$  (27), along with law (19), determine the dynamic properties and asymptotic stability of the closed system as a whole (15), (19), (26), (27), with regard to the orientation of the spacecraft. The control rules convert the initial conditions of a closed system into the intersection of two manifolds,  $\psi_4 = 0$  and  $\psi_5 = 0$  (equation 23), and then establish a stable orientation of the aircraft based on the decomposed equations (equation 18).

Fig. 2 shows the results of modeling the synthesized of the spacecraft orientation system with the following model parameters ( $\kappa\Gamma \cdot \text{m}^2$ ):  $A = 7, B = 8, C = 9$  and the regulator:  $T_1 = T_4 = T_5 = 1, \alpha_2 = \alpha_4 = \beta_2 = \beta_3 = 1, \alpha_3 = \beta_4 = -1$ .



a) Graphs depicting the transient processes of the projections of the angular velocity vector



b) Graphs depicting the transients of the values of coordinates  $x_2$  and  $x_3$ .

**Fig.2** Simulation results of the "SC-autopilot" control system

The computer modeling findings of the closed-loop control system "SC-autopilot" shown in Fig. 2 validate the fundamental theoretical propositions proposed. Therefore, the synthesized control rules guarantee the long-term stability of closed nonlinear systems and achieve the desired control objectives.

#### IV. CONCLUSION

This paper addresses the issue of analytically designing a generalized three-channel autopilot for a spaceship, utilizing its comprehensive nonlinear dynamic model. This text discusses the fundamental ideas and techniques of synergetic control theory. It is founded on the concept of implementing attractive invariant manifolds, which effectively coordinate the inherent qualities of the object (such as energy, mechanics, and thermal characteristics) with the control task needs. A control vector has been discovered through the use of the developed extended nonlinear spacecraft model, in line with the principles of synergetic synthesis. This control vector guarantees the successful transition of the spacecraft from any initial condition within a specific acceptable range to the desired state. Simultaneously, external disruptions are mitigated and the long-term stability of the control system's motion is ensured. The synthesis given here applies the ideas and methods of synergetic control theory to manage the spatial motion of a spacecraft. This approach guarantees the realization of the desired motion modes and considers the inherent nonlinear characteristics of their mathematical models. This assignment is a crucial component of the practical study of controlling airplanes used in numerous fields. The authors will conduct more research focused on using synergetic control to manage the orbital movement of a low-thrust spacecraft.

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