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Specifics of Applying Multi-threshold Decoding Methods to Correct Errors in Fading Communication Channels

Abstract: - This research investigates the application of multi-threshold decoding (MTD) methods for error correction in communication channels prone to fading phenomena. The study emphasizes the preservation or minimal increase in the implementation complexity of the original MTD method, crucial for maintaining high decoding rates exceeding tens to hundreds of Gbit/s. Results highlight MTD's efficacy in mitigating errors in channels affected by diverse fading types, including multipath propagation with intersymbol interference in conjunction with Orthogonal Frequency Division Multiplexing (OFDM). The capability of MTD to decode extensive codes with low computational complexity significantly enhances its utility in addressing complex channel impairments. Conversely, conventional error-correction codes often falter due to inadequate code lengths in such environments. Consequently, the adoption of MTD proves pertinent in modern high-speed digital data transmission systems, ensuring robust error correction amidst challenging channel conditions.

Keywords: fading channels, OFDM, channel coding, communications, decoding, error correcting codes.

I. Introduction

Noise-resistant coding or forward error correction coding is used in communication systems, where there is no a reverse channel for requests retransmission or it is unavailable, the channel delays in retransmission requests appear to be unacceptably large, or finally, the interference level is so high that the number of retransmissions becomes extremely large [1]. Currently, there are many noise-resistant codes, such as Bose-Chaudhuri-Hocquenghem block codes [1], Reed-Solomon codes [1], convolutional codes [1], Low-Density Parity-Check (LDPC) codes [2], polar codes [3], turbo codes [4], and others. It should be noted, that along with high corrective ability, producibility of codec is important for developers of data communication systems, which assumes possibility of simple high-speed realization of encoder and, the most of all, of noise-resistant code decoder. Such producibility can be found in multi-threshold decoding methods (MTD) of self-orthogonal codes (SOC) [5, 6] that our collective have been developing within the framework of optimization theory of coding [7, 8] since 70-s of the last century [9].

MTD represent a development of the simplest threshold decoder Massey [10], which has been modified to arrange the iterative decoding of SOC in such a way that during its work process at each change in a decoded symbol, strict approximation of the MTD solution to the solution of optimal decoder is guaranteed [7]. It should be noted that attempts to modify the Massey decoder due to simplicity of its realization have been made by other researchers [11-16]. Ideas of multi-threshold decoding have also been developed in the works of other research teams [17, 18].

Today, characteristics of MTD are widely investigated for the channels with independent errors or erasures, in which these methods ensure near-optimal decoding of even very long codes with only complexity of realization that are linearly depend on the code length [6-8]. For such channels, a number of methods are known to improve the efficiency of MTD, which allow for approaching the area of effective MTD operation to the channel capacity [19-25]. As a result, both hardware and software versions of MTD appear to be capable of ensuring levels of power

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gain comparable with the gain of the best methods of decoding turbo and low-density codes with ten-fold processing speed [6-8]. At the same time, it should be noted that modern communication networks operate under much more complex conditions arising from multipath propagation of signals, Doppler shift and many other reasons [26, 27]. As a result, errors occurring in the channel are grouped into packets. In such conditions, the effect of coding is much greater than in channels with independent errors, since here, in some cases, it is impossible to increase the reliability of transmitted data only due to an increase in the transmitter power. The goal of this paper is to study the issues of the MTD application in channels with clustering errors. Hence, it is especially important to preserve or only insignificantly increase complexity of realization of initial MTD, since only the simplest methods of error correction can ensure processing speeds that are required today for decoding of digital streams amounting to tens and hundreds Gbit/s.

II. Proposed Method

Multi-Threshold Decoders of Self-Orthogonal Codes. Typically, MTD is used to decode binary linear systematic block or convolutional self-orthogonal codes (SOC) [7, 8]. These codes are characterized by the fact that the system of all checks controlling some information symbol is orthogonal by itself with respect to this symbol. Usually, SOCs are prescribed using generator polynomials $g(x)$ whose differential triangles (a set of differences between all degrees with non-zero coefficients) do not contain identical elements. SOCs are characterized by a coding distance d, equal to the number of non-zero components of the generator polynomial increased by 1. The number of information symbols k in a block SOC with code rate R=1/2 amounts to at least $2m+1$, where m is the maximum degree of the generator polynomial. The length n of such a code will be equal to $2k$.

An example of a block SOC encoder is shown in Fig. 1 [19]. This code is characterized by the parameters of n=26, k=13, R=1/2, d=5, $g(x)=1+x+x^4+x^6$.

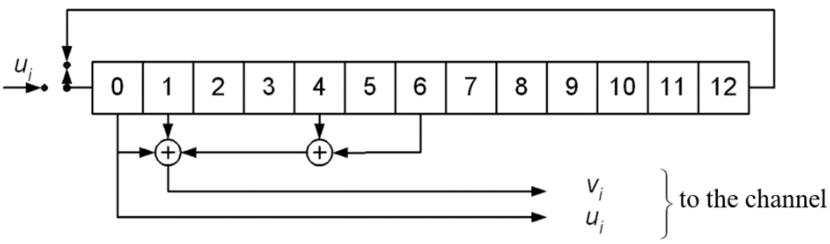

Fig. 1. Example of Binary Block Self-Orthogonal Code Encoder

The multi-threshold decoder is an iterative method for decoding self-orthogonal block or convolutional codes [19]. An example of a multi-threshold decoder diagram is shown in Fig. 2.

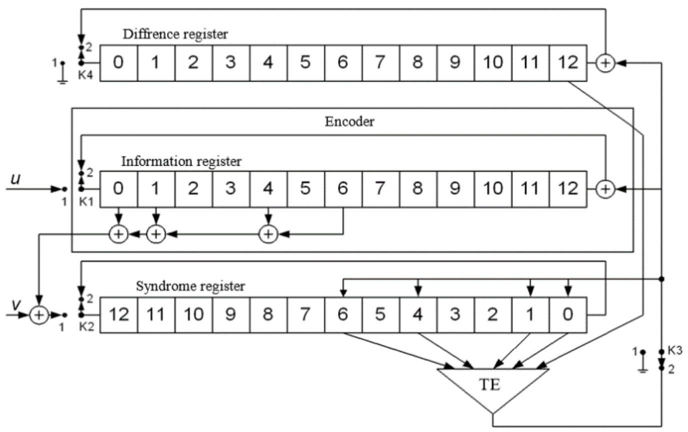

Fig. 2. Multi-Threshold Decoder of Block SOC with R=1/2, d=5 and n=26

As it can be seen from the presented diagram, each MTD iteration differs from the usual threshold decoder, which is the basis of this diagram, only by the presence of a difference register, in which the information symbols changed at the threshold element are marked. It is essential that the solutions of the threshold element from the difference register are then utilized by another threshold element in the next decoding iteration. At the threshold element of

each MTD iteration, the following operations are performed when decoding the information symbol u_k while using a rigid modem (corresponding to the case when the demodulator evaluates only the values of the received bits).
1. The sum of the checks (each of which is 0 or 1 in case of the hard modem) is calculated, i.e. function

$$L_k = \sum_{m=1}^{J} S_{g_m} + r_k \quad (1)$$

where $J = d–1$ is the number of checks (zero elements of the code seed g); r_k is a symbol of the difference register that belongs to the decoded symbol u_k (equal to 0 or 1); S_m is m-th element of the syndrome register, which is a part of the checks array relative to the decoded symbol u_k.
2. If $L_k > T$, where $T \geq (d–1)/2$ is the threshold value of the threshold element, than symbol u_k, all related to it checks $\{S_{g_m}\}_{m=1,J}$ and symbol r_k are inverted.
3. Switch to decoding of the next symbol (p.1).
In that case if the SOC applied in the data communication system are chosen according to the criterion of minimization of error propagation and parameters of MTD are adjusted correctly [7], it can be expected that with high enough noise level, the MTD solutions tend to the solution of the optimal decoder, the error probability of which can be estimated for SOC with the help of expression

$$P_b = \begin{cases} \sum_{i=(d+1)/2}^{d} C_d^i \, p^i (1-p)^{d-i} \text{ – for DSC with odd } d; \\ \frac{1}{2} C_d^{d/2} p^{d/2} (1-p)^{d/2} + \sum_{i=(d/2)+1}^{d} C_d^i p^i (1-p)^{d-i} – \\ \text{– for a channel of DSC type with even } d; \\ Q\left(\sqrt{2\, d\, E_s / N_0}\right) \text{– for a channel with AWGN,} \end{cases} \quad (2)$$

where d is code distance; p is error probability at the decoder output; E_s/N_0 *is* signal-noise ratio in the channel; $Q(x)$ is a function determined by the expression

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \quad (3)$$

It should be noted that both MTD, and the usual threshold decoder are easily modified for summarizing checks in (1) with some coefficients, in particular, while working with quantized by several levels solutions of a soft modem (i. e. when operating in a channel with AWGN), additional output bits of which determine the reliability of its judgment. Using soft solutions of the demodulator allows achieving results that are by 1.4...1.7 dB better than when using only hard demodulator solutions. In this case, the expression (1) for calculating the likelihood function L_j takes the following form

$$L_j = \sum_{m=1}^{J} s_{j_m} + d_j w_j \quad (4)$$

where w_{j_m} is a coefficient reflecting the reliability of the check s_{j_m}; w_j is a coefficient reflecting the reliability of the received symbol u_j.
Complexity of the MTD software implementation approximately equals to $N_{\text{MTD}} \approx I\,(d+2)+d–1 \approx (I+1)(d+2)$, where I is the number of decoding iterations, d is the code distance. While in most cases it is possible to reduce the total number of operations down to a value of $N_{\text{MTD}} \approx 4d+3I$.
It should be noted that to maintain the effective MTD operation at as high level of noise as possible, and also to minimize the probability of decoding error, it is necessary to apply a number of original methods of increasing the MTD efficiency. These methods are aimed at building and optimizing structure of codes that are in the greatest degree are steady to error propagation while decoding, optimizing hundreds of parameters of the multi-threshold decoder, coordinating encoder and decoder, cascade schemes of error correction [6-8, 19-25]. It is also important

to note that at the same time SOCs can be block and convolutional, binary and symbolic, short and long, and have different code rates. This allows flexible adjustment of the applied coding system to the tasks to be solved.

III. Results

Multi-Threshold Decoders Use in Fading Radio Channels. The simplest fading channel models are Rayleigh and Rician fading channel models, which are obtained when there are multiple signal propagation paths. There is no direct line of sight between the transmitter and receiver in the Rayleigh channel, while it presents in the Rician channel.

To estimate the MTD capabilities in such channels, let us obtain the lower limit for the decoding error probability. It should be noted that for MTD, the lower estimate of error probability in DSC is determined by the error probability of the optimal decoder, which is calculated in accordance with (2).

For the uncorrelated Rayleigh channel, the expression for the error probability is known

$$p=\frac{1}{2}\left(1-\sqrt{\frac{E_S/N_0}{1+E_S/N_0}}\right), \quad (5)$$

where E_S/N_0 - symbolic (dimensionless) signal/noise ratio. It should be noted that this estimate is valid when BPSK-type modulation, AWGN and Doppler shift frequency F_d = 0 are used in transmission. In the case of a channel with Rician fading, the error probability is defined as

$$p=erfc\left(\sqrt{\frac{kE_S/N_0}{k+E_S/N_0}}\right), \quad (6)$$

where k - Rice coefficient, and function *erfc*() is determined as $erfc(x)=2Q(\sqrt{2}x)$, where $Q(x)$ is error integral. In accordance with expressions (5) and (6), dependency graphs (curve *3* in Figures 3 and 4) for the bit error probability P_b and signal/noise ratio E_b/N_0 were obtained.

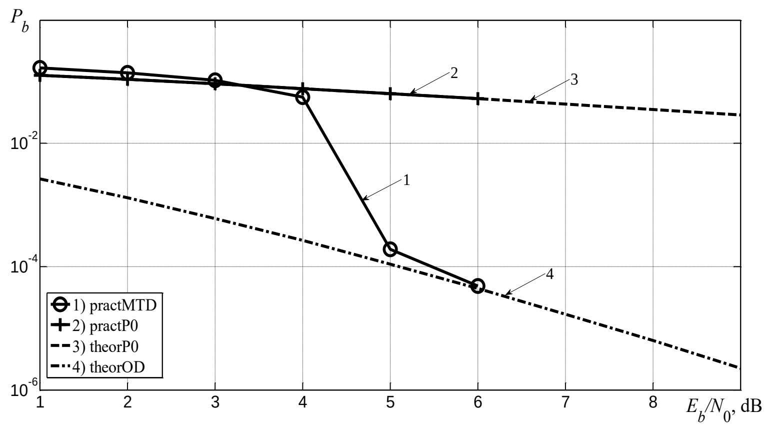

Fig. 3 Analytical Estimation of Error Probability and Simulation Results for MTD in the Rayleigh Channel

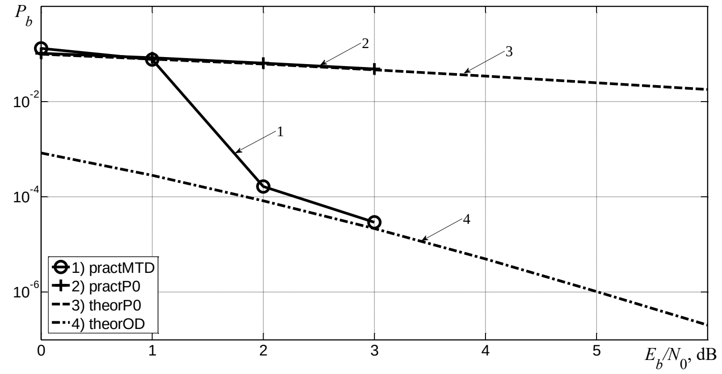

Fig. 4. Analytical Estimation of Error Probability and Simulation Results for MTD in the Rician Channel

Using the known fact that MTD is able to operate almost as an optimal decoder even at high enough noise level in the channel, we substitute (5) into (2) and, having made an assumption that the minimum code distance d is odd, we obtain a lower estimate for the bit error probability for MTD in the Rayleigh channel:

$$P_b = \sum_{i=(d+1)/2}^{d} C_d^i \frac{1}{2}\left(1-\sqrt{\frac{E_S/N_0}{1+E_S/N_0}}\right)^i \left(1-\frac{1}{2}\left(1-\sqrt{\frac{E_S/N_0}{1+E_S/N_0}}\right)\right)^{d-i} \tag{7}$$

Under the same conditions and with the same assumptions, the bit error probability in the Rician channel is defined as

$$P_b = \sum_{i=(d+1)/2}^{d} C_d^i Q\left[\sqrt{\frac{2kE_S/N_0}{k+E_S/N_0}}\right]^i \left(1-Q\left[\sqrt{\frac{2kE_S/N_0}{k+E_S/N_0}}\right]\right)^{d-i} \tag{8}$$

These expressions were used to calculate the lower estimates of the MTD error probabilities in the uncorrelated Rayleigh and Rician channels shown in Fig. 3 and 4 by curve *4*.

Then, simulations were performed to obtain error rates in the Rayleigh and Rician channels (Rice coefficient k=5) without use of coding (curves *2* in Fig. 3 and 4), as well as error frequencies when using MTD (curves *1* in Fig. 3 и 4). The block SOC with a length of 20,000 bits, code rate R=2/4 and minimum code distance d=9 was used as an noise-resistant code.

By comparing curves *2* and *3* in these figures, it can be concluded that the analytical estimate of the channel error probability agrees well with the experimental one. This allows for using these estimates in developing the error probability lower limits for MTD. And curves *1* and *4* allow stating that in the channel with Rayleigh fading, MTD with current parameters is able to approach closely to the area of optimal decoder operation starting from E_b/N_0 = 5 dB, and in the channel with Rician fading - starting from E_b/N_0 = 2 dB with chosen parameters of encoder and decoder.

Next, we consider the process of modeling MTD in Rayleigh and Rician channels and present the simulation results. Fig. 5 shows a diagram of the data communication system model for the Rayleigh or Rician channels used when obtaining the results. Unlike the Gaussian channel, a multiplicative noise component appears here, which is necessary for the demodulator to obtain both hard and soft decisions about the received bits. Let us note that this paper assumes that this multiplicative component in the demodulator is known exactly.

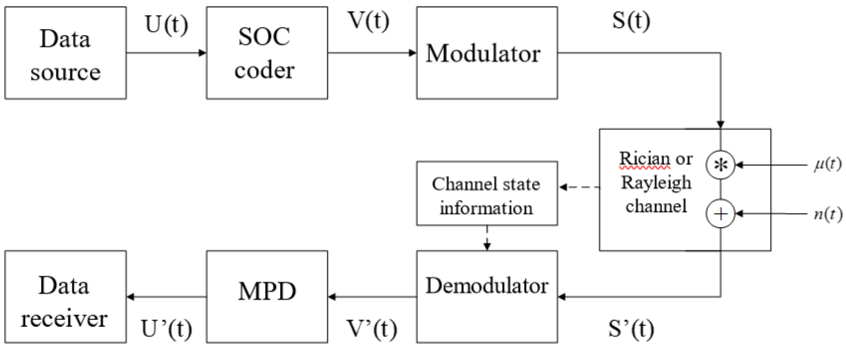

Fig. 5. Structural Diagram of the Digital Information Transmission System with Rayleigh or Rician Channels Use

While in the demodulator, the effect of multiplicative noise is first eliminated, followed by the demodulation used in the Gaussian channel. This paper does not study the use of more sophisticated demodulation algorithms that ensure better soft decisions.

In obtaining the characteristics presented below, the MPD for self-orthogonal code with code rate R=8/16 and parallel cascading, code distance d=17 and length of about n=32,000 bits, modulation of QPSK type and demodulator forming only hard decisions with respect to the received bits were used. And the MTD parameters were optimized for the given channels and used code. The simulation results are obtained using the developed software tools for data communication system modeling.

In Fig. 6, curve *1* represents an example of dependency of the bit error probability P_b of the MTD on the noise level in the channel (E_b/N_0) with AWGN (without fading). We will further compare his curve to the simulation results obtained for the fading channels. Curves *2* and *3* in this figure show the MTD characteristics in the channel with correlated Rayleigh fading with Doppler frequency F_d=50 and 100 Hz, respectively. In this case, only reflected rays come to the receiver from the transmitter, i.e. there is no direct line of sight between them. Let us note that in this case, there is a significant characteristics degradation if compared to the channel with AWGN. At the same time, faster fades are handled better by the MTD. This is explained by the fact that with slow fading, quite long sequences of adjacent bits distorting a significant part of the code block often appear to be erroneous. And MTD methods, as well as any other error correction methods, cannot cope with such share of distorted bits. One of the possible ways to improve the MTD efficiency under such conditions is to use an additional interleaver, the length of which shall amount to hundreds of thousands of bits.

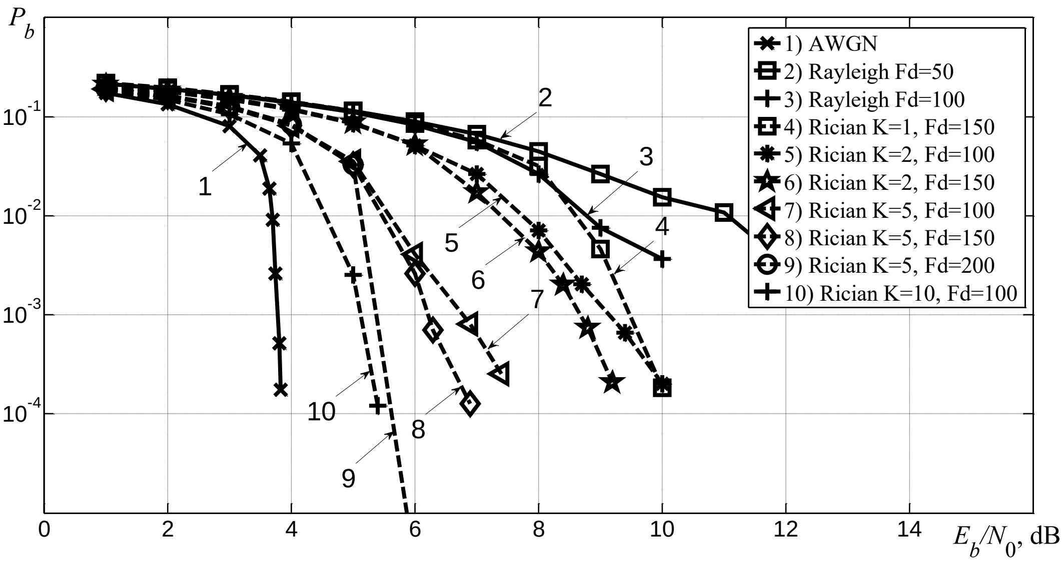

Fig. 6. MTD Characteristics in a Fading Channel

The remaining curves correspond to the case of MTD operation in a Racian fading channel, when there is a direct line of sight between the transmitter and receiver. Curve *4* corresponds to the option with the Rice coefficient K=1 and Doppler frequency F_d=150 Hz. When obtaining the curves *5* and *6*, the channel had K=2 and F_d=100 and 150 Hz. The Rice coefficient values K=5 and Doppler frequency F_d=100, 150 and 200 Hz correspond to curves *7, 8* and *9*. And, finally, K=10 и F_d=100 Hz correspond to curve *10*. Comparing the above graphs with each other and with the previously discussed graphs, we see that the greater the Rice coefficient, i.e. the greater the power of the direct beam, the better the provided noise-resistance characteristics. However, even with K=10 (i.e. at ten-fold predominance power of the direct beam over the power of the reflected beams) there is a loss in energy of about 2 dB compared to the Gaussian channel. In addition, it can be seen that in this case too, the MTD is better at handling faster fading, since it is less likely that a significant portion of the code block is subject to severe fading.

It should be noted that with block SOCs, serial or parallel communication of bits with respect to each branch is possible. In other words, if a set of all information and check registers, which are the output of the channel encoder, is represented as a matrix (and each row is a separate information or check register), and the input of the modulator as a codeword in the form of a vector, then the process of reading data from the matrix into the vector can be arranged by rows, columns or in other more complex ways. It should be noted that the previously obtained results correspond to the case of line-by-line reading of data from the matrix. Next, we present some characteristics obtained under the same conditions as for Fig. 6, but when reading data by column.

In Fig. 7 for comparison, curves *2, 4* and *6* show simulation results for Rician and Rayleigh fading with sequential bit communication. The MTD was used here for the same code as in Fig. 6. Curves *1, 3,* and *5* show the simulation results corresponding to parallel bit communication. From the analysis of the graphs, it can be seen that case uses the bits communication by column in the channel with Rician fading, the increase in err0r-correcting power is most noticeable and, for example, with a signal/noise ratio of 7 dB at K=5 and F_d=150, it is possible to improve the performance by two decimal orders of magnitude. This is explained by the fact that in this case the code with parallel cascading was used and whole branches with higher dimensionality of checks could be distorted when fading occurred. And in the case of parallel communication, errors appear to be dispersed on different branches, which prevents distortion of whole branches with higher dimensionality of checks.

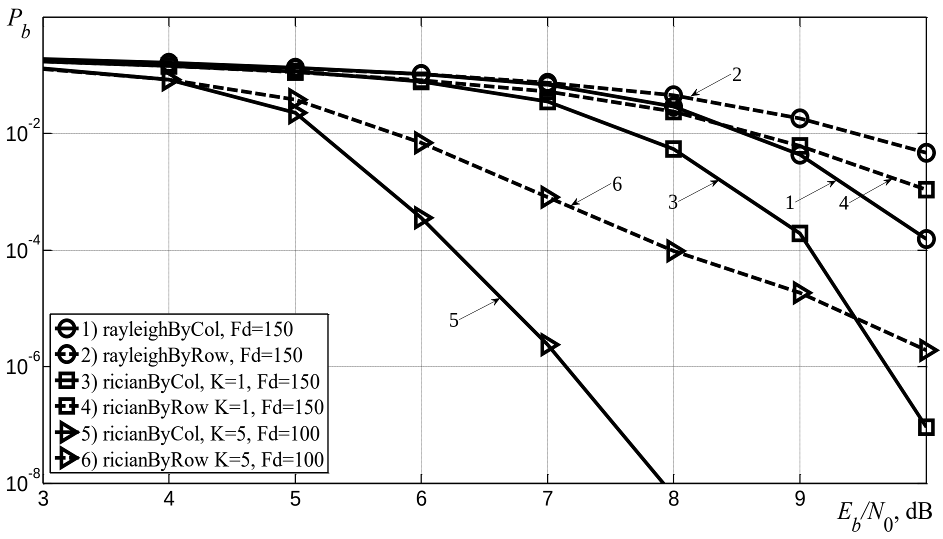

Fig. 7. MTD Characteristics in a Fading Channel with Serial and Parallel Bit Communication

It should also be noted that the discussed cases of MTD application allow to successfully deal only with fading, in which there is no intersymbol influence. If intersymbol influence is present, additional means, such as orthogonal frequency division multiplexing (OFDM), shall be used to combat such fading [26, 27].

In case of wireless data communication, fading with intersymbol influence (ISI) occurs when the multipath effect occurs in the spread of ray delays relative to each other, which exceed the symbol duration. To describe such multipath channel, so called tapped delay line model is usually used. A typical example of such a line is shown in Fig. 8.

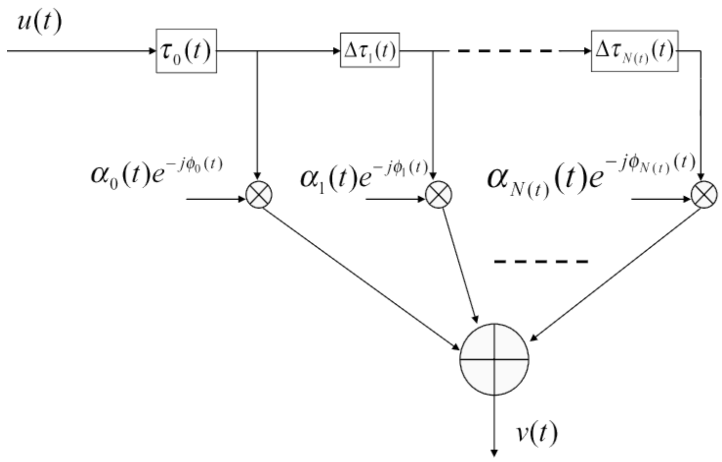

Fig. 8. Multipath Channel Model in the Form of Tapped Delay Line

In this figure, the communicated signal $u(t)$ passes through N+1 delay blocks (this number equals to the number of beams of the multipath model), delayed in each by $\Delta\tau_i$ seconds. The signals from the output of each delay are distorted by a random multiplicative factor $\alpha_i(t)e^{-j\phi_i(t)}$ and summed up with each other, yielding a multipath channel output $v(t)$. This output is further distorted by the AWGN and fed to the receiver.

It was mentioned above that along with MTD, OFDM modulation scheme can be used in such channels to combat ISI. The basic idea of OFDM is to divide one rapidly modulated wideband signal into many slowly modulated narrowband signals. The low symbol rate makes it possible to utilize guard interval between symbols, which helps to cope with temporal scattering and eliminate ISI.

In order to correct remaining errors left after the OFDM demodulator, noise-resistant coding is usually applied. It is expected that multi-threshold decoders, which have proved themselves remarkably well when operating in Gaussian channels, will prove to be equally effective under these conditions. Therefore, it is necessary to simulate the digital information communication system when MTD and OFDM are used together, the structural diagram of which is shown in Fig. 9.

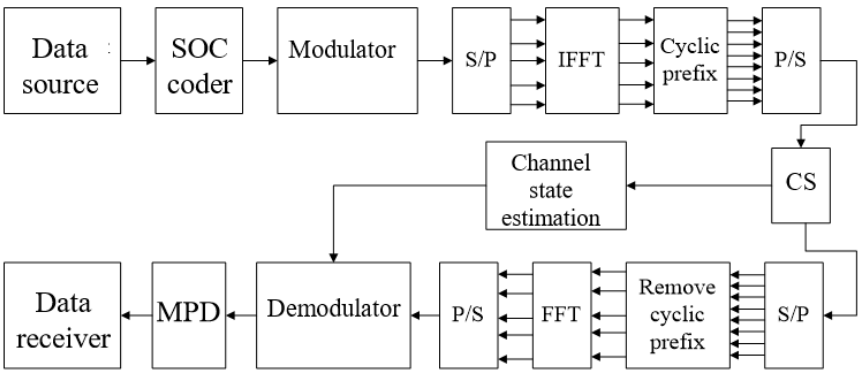

Fig. 9. Structural Diagram of Digital Information Communication using OFDM

The input data used in the simulation were: MTD for one of the best self-orthogonal codes with code rate R=1/2, code distance d=17 and length of about n=32,000 bits; a six-path channel model with beam delays [0, 800, 1600, 3200, 4900, and 7300] ns and power profiles [0, -1, -9, -10, -15, and -30] dB; QPSK, 8PSK, 16APSK or 32APSK modulation type; demodulator forming only hard solution in relation to the received bits; OFDM with the number of carriers N_{fft}=1024 at digitization rate F_s=7.5·10^6 Hz. A value of 1/16 of the OFDM symbol length is selected as the length of the guard interval (cyclic prefix). This is related to the fact that the cyclic prefix duration shall be longer than the maximum lag time. In our case, time of one OFDM symbol is T_s=1/7.5·10^6 s, thus cyclic prefix duration amounts to 1024/(16·7.5·10^6) = 8522 ns. It is also assumed that the system organizes data interleaving at the OFDM symbol level. I.e.. in the model, formation of a random process for arranging fading started anew with a random initial state for each OFDM symbol.

In Fig. 10, curve *1* shows an example of the dependence of the MTD decoding error probability on the noise level in the channel for quadrature phase-shift-keying (QPSK). It can be seen that for a noise level of 20 dB, the error probability is 10^{-6}. When 8PSK (curve *2*) is used, the characteristics degrades by about an order of magnitude, but the bit rate in this case increases by a factor of 1.5 compared to QPSK. When even higher order modulation (16APSK and 32APSK) is used, the characteristics shown by curves *3* and *4* in Fig. 10 are obtained, respectfully. Note that compared to QPSK, the 32APSK modulation scheme can increase the overall bit rate by 2.5 with an energy loss of about 6 dB. The figure shows that even with such complex manipulations the MTD is able to provide high reliability of data communication with a high level of errors in the channel.

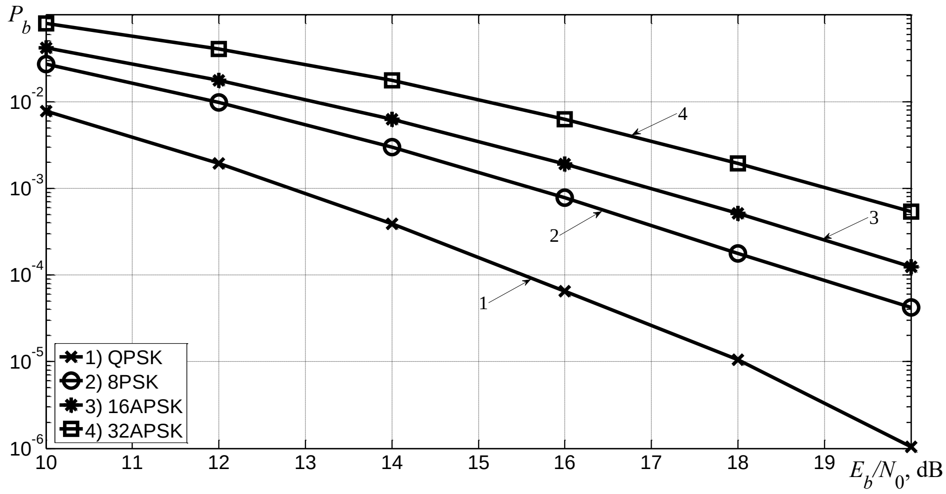

Fig. 10. MTD Characteristics with Different Types of Modulation

IV. Conclusion

The obtained results have shown that MTD due to its ability to almost optimally decode even very long codes with low computational complexity ensures high efficiency in channels with different types of fading, including multipath channels with ISI, when it is used jointly with OFDM. Under these conditions, many noise-resistant codes are unfit for service due to their small length. All this allows MTD application in modern high-speed systems of digital data communication with complex nature of errors.

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