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On Fuzzy Antiideals of a Semiring

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Abstract: - In this paper, we expand the concept of fuzzy antideals in semigroups by extending it to the topic of semirings. Starting with the definition of left and right antideals in a semiring, we illustrate our concept with nontrivial examples and investigate their fundamental properties. Next, we present fuzzy antideals inside semirings and provide subtle insights into their structural properties. Finally, we create sig- nificant links between semiring algebraic structures and fuzzy antideals, enhancing our comprehension of their theoretical consequences.

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I. Introduction

In 1965, Zadeh [24] introduced the concept of fuzzy sets as an extension of classical sets, which has only two possibilities: either an element is a member, or it is not. However, in a fuzzy set the element occurs inside the closed unit interval where the degree of belonging of an element plus its degree of non-belongingness adds up to one. Many problems in daily situations are handled by the new concept.

In 1971, Rosenfeld [21] presented the ideas of fuzzy subgroups of a group, combining the theories of fuzzy sets with algebraic structures. This marked the beginning of fuzzy algebraic structures as a recognized and productive study field.

Semirings are mathematical concepts that extend the concepts of rings and semi- groups. Moreover, it is a mathematical structure that combines a set with two binary operations, commonly referred to as addition and multiplication., which follow certain axioms. Semirings, unlike rings, do not always need the presence of additive inverses, which makes them well-suited for representing a wide range of mathematical and real- world events.

Ideals are crucial in the examination of algebraic structures like rings and semirings. An ideal of a semiring is a subset that satisfies closure under addition and absorbing un- der multiplication. Ideals give a method to analyze the algebraic structure of semirings, providing a valuable understanding of factorization and decomposition features.

Antideals are an important concept in semigroup theory, along with ideals. An antideal is a subset of a semigroup that is absorbed under the semigroup’s operation. Antideals are a broader concept than ideals in semigroups, including subsets that are ”almost absorbing” inside the structure of the semigroup. This paper takes the idea of antideals from the study of semigroups and applies it to semirings. We talk about left and right antideals in a semiring and look at their characteristics and how they can be used in algebraic theory and other fields. By looking at the structure of antideals, we can learn more about how semirings work algebraically and what they might be used for. For more details about these topics, we refer to [1]–[22].

II. Antiideals Of A Semiring

This section explores the notion of antiideals within the structure of semirings, providing information about their properties and significance.

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Definition 2.1. Let $(A, \cdot)$ be a semigroup and $\emptyset$ =/

1. a left antideal of A if $AI \cap I = \emptyset$
2. a right antideal of A if $IA \cap I = \emptyset$

$I \subseteq A$. Then I is

3. an antideal of A if I is a left and right ideal of A.

Example 1. Let $(2Z^{+}, +)$ be the semigroup of non-negative even integers under stander addition. Then $I = \{2\}$ is antiideal of $2Z^{+}$.

Definition 2.2. Let $S \neq \emptyset$ be a set with binary operations + and $\cdot$. Then $(S, +, \cdot)$ is a semiring if

1. $(S, +)$ *is a commutative semigroup.*
2. $(S, \cdot)$ *is a semigroup.*
3. $x \cdot (y + z) = x \cdot y + x \cdot z$ *and* $(y + z) \cdot x = y \cdot x + z \cdot x$ *for all* $x, y, z \in S$.

Definition 2.3. Let $(A, +, \cdot)$ be a semiring and $\emptyset \neq I \subseteq A$. Then I is

1. a left antideal of A if $AI \cap I = \emptyset$
2. a right antideal of A if $IA \cap I = \emptyset$
3. an antideal of A if I is a left and right ideal of A.

Example 2. Let $(A, +, \cdot)$ be the semiring of non-negative integers excluding 1 and under stander addition and multiplication of integers. Then $I = \{2, 3\}$ is antiideal of A.

Definition 2.4. Let $(A, +, \cdot)$ be a semiring and $\emptyset \neq I \subseteq A$. Then I is a strongly left(right) antiideal of A if I is left(right) antiideal of A and an antiideal of $(A, +)$.

Remark 1. A ring A doesn't have strong antiideal , this is clear as $0 \in A$ and $I + A \supseteq I$.

Example 3. Let $(M, +, \cdot)$ be a semiring of the even integers. Then $I = \{2\}$ is a

strongly antiideal of M .

Example 4. Let $(3Z, +, \cdot)$ be the ring of integers that are multiple of 3. Then $I = \{3\}$

is an antiideal of 3Z.

Proposition 2.5. *A ring with unity doesn't have antiideals.*

Proof. Let R be a ring with unity 1. Assume that A is an antiideal of I. Then $A+I \supseteq A$. Since $1 \in I$, we have $1 \in A+I$, Hence, $A+I = I$. This does not align with the definition of an antiideal.

Corollary 2.6. Every integral domain doesn't have antiideals. □

Proof. Since every integral domain is a ring with unity, According to Proposition 2.5, there are no antiideals in a ring with unity. It follows that all integral domains has no antiideals.

Proposition 2.7. *A nonempty subset M of a right (left) antiideal (strongly antiideal) of A is a right (left) antiideal (strong antiideal) of A.* □

Proof. Let I be a right (left) antiideal (strongly antiideal) of A. Let M be a non-empty subset of I. We wish to prove that M is a right (left) antiideal (strong antiideal) of A.

Assume that M is a subset of a right antiideal I of A. Hence for all $x \in A$ and $m \in M$, $as \notin I$. Since $M \subseteq I$, we have $mx \notin M$ for all $x \in A$ and $m \in M$. Thus, M is a right antiideal of A. Similary we can prove that M is a left antiideal.

Now, if I is a strongly antiideal of A, then for all $x \in A$ and $m \in M$, $xm \notin I$ and $mx \notin I$. Since $M \subseteq I$, we have $xm \notin M$ and $mx \notin M$ for all $x \in A$ and $m \in M$. Thus, M is a strong antiideal of A.

□

Corollary 2.8. Let I be a left (right) antiideal of A, and let S be any subset of A. Then $\emptyset \neq I \cap S$ is a left (right) antiideal of A.

Proof. It's obvious to see that $I \cap S$ is non-empty since I is non-empty. Furthermore, for any $x \in A$ and $s \in I \cap S$, we conclude that $s \in I$ since $s \in I \cap S$, and $xs \notin I$ since I is a left antiideal. In the same fashion, $s \in S$ implies $xs \notin S$ for all $x \in A$.

Therfore, $I \cap S$ is a left antiideal of A. Similary we can prove that $I \cap S$ being a right antiideal. □

Remark 2. The above result doesn't hold for the union case.

Proof. We will show that by a counterexample. Consider the abelian group$(S, +)$ and $(S, \cdot)$ is an abelian semigroup. Let I be a left antiideal of A, and let A be a subset of S such that $I \cup A$ is not a left antiideal of S.

Since I is a left antiideal of S, for any $s \in S$ and $i \in I$, we have $si \notin I$. Similarly, for any $a \in A$, we must have $sa \notin I$, otherwise $I \cup S$ would be a left antiideal.

Now, consider an element $x \in I \cup A$. There are two cases:

1. If $x \in I$, then for any $s \in S$, $sx \notin I$ because I is a left antiideal. However, $sx \in I \cup A$, so $I \cup A$ is not a left antiideal of S.

2. If $x \in S$, then for any $s \in S$, $sx \notin I$ because I is a left antiideal. However, $sx \in I \cup A$, so $I \cup A$ is not a left antiideal of S.

It tends out, we've proof that for the union of a left antiideal I and any subset A of S, $I \cup A$ may not be a left antiideal of S. □

Theorem 2.9. *Let $(A,)$ be a semigroup and $\emptyset \neq I$ be a subset of A. Then I is a left antideal of A if and only if for all $a \in A$ and $i \in I$, $ai \notin I$.*

Proof. (⇒) Suppose I is a left antideal of A. Then by definition, $AI \cap I = \emptyset$. This means for any $a \in A$ and $i \in I$, $ai \notin I$, otherwise, we would have $ai \in AI \cap I$, contradicting the assumption.

(⇐) Conversely, suppose for all $a \in A$ and $i \in I$, $ai \notin I$. We want to show that $AI \cap I = \emptyset$. Suppose for contradiction that there exists $x \in AI \cap I$. Then there exist $a \in A$ and $i \in I$ such that $x = ai$. But this contradicts our assumption that $ai \notin I$. Hence, $AI \cap I = \emptyset$, which means I is a left antideal of A. □

Theorem 2.10. *Let $(A, \cdot)$ be a semigroup and $\emptyset \neq I$ be a subset of A. Then I is a right antideal of A if and only if for all $a \in A$ and $i \in I$, $ia \notin I$.*

Proof. (⇒) Suppose I is a right antideal of A. Then by definition, $IA \cap I = \emptyset$. This

means for any $a \in A$ and $i \in I$, $ia \notin$

contradicting the assumption.

I, otherwise, we would have $ia \in IA \cap I$,

(⇐) Conversely, suppose for all $a \in A$ and $i \in I$, $ia \notin I$. We want to show that $IA \cap I = \emptyset$. Suppose for contradiction that there exists $x \in IA \cap I$. Then there exist $a \in A$ and $i \in I$ such that $x = ia$. But this contradicts our assumption that $ia \notin I$. Hence, $IA \cap I = \emptyset$, which means I is a right antideal of A. □

Theorem 2.11. *Let $(A, \cdot)$ be a semigroup and $\emptyset \neq I$ be a subset of A. Then I is an antideal of A if and only if it is both a left and right antideal of A.*

Proof. (⇒) Assume that I is an antideal of A. Then by definition, I is a left and right ideal of A. Therefore, $AI \cap I = \emptyset$ and $IA \cap I = \emptyset$, implying I is a left and right antideal of A.

(⇐) Conversely, suppose I is a left and right antideal of A. This means $AI \cap I = \emptyset$ and $IA \cap I = \emptyset$, implying I is a left and right ideal of A. Therefore, I is an antideal of A. □

Example 5. Consider a semiring of a non-negative even integers. Let $I = \{2\}$ and $J = \{4\}$ are antiideal of A. Its clear that $I \cup J$ is not an antiideal of A.

We present an example of a left antiideal that is not a right antiideal.

Example 6. Consider M = { m1   0 : m_1, m_2, $m_3 \geq 2$} be the semiring under stander addition and multiplication where I = { 2 0 , 4 0 }. It is obvious That I is a left antiideal of M However, its not a right antiideal of M .
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III. FUZZY ANTIDEALS OF A SEMIRING

In this section, we investigate fuzzy antideals in semirings, which sheds insight into algebraic structures.

Definition 3.1. Let $(A, +, \cdot)$ be a semiring and $\emptyset$ ¡ N be a fuzzy set of A. Then N is a

1. Fuzzy left antiideal of A if $N(rx) \wedge N(x) = 0$
2. Fuzzy right antiideal of A if $N(xr) \wedge N(x) = 0$
3. Fuzzy antiideal of A if it is left and right antiideal

Definition 3.2. Let $(A, +, \cdot)$ be a semiring and $\emptyset$ ¡ N be a fuzzy set of A. Then N is a strong fuzzy antiideal if N (a + x) ∧ N (x) = 0.

Theorem 3.3. *If $(A, +, \cdot)$ is a semiring and anonempty set N be a fuzzy set of A. Then N is a fuzzy left antiideal of A if and only if $N(mx) \wedge N(x) = 0$ for all $m \in A$ and $x \in A$.*

Proof. ⇒ Suppose that N is a fuzzy left antiideal of A. Then by definition, for any $m \in A$ and $x \in A$, we have $N(mx) \wedge N(x) = 0$.

⇐ On the other side, suppose $N(mx) \wedge N(x) = 0$ for all $m \in A$ and $x \in A$. suppose that $m \in A$ and $x \in A$. Then $N(mx) \wedge N(x) = 0$, this means N is a fuzzy left antiideal of A.

□

Theorem 3.4. *Let $(A, +, \cdot)$ be a semiring and $\emptyset$ /= N be a fuzzy set of A. Then N is a strong fuzzy antiideal of A if and only if $N(a + x) \wedge N(x) = 0$ for all $a, x \in A$.*

Proof. ⇒ Assume that N is a strong fuzzy antiideal of A. Then by defintion we have $N(a + x) \wedge N(x) = 0$, for all $a, x \in A$

⇐ In the other direction, suppose that $N(a + x) \wedge N(x) = 0$ for all $a, x \in A$. Then $N(a + x) \wedge N(x) = 0$, this means that N is a strong fuzzy antiideal of A.

□

IV. CONCLUSION

This paper examines the concept of fuzzy antideals in semirings in detail, extending it from semigroup theory to the simiring realm. By diligently examining left and right antideals in simirings and introducing fuzzy antideals, we have improved our understanding of the algebraic structures that these mathematical objects hold. In this field, we have advanced theoretical and applied research by clarifying the properties and relationships between fuzzy antideals and semirings.

Moreover, there are still exciting directions to explore. Further investigations of fuzzy antideals' behavior and applications in a variety of mathematical and real-world situations are made possible by research into them. Furthermore, investigating the relationships between fuzzy antideals and other algebraic structures may result in new discoveries and cross-disciplinary cooperation.

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