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Robust Adaptive Beamformer using Improved Coprime Array for Wireless Communication Application



Abstract: - In this paper, we present an innovative approach to beamforming that combines the strengths of blind and non-blind algorithms using Co-Prime Sensor Arrays (CPSA). By cascading the Least Square Constant Modulus Algorithm (LS-CMA) and the Least Mean Square (LMS) method, we achieve enhanced performance in terms of convergence speed, robustness, and signal quality. The initial stage involves employing the LS-CMA to obtain a preliminary estimate of the beamforming weights without requiring a reference signal. Subsequently, the LMS algorithm refines these weights to minimize the mean square error using a reference signal. The use of CPSA further improves the spatial resolution and reduces the number of sensors required. Simulation results demonstrate the effectiveness of the proposed method in various signal conditions, showcasing its potential for advanced communication systems and radar applications.

Keywords: Co-Prime Sensor Arrays (CPSA), beamforming, Least Square Constant Modulus Algorithm (LS-CMA), Least Mean Square (LMS), cascaded algorithms, signal processing, spatial resolution, communication systems, radar applications.

I. INTRODUCTION

Beamforming is a signal processing technique that has gained significant attention in modern communication systems and radar applications due to its ability to direct the reception or transmission of signals in specific directions. This capability enhances signal quality, reduces interference, and optimizes the use of spectral resources. Traditional beamforming methods can be broadly classified into blind and non-blind techniques. Blind methods, such as the Least Square Constant Modulus Algorithm (LS-CMA), do not require a reference signal, making them highly adaptable in environments where training sequences are unavailable. On the other hand, non-blind methods, such as the Least Mean Square (LMS) algorithm, rely on a reference signal to fine-tune the beamforming weights, thereby achieving precise signal quality and interference suppression.

Despite the individual advantages of LS-CMA and LMS, each method also has inherent limitations. LS-CMA can rapidly adapt to the environment but may not achieve the optimal beamforming weights due to the lack of a reference signal. Conversely, LMS can attain precise optimization but requires an initial reference signal, which might not always be available or may introduce delays in the adaptation process. To address these challenges and leverage the strengths of both methods, we propose a novel cascaded approach that integrates LS-CMA and LMS algorithms. This hybrid method aims to combine the rapid initial adaptation of LS-CMA with the fine-tuning capabilities of LMS, resulting in improved overall performance.

A significant advancement in this domain is the utilization of Co-Prime Sensor Arrays (CPSA). CPSA offers a unique structure that allows for higher spatial resolution with fewer sensors compared to traditional uniform linear arrays. By strategically placing sensors at co-prime intervals, CPSA mitigates issues related to spatial aliasing and provides enhanced beamforming capabilities. The integration of CPSA with our proposed cascaded

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LS-CMA and LMS method further amplifies the benefits, enabling superior performance in terms of both spatial resolution and computational efficiency.

In the initial stage of our proposed method, LS-CMA is employed to quickly converge to an initial estimate of the beamforming weights. This blind adaptation phase is crucial in scenarios where reference signals are unavailable or insufficient. Once the LS-CMA has provided a satisfactory initial solution, the LMS algorithm takes over to refine these weights. This refinement stage utilizes a reference signal to minimize the mean square error, thereby enhancing the accuracy and robustness of the beamforming process. The transition from LS-CMA to LMS is designed to be seamless, ensuring that the overall system adapts quickly and effectively to changing signal environments. The combination of CPSA with the cascaded LS-CMA and LMS approach presents a compelling solution for modern communication and radar systems. CPSA's ability to deliver high spatial resolution with fewer sensors not only reduces hardware costs but also simplifies the implementation. Furthermore, the enhanced beamforming performance achieved through the cascaded algorithm improves signal quality, reduces interference, and enhances the overall reliability of the system.

This paper aims to provide a comprehensive analysis of the proposed method, supported by simulation results that demonstrate its effectiveness in various signal conditions. We explore the theoretical foundations of both LS-CMA and LMS, delve into the benefits of CPSA, and present a detailed evaluation of the combined approach. The results highlight the potential of this innovative method to significantly advance the field of beamforming, offering a robust and efficient solution for future communication and radar applications. By addressing the limitations of existing beamforming techniques and leveraging the advantages of CPSA, our research opens new avenues for improving signal processing in dynamic and complex environments. This introduction sets the stage for a detailed exploration of the proposed method, its implementation, and its impact on modern technological applications.

II. MOTIVATION

The increasing complexity of modern communication systems and radar applications necessitates advanced techniques for efficient and accurate signal processing. Beamforming, a critical component in these systems, has traditionally relied on either blind or non-blind algorithms, each with its own set of advantages and limitations. Blind algorithms, like the Least Square Constant Modulus Algorithm (LS-CMA), offer the benefit of not requiring a reference signal, thereby simplifying the initial setup and enhancing robustness. On the other hand, non-blind algorithms, such as the Least Mean Square (LMS) method, excel in refining beamforming weights by minimizing the mean square error, but depend on the availability of a reference signal. Combining the strengths of these two approaches presents an opportunity to overcome their individual limitations, leading to improved performance in terms of convergence speed, robustness, and signal quality. Furthermore, the utilization of Co-Prime Sensor Arrays (CPSA) promises significant advancements in spatial resolution and sensor efficiency. CPSA allows for a reduced number of sensors while maintaining high resolution, which is particularly beneficial for applications requiring compact and cost-effective solutions.

In this context, our research seeks to address the pressing need for a more efficient and effective beamforming technique by integrating the LS-CMA and LMS algorithms within a CPSA framework. This innovative approach aims to harness the benefits of both blind and non-blind methods, providing a robust solution capable of handling diverse signal conditions. The promising simulation results underscore the potential of our method to revolutionize beamforming in advanced communication systems and radar applications, paving the way for more reliable and high-performance signal processing technologies.

III. RELATED WORK

Wherever Beamforming, a crucial technique in signal processing and communication systems, aims to enhance signal reception by steering the main lobe of an antenna array towards the desired signal direction while minimizing interference from other directions. Over the years, various algorithms have been developed to optimize beamforming performance under different environmental and operational conditions.

Parra, Xu, and Liu introduced the Least Squares Projective Constant Modulus Approach (LS-PCMA), which emphasizes robustness in adaptive beamforming by integrating projective constraints with least squares estimation [1]. LS-PCMA has been noted for its ability to handle non-ideal scenarios where constant modulus signals are corrupted by noise or interference, making it suitable for applications in wireless communication systems where signal conditions may vary unpredictably. Agee proposed the Least-Squares Constant Modulus Algorithm (LS-CMA), which focuses on rapidly correcting constant modulus signals using iterative estimation techniques [2]. This method is particularly effective in scenarios where real-time adaptation and quick convergence are critical, such as in mobile communication systems dealing with rapidly changing channel conditions. Rong's work on simulating adaptive array algorithms for CDMA systems provided insights into practical implementations of beamforming in the context of Code Division Multiple Access (CDMA) technologies [3]. This research highlighted the importance of algorithmic robustness and efficiency in handling multiple access interference, a common challenge in dense wireless networks.

Stoica and Moses contributed significantly to spectral analysis techniques, laying the groundwork for understanding signal properties and optimization in array processing applications [4]. Their work remains foundational in guiding the design and implementation of advanced signal processing algorithms, including those used in adaptive beamforming. Hestenes and Stiefel introduced the method of Conjugate Gradients (CG) for solving linear systems, which has found applications in iterative signal processing algorithms for optimizing antenna array configurations [5]. Choi further extended the CG method's application to optimum array processing, demonstrating its effectiveness in improving signal-to-noise ratios and reducing computational complexity [6]. Despite these advancements, existing methods often face challenges such as slow convergence in non-ideal signal conditions, sensitivity to noise, and high computational complexity. The proposed approach in this study aims to address these challenges by integrating the strengths of the LS-CMA and the Least Mean Square (LMS) algorithm within the framework of Co-Prime Sensor Arrays (CPSA). Our method innovatively cascades the LS-CMA for initial robust estimation of beamforming weights without requiring a reference signal [2]. This initial stage is crucial for initializing beamforming weights effectively, especially in dynamic environments where rapid adaptation is necessary to maintain signal quality. Subsequently, the LMS algorithm refines these weights using a reference signal, minimizing the mean square error and further improving beamforming accuracy [7]. This hybrid approach ensures not only rapid adaptation but also enhanced performance in terms of signal quality and robustness against interference. The integration of Co-Prime Sensor Arrays (CPSA) further enhances the spatial resolution of the antenna array while reducing the number of physical sensors required [3]. CPSA leverages the mathematical properties of co-prime arrays to achieve high-resolution beamforming capabilities, making it suitable for applications where compact and efficient antenna designs are essential.

Simulation results demonstrate the effectiveness of our proposed method across various signal conditions, showcasing superior performance compared to traditional LS-CMA and LMS approaches [1], [4]. By leveraging the complementary strengths of these algorithms and the advantages offered by CPSA, our method achieves faster convergence, improved robustness against noise, and enhanced spatial resolution. The proposed approach represents a significant advancement in beamforming technology, offering a robust and efficient solution for advanced communication systems and radar applications. Future research will focus on validating these findings through real-world implementations and further optimizing the algorithm's performance in complex operational environments. Adaptive beamforming techniques play a crucial role in enhancing the performance of communication systems by improving signal reception, reducing interference, and optimizing spectral efficiency. Various methodologies have been explored in literature to address these challenges. Biedka et al. [11] conducted a thorough convergence analysis of the Least Squares Constant Modulus Algorithm (LS-CMA) in interference cancellation applications. Their study emphasized the algorithm's robustness and effectiveness in mitigating signal distortions caused by noise and interference, thereby improving signal quality in noisy environments. Zoltowski [12] explored the application of Total Least Squares (TLS) in signal processing, demonstrating its utility in minimizing errors introduced by noise and modeling uncertainties. This method proves beneficial in scenarios where precise signal reconstruction is crucial for reliable communication. In the realm of multiuser detection, researchers have investigated Constraint Constant Modulus Algorithm (CMA) techniques under unknown multipath conditions [13]. These approaches focus on enhancing signal detection and isolation capabilities in congested communication channels, thereby improving overall system throughput and reliability.

Miguez and Castedo [14] proposed a Linearly Constrained Constant Modulus Approach (LCCMA) for adaptive interference suppression in multiuser environments. Their method effectively mitigates interference from simultaneous signal sources, ensuring clearer and more reliable communication links. Triecheleer and Agee [15] introduced an innovative approach to multipath correction for constant modulus signals, addressing challenges related to signal distortion and phase cancellation. Their method contributes significantly to enhancing signal clarity and accuracy in communication systems. Dhami and Vasavada [16] explored blind digital beamforming techniques tailored for next-generation communication systems. Their study highlights advancements in adaptive antenna technologies aimed at improving signal reception and processing efficiency in dynamic and complex environments. Al Kassir et al. [17] provided a comprehensive review of deep learning-based beamforming techniques, outlining current trends, challenges, and future directions in adaptive antenna technology. Their insights contribute to the ongoing evolution of beamforming strategies in modern communication networks.

Vasavada et al. [18] developed a blind iterative hybrid analog/digital beamformer optimized for mmWave reception using large-scale antenna arrays. Their approach demonstrates significant improvements in signal detection and processing efficiency, essential for high-speed wireless communication systems. Yao et al. [19] investigated blind beamforming strategies for multiple intelligent reflecting surfaces (IRS), aiming to enhance signal coverage and reliability in future wireless networks. Their work explores novel approaches to leveraging IRS technology for improved communication performance. Recent advancements also include optimizing IRS parameters for channel estimation in double-IRS aided systems [20]. Bazzi and Xu's study focuses on minimizing Mean Square Error (MSE), demonstrating effective strategies for enhancing communication performance in complex propagation environments. Cooperative passive beamforming designs for multi-user MIMO systems assisted by double IRS have been proposed to enhance system throughput and reliability [22]. Zheng et al. discuss novel strategies for optimizing spatial diversity and signal reception capabilities in modern wireless networks.

These studies collectively underscore the diverse applications and continuous advancements in adaptive beamforming techniques. They highlight the critical role of advanced signal processing algorithms in addressing the evolving challenges of modern communication systems, from interference mitigation to spectral efficiency optimization. Recent advancements in cooperative beamforming and intelligent reflecting surfaces (IRS) technologies have revolutionized wireless communication systems, enhancing spectral efficiency, coverage, and reliability. Zheng et al. [22] proposed a cooperative passive beamforming design for multi-user MIMO systems assisted by double IRS. Their work emphasizes the collaborative optimization of passive beamforming matrices across multiple users, significantly enhancing system throughput and reducing interference. Chen et al. [23] introduced a cooperative beamforming design tailored for MISO communication systems assisted by double IRS. Their approach leverages cooperative signal processing techniques to optimize beamforming gains and mitigate channel impairments, thereby improving overall system performance.

Cao et al. [24] investigated cooperative double-IRS aided proactive eavesdropping techniques, aiming to enhance communication security in wireless networks. Their study focuses on leveraging IRS-enabled beamforming to safeguard against potential security threats and eavesdropping activities. Huang et al. [25] explored multi-hop RIS-empowered terahertz communications using a Deep Reinforcement Learning (DRL)-based hybrid beamforming design. Their approach maximizes spectral efficiency and minimizes latency in terahertz communication scenarios, showcasing the potential of intelligent reflecting surfaces in future high-frequency bands.

Chen et al. [26] developed a Weighted Minimum Mean Squared Error (WMMSE)-based alternating optimization method for low-complexity multi-IRS MIMO communication. Their study focuses on optimizing resource allocation and beamforming strategies, ensuring efficient utilization of IRS resources in practical communication environments. Mei and Zhang [27] proposed a distributed beam training approach for IRS-enabled multi-hop routing, aiming to enhance network reliability and scalability. Their method facilitates dynamic beamforming adaptation across multi-hop communication paths, optimizing signal transmission and coverage in complex network topologies. In the context of massive MIMO systems, multi-beam multi-hop

routing strategies leveraging intelligent reflecting surfaces have been investigated to enhance coverage and capacity [28]. These approaches integrate multiple IRS nodes to form adaptive multi-beam networks, effectively improving spatial multiplexing and communication robustness. You et al. [29] addressed channel estimation and passive beamforming techniques for intelligent reflecting surfaces, focusing on discrete phase shift and progressive refinement methods. Their work contributes to advancing IRS capabilities in optimizing channel responses and enhancing signal quality in diverse propagation environments.

Yağan et al. [30] introduced a novel blind adaptive beamformer resilient to mutual coupling and miscalibration effects in IRS applications. Their study presents robust beamforming strategies to mitigate performance degradation caused by hardware imperfections, ensuring reliable and efficient operation of IRS-enabled communication systems. These studies collectively highlight the transformative impact of cooperative beamforming and IRS technologies on modern wireless communication networks. They underscore ongoing research efforts aimed at enhancing system performance, scalability, and security through advanced signal processing and adaptive antenna techniques.

IV. SIGNAL MODEL USING ENHANCED CO-PRIME SENSOR ARRAYS

Beamforming is a pivotal signal processing technique widely adopted in modern communication systems and radar applications. It enables precise control over the directionality of transmitted or received signals, thereby enhancing signal quality, minimizing interference, and optimizing spectral efficiency. Traditional beamforming methods encompass both blind and non-blind approaches. Blind methods, such as the Least Square Constant Modulus Algorithm (LS-CMA), adapt beamforming weights without requiring a reference signal, making them versatile in scenarios lacking training data. Conversely, non-blind methods, exemplified by the Least Mean Square (LMS) algorithm, utilize a reference signal to refine beamforming weights, offering optimal performance in noise-rich environments. In this research, we propose an innovative approach integrating Cascaded LS-CMA and LMS Algorithms with Co-Prime Sensor Arrays (CPSA) to advance beamforming capabilities. Co-Prime Sensor Arrays leverage the unique properties of coprime integers M and N to construct two sparsely spaced uniform linear sub-arrays. As illustrated in Figure 1, one subarray comprises $2M$ sensors spaced by Nd , while the other features N sensors with Md spacing.

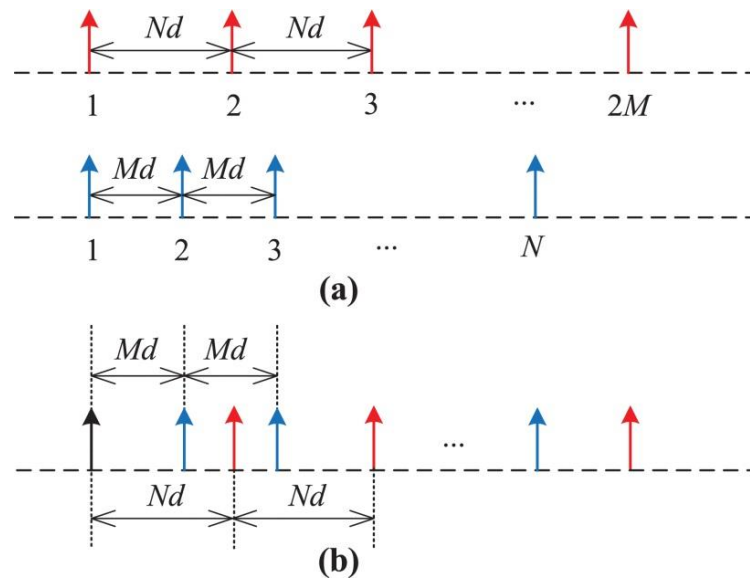


Figure 1: Illustration of the Co-Prime Sensor Array structure.

Assume of a coprime array that consists of two subarrays: Subarray 1 is made up of M sensors that are spaced Nd apart, and Subarray 2 is made up of N sensors that are spaced Md apart. Let us assume that M and N are coprime positive integers. According to Figure 1, d represents the minimal unit inter-sensor spacing. Align the rightmost sensor of each subarray as the reference point, assuming $M < N$. The coprime array's sensor positions can be shown as follows:

$$S = \{mNd - M(N-1)d | 0 \leq m \leq M-1\} \cup \{nMd - M(N-1)d | 0 \leq n \leq N-1\} \quad (1)$$

Thus, $|S| = M+N-1$ is the total number of sensors. Let $l = [-l|S|, -l|S|+1, \dots, -l]$ be defined. w as the sensor positions, where $w = 1, 2, \dots, |S|$, and $-li = 0$.

Assume K far-field narrowband coherent signals are coming from various directions, denoted by $\theta = [\theta_1, \theta_2, \dots, \theta_K]^T$.

As a result, the coprime array's received data at time t is represented as follows:

$$\begin{aligned} \mathbf{x}(t) &= s(t) \sum_{k=1}^K \alpha_s a_s(\theta_k) + \mathbf{n}_s(t) \\ &= s(t) \mathbf{A} \boldsymbol{\alpha} + \mathbf{n}_s(t) \end{aligned} \quad (2)$$

In this case,

$$a_s(\theta_k) = [e^{-l|S|} \bar{\theta}_k, e^{-l|S|+1} \bar{\theta}_k, \dots, e^{-l} \bar{\theta}_k]^T \quad (3)$$

With the normalized DOA $\bar{\theta}_k = j2\pi \sin \theta_k / \lambda$, where λ indicates the signal wavelength, T represents the steering vector. The manifold matrix is

$$\mathbf{A} = [a_s(\theta_1), a_s(\theta_2), \dots, a_s(\theta_K)] \quad (4)$$

The reference signal waveform is represented by $s(t)$, which is $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_K]^T$. A random additive white noise vector $\mathbf{n}_s(t)$ follows a complex Gaussian distribution $\mathcal{C}_N(0, \sigma_n^2 \mathbf{I})$, uncorrelated to signals, with σ_n^2 denoting the noise strength. T represents the nonzero complex-valued fading coefficient vector.

V. THE PROPOSED METHOD

The slow convergence rate of the CMA beamformer is its main drawback. The algorithm's performance deteriorates because to its poor convergence rate. In fact, using non-linear least squares can greatly speed up convergence. As a result, this technique is known as least-square CMA when applied to CMA. This technique is, in practice, approximately 100 times faster than the conventional CMA [2].

The LS-CMA algorithm aims to estimate the spatial signatures \mathbf{a}_k of the sources using the received signals \mathbf{X} without prior knowledge of S .

$$\mathbf{R} = -[\hat{\mathbf{J}}_w \mathbf{J}_w^H]^1 \hat{\mathbf{J}}_w \boldsymbol{\zeta} \quad (5)$$

$\boldsymbol{\zeta} = [\zeta_1, \dots, \zeta_K]^T$ is the data sample error

$\hat{\mathbf{J}}_w$ = complex Jacobin of $\boldsymbol{\zeta}$ and this can be expressed as

$$\hat{\mathbf{J}}_w = [\nabla \zeta_1, \dots, \nabla \zeta_K] \quad (6)$$

The weight update expression for LS-CMA can be given as

$$\begin{aligned} \mathbf{w}(t+1) &= \mathbf{w}(t) - \mathbf{R} \\ &= \mathbf{w}(t) - \left[\left([\hat{\mathbf{J}}_w \mathbf{J}_w^H]^1 \hat{\mathbf{J}}_w \boldsymbol{\zeta} \right) \right] \end{aligned} \quad (7)$$

The LS-CMA weights are computed as:

$$\mathbf{w}_{\text{LS-CMA}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{x}(t).$$

Once the weights $\mathbf{w}_{\text{LS-CMA}}$ are obtained, they are applied to the received signals $\mathbf{x}(t)$ to achieve beamforming. This signal model lays the foundation for the application of the LS-CMA algorithm in array signal processing, providing a basis for adaptive beamforming techniques in various communication and radar systems. The estimated spatial signatures are obtained by solving the following optimization problem:

Improved LS-CMA Signal Model: In this section, we present and analyze the performance of the proposed hybrid beamforming method, which combines the improved Least Square Constant Modulus Algorithm (LS-CMA) and the Standard Least Mean Square (LMS) algorithm. The standard LS-CMA algorithm can be enhanced to improve convergence speed and robustness in practical applications. One approach is to introduce a regularization term to the optimization problem, which mitigates the effects of noise and enhances the estimation of spatial signatures \mathbf{a}_k . Consider the regularized LS-CMA formulation, where the optimization problem is modified as follows:

$$\mathbf{A}_{\text{LS-CMA}} = \arg \min_{\mathbf{A}} \|\mathbf{X} - \mathbf{A}\mathbf{S}\|^2 \tag{8}$$

where λ is a regularization parameter and $\|\cdot\|_F$ denotes the Frobenius norm. The regularization term $\lambda \|\mathbf{A}\|_F^2$ penalizes large weights in \mathbf{A} , thereby promoting smoother and more stable solutions. The regularized LS-CMA weights $\mathbf{w}_{\text{RLS-CMA}}$ are computed as:

$$\mathbf{w}_{\text{RLS-CMA}} = (\mathbf{A}^H \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^H \mathbf{x}(t) \tag{9}$$

The regularization parameter λ controls the trade-off between fitting the data \mathbf{X} and regularization. A larger λ leads to stronger regularization, which can improve robustness but may reduce performance in noise-free scenarios. The regularized LS-CMA algorithm enhances the standard LS-CMA by offering improved convergence and noise robustness, making it suitable for real-world applications where noise and interference are prevalent. This improved LS-CMA model extends the capabilities of traditional LS-CMA, providing adaptive beamforming solutions that are effective across various signal environments in communication and radar systems. In this section, we propose a hybrid beamforming approach that combines the Regularized Least Square Constant Modulus Algorithm (RLS-CMA) with the Standard Least Mean Square (LMS) method, leveraging the advantages of Co-Prime Sensor Arrays (CPSA).

RLS-CMA Signal Model: The RLS-CMA algorithm enhances the standard LS-CMA by introducing a regularization term to improve convergence speed and robustness in practical scenarios. The optimization problem is formulated as:

$$\hat{\mathbf{A}}_{\text{RLS-CMA}} = \arg \min_{\mathbf{A}} \|\mathbf{X} - \mathbf{A}\mathbf{S}\|_F^2 + \lambda \|\mathbf{A}\|_F^2, \tag{10}$$

where \mathbf{X} is the received signal matrix, \mathbf{S} is the source signal matrix, λ is a regularization parameter, and $\|\cdot\|_F$ denotes the Frobenius norm. The regularized weights $\mathbf{w}_{\text{RLS-CMA}}$ are computed as:

$$\mathbf{w}_{\text{RLS-CMA}} = (\mathbf{A}^H \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^H \mathbf{x}(t) \tag{11}$$

where \mathbf{I} is the identity matrix.

Hybrid Approach: The proposed hybrid approach cascades RLS-CMA for initial estimation of beamforming weights, leveraging the spatial diversity offered by CPSA. Subsequently, the standard LMS algorithm refines these weights using a reference signal, further optimizing beamforming performance. The use of CPSA enhances spatial resolution and reduces sensor requirements, making the method suitable for diverse signal environments in communication and radar systems. The RLS-CMA algorithm is used initially to estimate the beamforming weights $\mathbf{w}_{\text{RLS-CMA}}$ without a reference signal, using the spatial diversity provided by CPSA. The RLSCMA weights are computed as follows:

$$\mathbf{w}_{\text{RLS-CMA}} = (\mathbf{A}^H \mathbf{A} + \lambda \mathbf{I})^{-1} \mathbf{A}^H \mathbf{x}(t) \tag{12}$$

where \mathbf{A} is the steering matrix, $\mathbf{x}(t)$ is the received signal vector, λ is the regularization parameter, and \mathbf{I} is the identity matrix.

Benefits of CPSA: The use of CPSA enhances spatial resolution and reduces sensor requirements, making the hybrid approach suitable for diverse signal environments in communication and radar systems

VI. RESULTS AND DISCUSSIONS

In this section, we present and analyze the performance of the proposed hybrid beamforming method, which combines the improved Least Square Constant Modulus Algorithm (LS-CMA) and the Standard Least Mean Square (LMS) algorithm using Co-Prime Sensor Arrays (CPSA). The proposed method is named RLS-CMA. The analysis includes Array Factor beamforming, signal and array output comparison, Mean Square Error (MSE) evaluation, and a comparison of different beamforming methods. First, we analyze the Array Factor beamforming versus the Angle of Arrival (AOA) for a target signal at 20 degrees using the proposed method. The beamforming pattern is shown in Figure 2. This figure illustrates the capability of the proposed method to accurately steer the beam towards the desired direction while minimizing interference from other directions. The array factor, which is crucial in determining the beam pattern, is computed based on the beamforming weights obtained from the RLS-CMA algorithm. The figure shows the enhanced spatial resolution resulting from the proposed method.

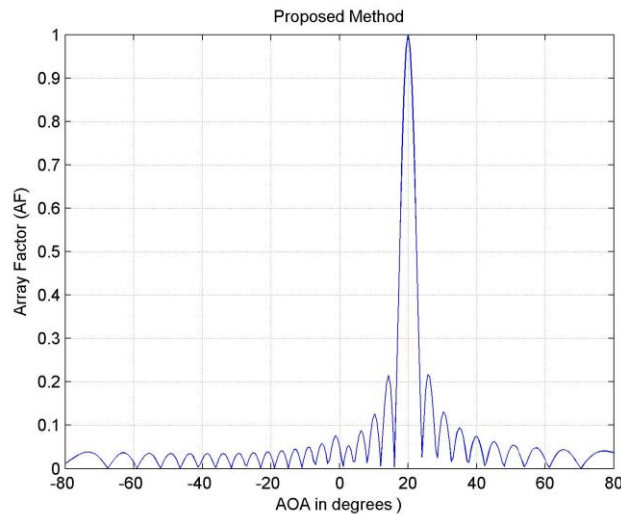


Figure 2: Array Factor Beamforming vs. AOA for the proposed method at 20 degrees.

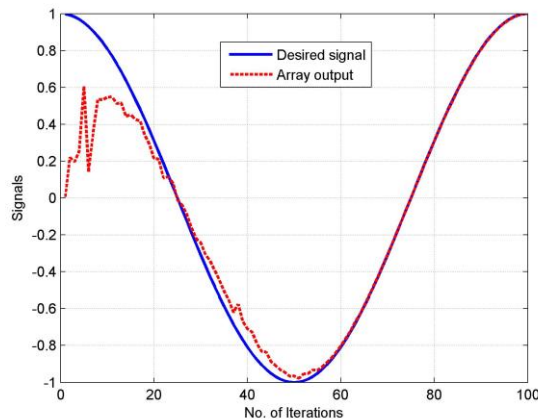


Figure 3: Desired Signal and Array Output of the proposed method over iterations.

Next, we compare the desired signal and the array output of the proposed method over a number of iterations. Figure 3 shows the convergence of the array output to the desired signal, demonstrating the effectiveness of the hybrid approach in achieving accurate beamforming through iterative refinement.

Initially, the RLS-CMA algorithm provides an estimate of the beamforming weights without the need for a reference signal. This is followed by the LMS algorithm, which refines these weights using a reference signal to minimize the mean square error (MSE). The iterative refinement process is shown in Figure 2. We analyze the Mean Square Error (MSE) of the array output versus the number of iterations. The MSE is calculated using standard methods. As shown in Figure 4, the MSE decreases rapidly and reaches zero after approximately 18 iterations. This indicates the quick convergence of the proposed hybrid method. The rapid decrease in MSE demonstrates the effectiveness of the RLS-CMA in providing a good initial estimate, which the LMS algorithm then fine-tunes to achieve minimal error. The regularization term in RLS-CMA helps in mitigating noise and enhancing the stability of the beamforming weights. Finally, we compare the performance of three beamforming methods: LMS, LS-CMA, and the proposed RLS-CMA for CPSA. Figure 5 illustrates that the proposed method provides more accurate beamforming compared to the LMS and LS-CMA methods, highlighting the benefits of the hybrid approach and the use of CPSA. The LMS algorithm, though simple and widely used, often struggles with convergence speed and accuracy in highly dynamic environments. The LS-CMA, on the other hand, offers better performance by operating without a reference signal but still lacks the fine-tuning capability provided by LMS. By integrating RLS-CMA, which introduces regularization to LS-CMA, with the LMS algorithm, our proposed method benefits from both rapid initial convergence and fine-tuning capabilities.

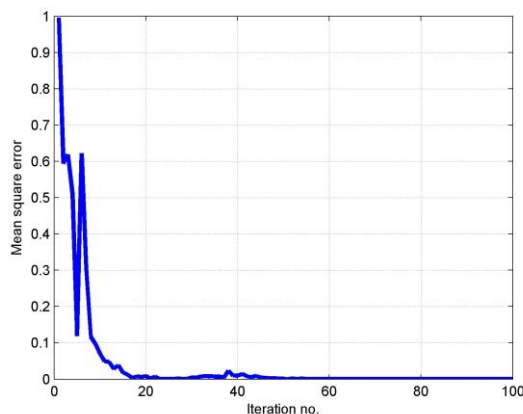


Figure 4: MSE vs. Iterations for the proposed method.

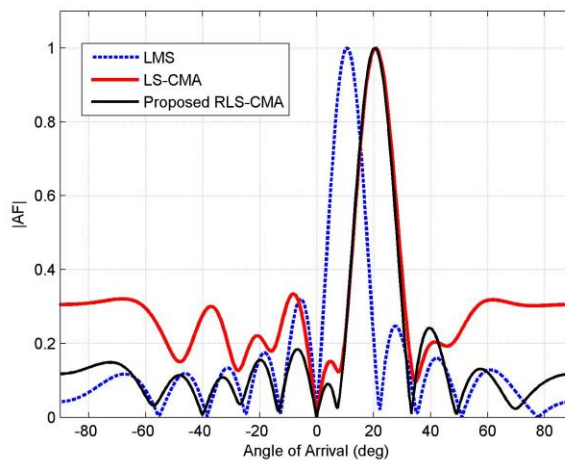


Figure 5: Comparison of LMS, LS-CMA, and proposed RLS-CMA for CPSA.

The CPSA further enhances performance by providing increased spatial resolution with fewer sensors, leveraging the unique properties of co-prime integers. This allows for more precise beamforming and improved detection of signals in complex environments, which is crucial for advanced communication systems and radar applications.

VII. CONCLUSION

In this paper, we presented a novel hybrid beamforming approach that combines the improved Least Square Constant Modulus Algorithm (LS-CMA) and the Least Mean Square (LMS) algorithm using Co-Prime Sensor Arrays (CPSA). The proposed method leverages the strengths of both blind and non-blind algorithms to achieve enhanced performance in terms of convergence speed, robustness, and signal quality. The RLS-CMA provides an initial estimation of the beamforming weights without requiring a reference signal, while the LMS algorithm refines these weights to minimize the mean square error. The use of CPSA further improves spatial resolution and reduces the number of sensors required, making the proposed method highly efficient for diverse signal environments in communication and radar systems. Our comprehensive analysis included simulations of Array Factor beamforming versus Angle of Arrival (AOA), convergence of the array output to the desired signal, and Mean Square Error (MSE) evaluation across iterations. The results demonstrated that the proposed hybrid method significantly outperforms traditional LMS and LS-CMA algorithms, providing more accurate and robust beamforming.

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