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Secure Vertex Edge Domination of Some Middle Graphs

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Abstract: - Graph theory is one of the fields associated with contemporary mathematics and computer science that has been growing the fastest. The most rapidly evolving branch of graph theory is domination. When there is a restricted amount of facilities (which include fire stations and hospitals) and every attempt is made to shorten the distance a person must travel in order to get to the nearest company, accessibility issues become dominant. A comparable issue arises when one tries to enforce a certain maximum distance to a facility while minimizing the number of facilities available to service individuals. Domination notions are also spotted in mapping, communication or electrical network surveillance, and challenges related to creating delegations. { J } [V (G)] is a secure vertex edge dominating set of G, Assume each edge, e [E (G)], a vertex exists, V { J } such that V defends edge e. That is, the edges incident on a vertex in {J}together with the edges adjacent to the incident edgeare protected by that vertex.A secure vertex edge dominating { J }G has the characteristic of being the dominant set in a graph where each vertex y [V – { J }] is adjacent to either a vertex , adjacent to a vertex as well an incident edge of y, x { J }which means ({J} - {x}) {y} is a dominating set. The phrase "secure vertex edge domination number" corresponds to lowest cardinality of secure vertex edge domination in G and corresponds to the lowest cardinality of secure vertex edge domination in G and is represented by . We kick off the current studyby investigating the novel parameter and determining the secure vertex edge dominance number of particular middle graph kinds that are members of particular graph families, includingpath, cycles, complete, wheel and friendship graphs.

Keywords: Middle graph, Dominating set, Secure dominating set, Domination number.

1. A PREFACE

Graph theory is a crucial mathematics topicutilized in structural models. These structural configurations of diverse items or technology inspire new discoveries and alter the environment to advance those fields. The Kongsberg Bridge problem marked the beginning of the field graph theory in 1735.Around 1960, the mathematical research of graph domination began. In graph theory, the concept of domination has generated a lot of research. A book on domination that lists 1222 works in this field was published in 1998 [1]. A group of vertices in D that have every vertex either in D or adjacent to a vertex in D is known as a dominant set in a graph. Researchers de Jaenisch [2] in 1862looked on the minimal quantity of queens needed to cover a chessboard of n by n.In 1892, chess players studied three basic types of problems, according to W. W. Rouse Ball [3].

The study of domination in graphs experienced additional growth in the late 1950s and early 1960s, starting with Claude Berge [4] in 1958. In a book he authored on graph theory, Berge first described the "coefficient of external stability," commonly referred to as the domination number of a graph. The phrases "dominating set" and "domination number" were used by Oystein Ore [5] in his 1962 book on graph theory. Yaglom brothers conducted a more in-depth study on the aforementioned issues about 1964 [6]. For kings, bishops, knights, and rooks respectively, some of these issues were resolved as a result of their research.

A decade later, in a review work, Cockayne and Hedetniemi [7] made history by using the notation $\gamma(G)$ to denote the graph G's domination number for the first time.Dominant in graphs has drawn a lot of attention since the publication of this paper, with many more research articles being published about it. Cockayne et al. [8] developed the idea of secure domination. In a graph G = ([V],[E]), each vertex in a subset S of V must have an enforcer assigned to it if we want to defend an unprotected vertex, u.

This new guard configuration also creates a powerful group. A secure dominating set of G is defined as follows: every vertex in V-S that is S secured by a vertex in S is considered to be a secure dominating set. This is the

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minimal cardinality of a secure dominating set. Certain conclusions about co-secure and secure dominations in graphs are given by Arumugam et al. (2014) [9]. In 2007, J. R. Lewis [10] presented a number of findings on vertex-edge and edge-vertex characteristics in graphs in his doctoral dissertation. A new parameter has been implemented by us, secure vertex edge domination of graphs, as a result of this. [11] In 2019, P.Roushini Leely Pushpam and Chitra Suseendran presented their findings regarding Secure Domination in Middle graphs. According to Farshad Kazemnejad et al. [12], the Domination number of Middle graphs has been calculated. In this work, we determine the secure vertex- edge domination number of different middle graphs, including complete, wheel, path, cycles, complete bipartite, and friendship graphs, that are members of particular graph families.

The Graph:

The edge set (E(G)) and vertex set (V(G)) of a graph G = (V, E) are its constituent parts. Let $n = |V(G)|$ indicate the G order. The degree of a graph vertices is calculated by computing the total number of edges that occur on the vertex, twice accounting for loops.The symbol deg(v) represents a vertex's degree. The symbols $\Delta[G] and\ \delta[G]$ indicate the graph's highest and least degree, respectively.

Neighbourhood:

A vertex V's open neighbourhood is represented by $N(v) = \{u \in V : uv \in E\}$, and its closed neighbourhood is denoted by $N[v] = N(v) \bigcup \{v\}$

Floor Function:

The largest integer that is equal to or lower than a real number y is its floor function, and it is represented by. $\lfloor y \rfloor$ Given that n is an integer, let's say that $\lfloor y \rfloor = m, m \leq y < m+1$.

The Ceiling Function:

The lowest integer higher than or equal to a real number y is its ceiling function, and it is represented by $\lceil y \rceil$. Given that n is an integer, let's say that $\lceil y \rceil = m,\ m-1 < y \leq m$.

The Domination:

Very stunning collection D is a set of vertices where each vertex of G has at least one neighbour who is also in D, or is either located in D. The smallest cardinality of such a set is shown by the domination number of G. It is represented as $\gamma(G)$.

Secure Domination:

Assume that the basic graph G = ([V],[E]) has order n. If each one vertex in V-F is next to a vertex in F, then { F } $\subseteq$V is a dominant set of G. Graph G shows a stable dominating set F is a dominant set with the characteristic of being next to each and every vertex u $\in$(V – F) so that (F- {u})$\cup$({v}) is a dominating set .The secure domination number, represented by $\gamma_s(G)$, is the minimal cardinality of a secure dominating set of G.

Middle Graph:

The graph that results from splitting each edge of a graph G exactly once and connecting all of these newly added vertices of neighboring edges of G is called the middle graph M[G] of a graph G.

Here is a basic definition of M[G].

V[G]$\cup$ E[G] is the vertex set of M[G]. If either

(i) u, v is in E[G] and u, v is adjacent in G or

(ii) u is in V[G], v is in E[G], and u, v is incident in G, Then these two vertices, u and v, in the vertex set of M[G], are adjacent in M[G].

Star Graph and Complete Bipartite Graph:

The following characteristics apply to the fundamental m-vertex graph, which is known as the star graph of order m, or Sm. Apart from the distinguishing vertex, which is the single unique vertex of degree m-1, all the other vertices are of degree 1.

If each one vertex in a star graph is part of one set and every other vertex is part of the other set, then the graph is considered complete bipartite. $K_{1, m-1}$ represents a complete bipartite network, and is a graph of stars with m vertices.

Friendship Graph:

The graph generated by having n = 2p+1 vertex $p \geq 1$ triangles all attached to a common vertex and it is denoted by Fn.

2. SECURE VERTEX EDGE DOMINATION OF SOME MIDDLE GRAPHS

Secure Vertex Edge Domination:

A dominating set { J } $\subseteq$ V(G) of a graph G is said to be a secure vertex edge dominating set of G if, for all edges, $e \in E(G)$ then there exists a vertex $x \in$ {J} such that x defends the edge e. That is, A vertex in { J }defends the edges incident on that vertex and the edges which are adjacent to that incident edges. A secure vertex edge dominating set { J } of a graph G is a dominating set with the property that each vertex $y \in$ V – {J} is either adjacent to a vertex or a vertex adjacent to the incident edges of y, x $\in$ {J }such that ({J} - {x}) $\cup$ ({y}) is a dominating set.

The secure vertex edge domination number in G is the lowest cardinality of a secure vertex edge domination and is expressed by $\gamma_{sve}(G)$.

Theorem - 1:

For every path graph's middle graph $\gamma_{sve}[M(P_n)] = \left\lceil \frac{2n+3}{4} \right\rceil, n >= 2$.

Proof:

To get M(Pn), let e1, e2, e3,.. en be the newly added vertices corresponding to the edges of Pn, and let v1, v2, v3,.. vn be the vertices of the path graph Pn.

Thus $V[M(P_n)] = [2n - 1]$ and $E[M(P_n)] = [3n - 4]$.

Now, let the theorem be proved by the principle of Mathematical Induction.

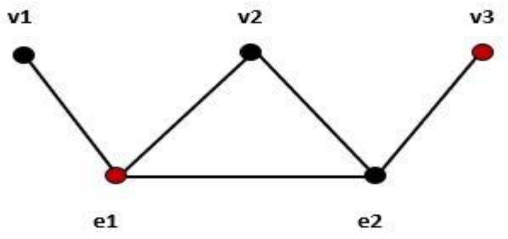
For n =2, $V[M(P_2)] = 3$ and $E[M(P_2)] = 2$

The secure vertex edge domination number is given as $\gamma_{sve}[M(P_2)] = \left\lceil \frac{2(2)+3}{4} \right\rceil = \lceil 1.75 \rceil = 2$.

In the above example, the two-vertex secure vertex edge dominating set will safeguard all vertices and edges of the path graph 2's middle graph.

Hence it is true for n = 2.

For n = 3, $V[M(P_3)] = 5$ and $E[M(P_2)] = 5$



M(P3)

For the graph M(P_3) the secure dominating set J = {e_1, v_3}. The vertices e_1 and v_3 secure all the five vertices and 5 edges of the graph.

Hence it is true for n = 3.

It is simple to confirm the outcome for $2 \le n \le 11$.

Consider that all paths of order smaller than n, where n > 11, have the same outcome.

The vertices of a middle graph of path graph order n are denoted as (v1, v2, v3,....., vn) and Graph $M(P_n')$ has (v_{11}, v_{12}, v_{13}, ...v_n) as it's vertices.

By the principle of mathematical induction,

$$\gamma_{sve}[M(P_n')] = \left\lceil \frac{2(n-10)+3}{4} \right\rceil$$

$$\gamma_{sve}[M(P_n)] = \gamma_{sve}[M(P_n')] + 5$$

$$= \left\lceil \frac{2(n-10)+3}{4} \right\rceil + 5$$

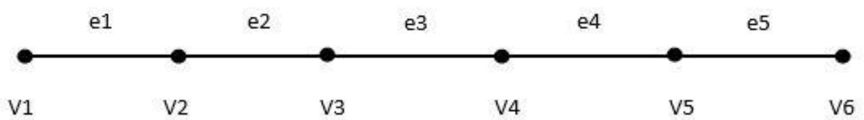
$$= \left\lceil \frac{2n-20+3+20}{4} \right\rceil$$

$$= \left\lceil \frac{2n+3}{4} \right\rceil$$

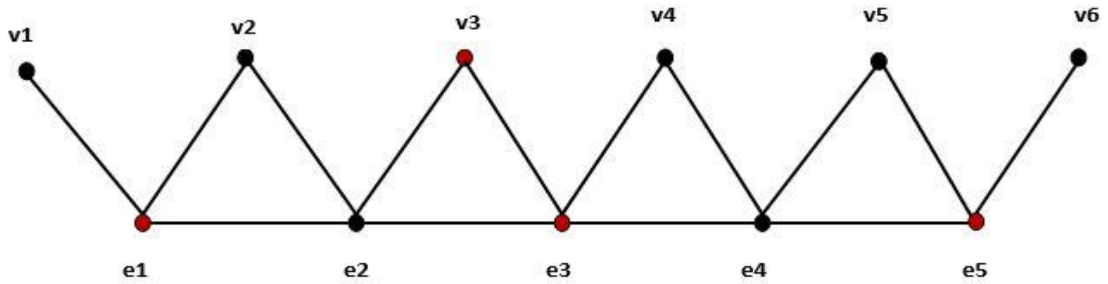
Hence the proof.

Example – 1 :

Consider a path graph with 6 vertices.



P_6



Middle Graph of Path Graph – M(P_6)

The middle graph of the path graph with 6 vertices contains 11 vertices and 14 edges.

The secure vertex edge domination number is given by $\left\lceil \frac{2n+3}{4} \right\rceil = \left\lceil \frac{2(6)+3}{4} \right\rceil = \lceil 3.75 \rceil = 4$.

The secure dominating set J = {e1, v3, e3, e5}. Every vertex within the set J will secure the edges that are next to it, the edges that are incident from it, and the vertices that are next to it.

As a result, every vertex and every edge in the middle graph of path graph, which has six vertices, will be secured.

Theorem – 2:

If Cm is the cycle graph with m vertices, then the middle graph of Cm's secure vertex edge domination is given by $\gamma_{sve}[M(C_m)] = \left\lceil \frac{m+1}{2} \right\rceil, m \geq 3$.

Proof:

To produce M(C_m), let e1, e2, e3,..., em be the freshly added vertices associated with the edges of C_m, and let the vertices of the cycle graph C_m be v1, v2, v3,..., vm. The middle graph of C_m's vertex and edge sets are denoted as [2m] and [3m], respectively.

Now we construct a secure vertex edge dominating set J of M(C_m) as

$J = \left\{ e_j \;\; such\ that\ 1 \leq j \leq \left\lfloor \frac{m-1}{2} \right\rfloor \right\}$ along with $|J| = \left\lceil \frac{m+1}{2} \right\rceil$. Considering M($C_m$) has edges that are all either incident on any vertex in { J }or adjacent to the incident edge on that any vertex in {J}.

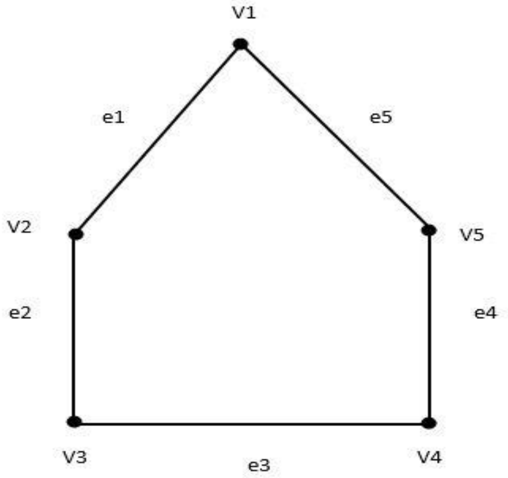
Thus, every vertex in $u_i \in J$ can be replaced by $v_j \in V - J \;\; such\ that\ u_i v_j \in E[M(C_m)]$ and the set $(J - \{e_i\}) \cup \{v_j\}$ is also a M(C_m)'s secure vertex edge dominant set.J is thus a secure vertex edge dominating set of M(Cm).

In V - J, each vertex has a degree of two or four, so every set in $T \subseteq V - J$ We are able to locate a set Swhich is non empty in J so that the subgraph that is generated by $T \cup S$ has been connected. Hence, J is a secure vertex edge dominant set of M(C_m).

Thus, $\gamma_{sve}[M(Cm)] = \left\lceil \frac{m+1}{2} \right\rceil,\ m \geq 3.$

Example –2 :

Let's examine a cycle graph that has five vertices.



$\mathbf{C_5}$

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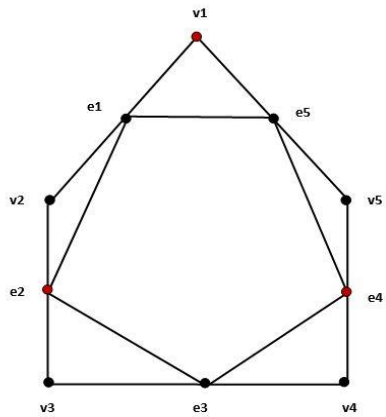


Middle Graph of Cycle Graph – M(C_5)

The middle graph of C_5 contains 10 vertices and 15 edges.

The secure vertex edge dominating number of M(C_5)'s is provided by $\left\lceil \frac{5+1}{2} \right\rceil = 3$.

Let J ={ v_1, e_2, e_4}be the M(C_5)'s secure vertex - edge dominating set. Every vertex in the graph, as well as the incident edges and edges adjacent to the incident edges M(C_5) are protected by the vertices in the collection {J}.

Theorem– 3 :

The secure vertex edge dominance for a middle graph of the complete graph Km can be determined by $\left\lceil \frac{m(\mathrm{m}+1)}{4} \right\rceil, m \geq 2$.

Proof:

To obtain M(K_m), let e1, e2, e3,..em be the most recently added vertices that correspond to the edges of K_m. Consider the vertices of the complete graph K_m to be v1, v2, v3,... vm. Furthermore, the middle graph of the complete graphs' s vertex and edge sets is provided by $\left[\frac{\mathrm{m}(\mathrm{m}+1)}{2} \right]$ *and* $\left[\frac{3m(\mathrm{m}-1)}{2} \right]$ respectively.

Let us prove this result by using the principle of mathematical induction.

It is evident that the middle graph of K_2 will contain three edges and vertices when m = 2, and that M[K_2]'s secure vertex edge domination number will be 2.All of the edges and vertices in the middle graph of K_2 are protected by the two vertices that make up the dominant set.

For n = 2, the result is hence true.

Verifying the outcome is simple when $2 \le n \le 5$.

With the assumption that If n equals l, then it is genuine, it can be $\gamma_{sve}[M(K_l)] = \left\lceil \frac{l(l+1)}{4} \right\rceil$.

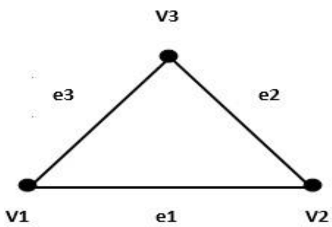
When n = l+ 1,

$$\begin{aligned}\gamma_{sve}[M(K_{l+1})] &= \gamma_{sve}[M(K_l)] + (l+1) \\ &= \left\lceil \frac{l(l+1)}{4} \right\rceil + l + 1 \\ &= \left\lceil \frac{l(l+1)+4(l+1)}{4} \right\rceil \\ &= \left\lceil \frac{(l+1)(l+2)}{4} \right\rceil\end{aligned}$$

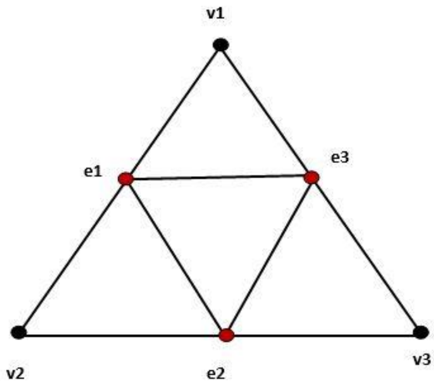
For l + 1, it is therefore true. Thus, proof exists.

Example – 3 :

Think of a complete graph that has three vertices.



The equivalent K_3 graph's middle graph is provided by



Middle Graph of Complete Graph - M(K_3)

The graph $M(K_3)$ has 6 vertices and 9 edges. The secure vertex edge domination number is given by

$$\gamma_{sve}[M(K_3)] = \left\lceil \frac{3(3+1)}{4} \right\rceil = \lceil 3 \rceil = 3.$$

The secure vertex edge dominating set J = {e_1, e_2, e_3}, which safeguards each vertex and edge in K_3's middle graph.

Theorem - 4:

Assume that W_n is the wheel graph with $n \geq 4$, and then $\gamma_{sve}[M(W_n)] = \left\lceil \frac{2n+3}{4} \right\rceil$.

Proof:

The Wheel graph W_n consists of the following vertices: v1, v2, v3,...vn. To obtain $M(W_n)$, add e1, e2, e3,..en as the recently included vertices that correspond to the edges of W_n. The middle graph's vertex and edge sets of wheel graphs are thereby accordingly, obtained using [3n – 2] and [18 + (n – 4).10].

We prove this by the principle of Mathematical Induction.

If $4 \leq n \leq 7$, by direct computation we get $\gamma_{sve}[M(W_n)] = \left\lceil \frac{2n+3}{4} \right\rceil$.

Assume now $n \geq 8$. The graph W_n has several sub graphs isomorphic to P_n, and hence $M(W_n)$ has subgraphs isomorphic to $M(P_n)$. Fix one of those and consider J a secure vertex edge dominating set of $M(P_n)$.

Considering that the conclusion is valid for n = k, $\gamma_{sve}[M(W_k)] = \left\lceil \frac{2k+3}{4} \right\rceil$.

Now n = k + 1. Since $M(W_n)$ is isomorphic to $M(P_n)$, It is trivial to determine the end results for

$2 \leq n \leq 11$.

Claim that all paths of degree smaller than n, where n > 11, have the same outcome.

Let us assume that the graph's $M(P_n')$ vertices are (v11, v12, v13,..... vn) and that the vertices of $M(P_n)$ are (v1, v2, v3,......, vn).

Applying the mathematical induction notion,

$$\gamma_{sve}[M(P_n'')] = \left\lceil \frac{2(n-10)+3}{4} \right\rceil$$

$$\begin{aligned}
\gamma_{sve}[M(P_n)] &= \gamma_{sve}[M(P_n'')] + 5 \\
&= \left\lceil \frac{2(n-10)+3}{4} \right\rceil + 5 \\
&= \left\lceil \frac{2n-20+3+20}{4} \right\rceil \\
&= \left\lceil \frac{2n+3}{4} \right\rceil
\end{aligned}$$

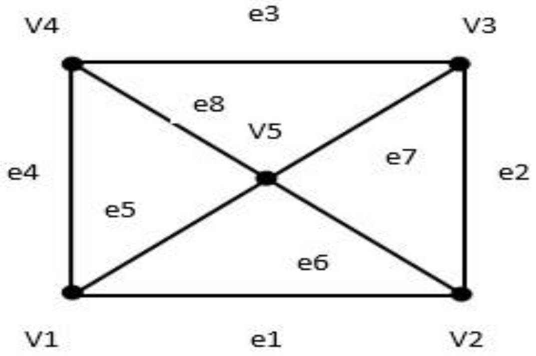
$$\therefore \gamma_{sve}\left[M(W_n)\right]=\left\lceil \frac{2n+3}{4} \right\rceil.$$

Hence the proof.

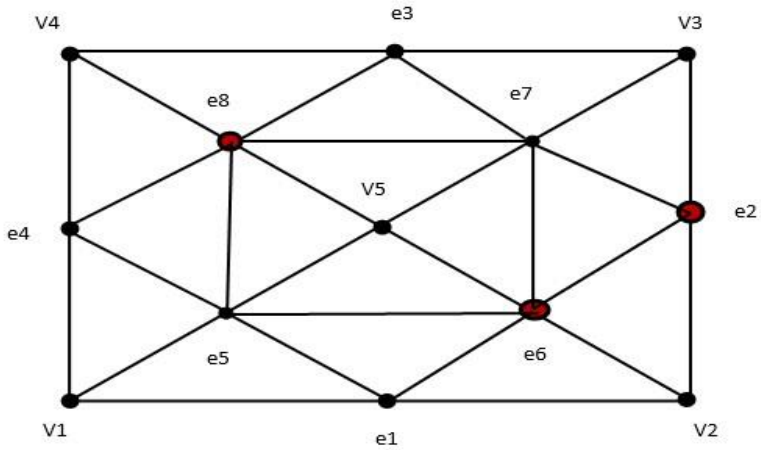
Example -4:

Consider a wheel graph with 5 vertices.

The five-vertex wheel graph and the accompanying wheel graph's middle graph are provided by



W_5



Middle Graph of Wheel Graph – M(W$_5$)

There are 13 vertices and 28 edges in the wheel graph (W5)'s middle graph. This graph's secure vertex edge domination number is provided by

$$\gamma_{sve}\left[M(W_5)\right]=\left\lceil \frac{(2\times 5)+3}{4} \right\rceil=\left\lceil \frac{13}{4} \right\rceil=\lceil 3.25\rceil=3.$$

The secure vertex dominating set J = {e_2, e_6, e_8}. Each one of the edges of the wheel graph's middle graph is protected by the vertices in set J.

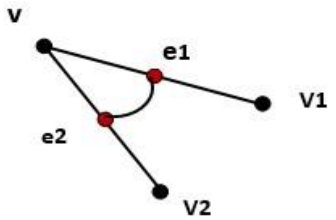
Theorem –5:

The middle graph's secure vertex edge dominance number of the complete bipartite graph K1,m-1 is determined by $\gamma_{sve}\left[M\left(K_{1,m-1}\right)\right]=\begin{cases}2, m=3\\ m-1, m\geq 4\end{cases}$.

Proof:

The complete bipartite network $K_{1, m-1}$ consists of the vertices v, v1, v2, v3, ... vm-1, respectively. To produce M ($K_{1, m-1}$), as newly created additional points, add e1, e2, e3, ... em-1 that line up with the edges of $K_{1, m-1}$. As a result, the vertex set of the middle graph of the complete bipartite graph is calculated by [2(m-1) +1], whereas the middle graph's edge set is produced by $\left[\frac{m(m+1)}{2}-1\right]$.

Case (i): Where m = 3,

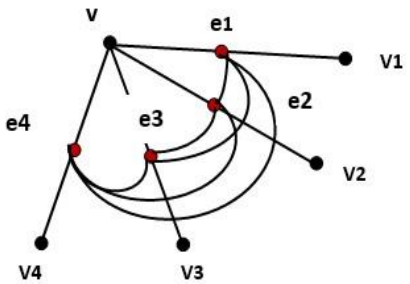


M [$K_{1,2}$]

When m = 3, the complete bipartite graph's middle graph. M($K_{1,2}$) is isomorphic to M(P_3), since by Theorem 1, $\gamma_{sve}[M(P_3)]=\left\lceil\frac{2n+3}{4}\right\rceil=\left\lceil\frac{2(3)+3}{4}\right\rceil=\lceil 2.25\rceil=2$.

$\therefore\ \gamma_{sve}[\mathrm{M}(\mathrm{K}_{1,2})]=2.$

Case (ii) For m ≥ 4,



M [$K_{1,4}$]

Let us consider the secure vertex edge dominating set $J=\{e_j, 1\leq j\leq m-1\}$ of M ($K_{1, m-1}$) with $|J|=m-1$. The deg(v_i) = 1, deg(v) = m-1 and deg(e_i) = m in M ($K_{1, m-1}$). For each vertex in J to be adjacent to a maximum of two vertices in [V - J]. For each, $e_j \in J$ the center vertex v in M($K_{1,m-1}$)'s V-J can be substituted, and the resultant new set $\left(J-\{e_j\}\right)\cup\{v\}\ or\ (J-\{e_j\})\cup\{v_j\}$ is likewise a secure vertex edge dominating set ($K_{1,\,m-1}$).

Therefore, J represents a secure vertex edge dominating set of M ($K_{1,\,m-1}$). For every set $T \subseteq V-J$ we can find a set S which is non empty in J so that the subgraph that is produced by

$T \cup S$ has been connected. Therefore, J represents the set of secure vertex edges that dominate M ($K_{1,\,m-1}$).

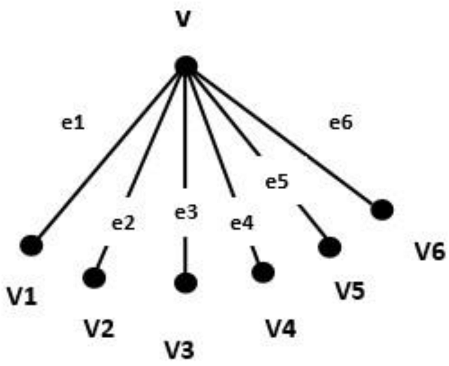
To verify that J is a M ($K_{1,\,m-1}$)'s minimum secure vertex edge dominant set with $|J| = m-1$. Under the assumption that {J} is a minimum secure vertex edge dominant set, suppose $|J| = m-2\ or < m-1$ so {J} by itself does not represent a set that dominates M ($K_{1,\,m-1}$). Accordingly, J is a minimum secure vertex edge dominant set that has $|J| = m-1$ of M ($K_{1,\,m-1}$).

$\therefore \gamma_{sve}[M(K_{1,m-1})] = m-1\ for\ m \geq 4$.

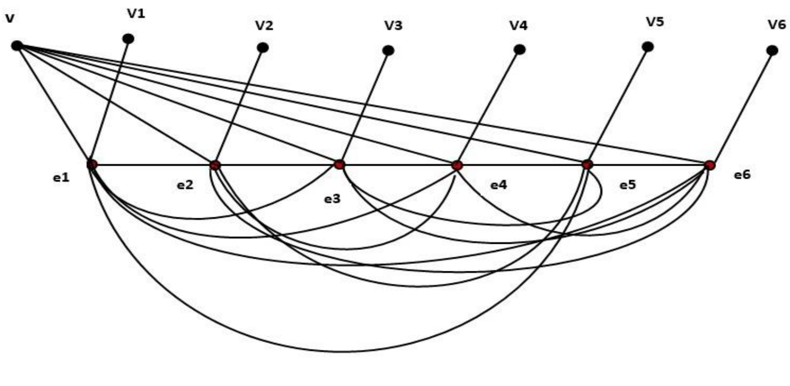
Hence the proof.

Example - 5:

Examine a complete bipartite graph's middle graph. $K_{1,6}$.



K 1,6



Middle Graph of K 1,6

There are 13 vertices and 27 edges within the middle graph of the complete bipartite graph $K_{1,6}$. This graph, secure vertex edge domination number is indicated by n -1 = 6. The secure vertex edge domination set, J = {e1, e2, e3, e4, e5, e6}, protects all 27 edges in this graph.

Theorem 6:

If F_n is the friendship graph, then the middle graph's secure vertex edge domination is as follows $\gamma_{sve}\left[M(F_n)\right]=\left\lceil \frac{4n}{3} \right\rceil,\ n\geq 2.$

Proof:

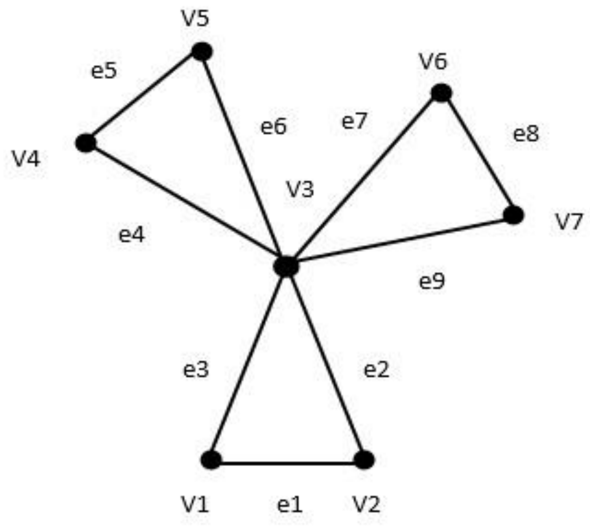
Given a friendship graph F_n, which consists of p > 1 copies of K_3 with n = 2p+1 vertex. The center vertex of friendship graph has degree 2n and the vertices has degree 2. Let v_1, v_2, v_3, …. v_n, where n = 2p +1be the vertices of the friendship graph and let e_1, e_2, e_3, …. e_n be the latest added vertices which correspond to the friendship graph F_n's edges in order to get $M(F_n)$. Let $V[M(F_n)]$ = 5p + 1, and $E[M(F_n)]$ = 9p + p, p triangles, this yields the vertex set and edge set of the friendship graph's middle graph.

The friendship graph's middle graph is produced by a replication of M[K3]. Let the secure vertex edge domination set J contains the center vertex and a vertex from each copy of $M[K_3]$, so that the vertices in J can defend the edges next to the incident edge and the edges incident to that vertex.As an outcome, the set of vertices in J will be able to protect every edge in the graph.If the center vertex is replaced by any other vertex in V-J, so M[Fn]'s secure vertex edge dominant set is not the new set.As a result, we determine that J is M[Fn]'s least secure vertex edge dominating set, with $\gamma_{sve}\left[M(F_n)\right]=\left\lceil \frac{4n}{3} \right\rceil,\ n\geq 2$ having at most one vertex from each copy of $M[K_3]$ and the center vertex.

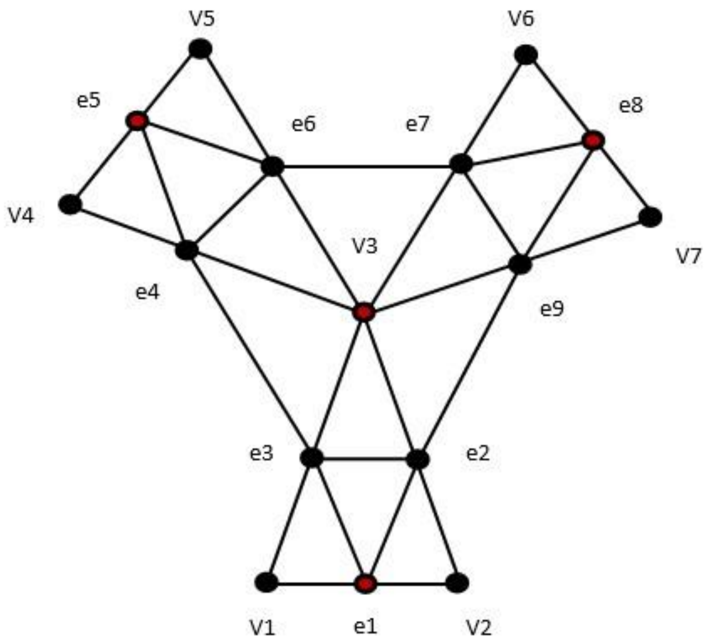
Thus, the proof that exists.

Example 6:

Observe the friendship graph F_3 together with its middle graph.



F_3



Middle Graph of Friendship Graph – M(F_3)

In the middle graph of friendship graph's F_3, it has 16 vertices and 30 edges. The secure vertex edge domination number for the graph M(F_3) is given by $\left\lceil \frac{4n}{3} \right\rceil = \left\lceil \frac{4(3)}{3} \right\rceil = 4$.

The set J = {v3, e1, e5, e8}is the minimum secure vertex edge dominating set of M(F_3) which defends all the 30 edges of the graph.

3. CONCLUSION

Domination is a fundamental idea in the study of graph theory, and it is used to analyze the sustainability of communication networks represented as graphs. Depending on the characteristics and configuration of the dominant sets, there are numerous kinds of dominations. The use of dominance has several applications. In this paper, we compute the secure vertex edge domination number of certain middle graphs, such as pathways, cycles, complete graphs, wheels, complete bipartite graphs, and friendship graphs, that belong to particular graph families.

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