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Multi Objective Hub Locations Problem for Medical Supply Chain Optimization (Solving Sample Problems in Large Sizes with MOPSO and NSGA II Algorithms)

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Abstract: - Today, the problem of facility location planning is mainly from the long-term and strategic operational level of large private organizations. Based on the hub location model for the green supply chain of medical and pharmaceutical equipment, it is possible to investigate the current condition of the facilities and significantly improve the demand coverage by spending acceptable costs. Therefore, after presenting the mathematical model, validation was performed in small dimensions and then sensitivity analysis was performed on the main parameters of the model. Next, Bender's analysis algorithm was used to analyze the NP-hardness of the model. Finally, by comparing the model solving time without implementing the Benders decomposition algorithm and by using it, it is clear that in high-dimensional example problems, the Benders decomposition algorithm reaches the solution in much less time than the normal case, and according to the answers, the performance can be acceptable. In addition, to show the efficiency of the model, two meta-heuristic algorithms NSGAII and MOPSO were developed. Then, based on the analysis, it can be seen that the computational time increases exponentially with the increase in the size of the sample problems, which is a reason for the NP-Hard of the problem. However, the MOPSO algorithm is better than the NSGA II algorithm in terms of computational time up to medium size problems.

Keywords: Multi-objective problem, hub location, green supply chain of medical and pharmaceutical equipment, robust uncertainty, MOPSO.

I. INTRODUCTION

Planning regarding the location of facilities is mainly from the long-term and strategic operational level of large private organizations, and the large costs related to the location and construction and operation of facilities have turned research into long-term decisions [1]. Therefore, the success or failure of construction centers in any sector of public and private activities depends entirely on their chosen location [2, 3]. The problem of location-allocation of the hub is one of the most used problems in the field of location of commercial facilities. The purpose of these types of issues is to reduce the level of operating costs of several new facilities among the existing facilities and the appropriate allocation of new facilities for the old facilities [4]. In commercial logistics, the design and planning for the logistics network includes the design of the system by which the goods reach from the supplier to the consumer [5, 6]. The main goal of the design and planning process of the supply chain network is to minimize the annual logistics costs according to limitations such as facility capacity, demand response time, country policies, etc. The imbalance in the workload, which includes the two categories of imbalance in the distance traveled by the vehicles and the amount of cargo carried by them, causes the mind of the centralized managers to distribute these items as quickly as possible and get the goods to customers on time, while minimizing the costs of the entire system. In order to speed up the distribution of important items, it comes to two logical issues, one is the creation of centers as an intermediary to collect, store and distribute goods near customers, and the other is the route that vehicles must take to get the goods. Distribution time is also one of the most important factors in the distribution of items in the supply chain. Supply chain management is a set of approaches used in the effective integration of suppliers, manufacturers, warehouses and stores, to be used in order to produce and supply goods with specific quantities, in appropriate and timely situations.Its purpose is to minimize system costs and it continues until all services and requirements are fully met. Supply chain management includes purchasing, inventory control, production, sales and distribution activities. So far, many studies and researches have been done in the field of facility location. In addition, in recent decades, it has been specialized on the issue of locating various facilities in the health field (both emergency and non-emergency facilities) [7]. However, a very important issue that has not been studied so far is the comprehensive examination of the optimality of the location of facilities, in such a way that during different years and at different times in each city, according to the existing needs and limitations, the decision makers proceed to locate a number of facilities. But what is important here is the comprehensive examination of the location of

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the facilities; this means that the decisions that have been made at any point in time should be examined in an integrated manner and in connection with each other and check whether the facility locations are comprehensively optimal or not. Therefore, the issue of integrated location of facilities is raised, in which it is tried to examine all the desired area in an integrated and comprehensive manner in terms of the optimality of the location of the facility, and if necessary, re-locating the location of the facility is done. This relocation can include establishing a new facility, removing an existing facility, increasing the capacity or reducing the capacity of a facility. If the re-location is not done on the desired area, we will face a number of local optimal solutions that each one was obtained at a specific time and not only may not give us the overall optimal answer, but also the changes made over time after the location in the demand in the covered areas are not considered in them.

II. REVIEW OF LITERATURE

One of the important concepts of the last few decades is the philosophy of supply chain management. The most important reason for paying attention to this issue is the increasing competitiveness and efforts to survive in organizations. In recent decades, this issue has led to the management of supply, production, and distribution processes to take a step towards the organization's competitive goal. Current supply chains operate in an environment that always seeks to improve and reduce costs and use solutions and strategies in this direction. Since supply chain members are mostly separate organizations and independent economic enterprises, despite the benefits of integrated decision-making, in practice, they do not have the desire to follow the decisions made for all members and try to optimize their goals instead of the goal of the whole system. So far, many researches have been presented in the field of planning, production scheduling or supply chain network design with different approaches and methods. For this purpose, some of the most recent and prominent researches conducted in recent years are categorized in Table 1. According to the evaluation and analysis of research literature carried out in the field of hub positioning in the supply chain, it can be seen that the issue of hub positioning plays a central role in reducing costs and increasing the productivity of the medical equipment chain, and the improvements made in this area lead to the provision of services in the supply chain. On the other hand, according to the analysis carried out in the text of this research, it was shown that the areas related to the covering radius of hub locations is a key issue that has been mentioned in most of the researches, and it is considered an urgent need in this area. There is a comprehensive analysis and evaluation in this field which is discussed in this research.

Table 1. Review of literature

Reference	Year	Model	Single purpose	Multi-purpose	Fuzzy uncertainty	Random uncertainty	Parameter	Approch	Exact solution 2
Jafari and Hadianpour [8]	2018	MOHLAP	✓				cost		✓
Husseinzadeh Kashan et al. [9]	2021	SAHLP	✓			✓	Capacity transport equipment		
Husseinzadeh Kashan et al. [10]	2021	Pareto-based grouping for humanitarian relief logistics		✓		✓	demand		
Husseinzadeh Kashan et al. [11]	2020	integrated decision making model in supply chain		✓		✓	demand		✓
Ghafari nasab	2022	HLPBD		✓		✓	demand		
Zhang	2022	MOHLAP		✓		✓	Demand, cost, time		✓ Benders

Oliviera et al.	2022	MAH Network Design	✓		✓		demand		
Ghafari nasab	2021	MILP		✓	✓		risk		✓
Erdogan et al.	2021	HLP	✓		✓		cost		✓
Mi Hu et al.	2021	HLP	✓		✓		demand		✓
Kayisoglu	2021	MAHLP	✓		✓		cost		✓
Mohtashami	2020	GSChain		✓			Environment and demand		✓
Moharejan	2020	Supply Chain			✓	✓	Response time and location of emergency equipment		
Okan et al.	2019	MPLAIP		✓		✓	Demand and greenhouse gas time	stable	✓
Chang and Ta	2019	sustainable seaport-dry port network design		✓		✓	Cost/ environmental effects	Possible stable	✓
Musazade et al.	2018	health service network		✓	✓		Cover/ service level	stable	✓
Navazi et al. [12]	2018	MPLAIP		✓	✓		Cost/ time	Possible stable	✓
Eidi et al.	2018	p-Hub Center Location	✓		✓		cost		
Asadnia et al.	2017	HLP		✓	✓		cost		✓
Ghaderi et al. [13]	2017	Supply chain network		✓		✓	Cost/ environmental effects		✓
Ferraro and Giordani [14]	2017	clustering		✓	✓		cost	Fuzzy stable	
Zhalechian et al. [15]	2016	MOHLP				✓	Cost/ time	Possible stable	
Mousazadeh et al. [16]	2016	green Log @ R Log M		✓	✓		Cost/ green chain	Scenario oriented	
Mousazadeh et al. [17]	2015	Supply chain network		✓		✓	Cost/ demand	stable	✓
Pishvaee et al.	2015	MOHLAP		✓	✓		Cost/ green chain	stable	✓

Charsoughi et al.	2014	HLP		✓	✓		cost		✓
Marros et al.	2014	Supply chain network	✓			✓	cost		
Pishvaei et al. [18]	2014	Supply chain network		✓	✓		Cost/ environmental effects/ social responsibility		
Atoei et al. [19]	2013	Supply chain network		✓		✓	Cost/ reliability		
Bashiri et al. [20]	2013	F-hub center problem	✓		✓		time	Fuzzy vector	
Yang et al. [21]	2013	P-hub center problem	✓			✓	time	stable	
Mohammadi et al. [22]	2013	Hub covering problem		✓		✓	Cost/ time	stable	
Aghhali et al.	2013	Hub covering problem				✓	Cost/ connections	Confidence level	
Pishvaee et al. [23]	2012	Supply chain network	✓		✓		Cost/ social responsibility	Possible stable	✓
Jabbarzadeh et al. [24]	2012	Supply chain network	✓			✓	cost		
Elomore et al.	2012	Supply chain network	✓			✓	cost	stable	✓
Taghipourian et al. [25]	2012	HLP	✓		✓		cost		✓
Zhai et al.	2012	Dynamic virtual hub location	✓			✓	Service level	stable	
Vasconcelos et al. [26]	2011	HLP	✓			✓	cost		✓
de Camargo et al. [27]	2011	HLP	✓				cost		

Contreras et al. [28]	2011	HLP	✓			✓	cost		
Mohammadi et al. [29]	2011	Hub covering location		✓			Cost/ time		
Davari et al. [30]	2011	maximal covering problem	✓			✓	time		
Sim et al.	2009	P-hub center problem	✓				time		✓
Yang [31]	2009	HLP	✓			✓	cost		✓
Camargo et al. [32]	2008	HLP	✓				cost		
da Graça Costa et al. [33]	2008	HLF		✓			Cost/ time		✓
Snyder et al. [34]	2007	Location model	✓				cost		✓
Marianov and Serra [35]	2003	HLF	✓				cost		

III. RESEARCH PROBLEM

In this proposed model, it is assumed that there are a number of facilities at different levels in the investigated area. Due to the fact that the existing facilities have been established over a long period of several years and also due to the fact that during these years there have been changes in the amount and concentration of the level of demand of subscribers for services, so it is necessary that all hubs should be examined in an integrated way. In this section, at first, the hubs in the area for the transfer of medical equipment are classified into three levels, and then the amount of demand covered by these facilities is checked, and if there is a change in the capacity or location of the facility, in which the cost is also justified, the changes are made in the location and capacity of existing facilities. Finally, a new structure of locating hubs is presented, in which the location and capacity of some hubs have been changed, but on the other hand, the coverage of the demand for medical goods has increased. Improving demand coverage is not done only by checking the capacity level of existing hubs and modifying the location of these hubs. In this article, three capacity levels are also considered for hubs, and if needed, the hub will be upgraded to a higher capacity level or a new hub will be located.

In order to improve the coverage quality of medical equipment customers' demand, coverage radius is also included for each hub. The coverage radius helps to allocate the demand to the hubs in such a way that the distance between the demand point and the hub is optimal and acceptable. In this article, the coverage radius is considered as a variable. Considering that the amount of demand coverage has a direct relationship with the distance between the demand point and the hub as well as the variable cost of coverage; It is tried to minimize the coverage radius of each hub. In improving the existing situation, not only changes in the existing hubs are addressed, but if there is a need to locate a new hub, the new hub is being established. In this issue, we are looking for the optimality of existing hubs in terms of coverage capacity and positioning of new hubs with the aim of reaching a higher demand coverage level, reducing costs and maximizing the level of greenness of hub positioning. For this purpose, in our review, considering economic efficiency, we will consider these measures: establishing a new hub, removing the previous or current hub, increasing and reducing the capacity of hubs.

Assumptions of mathematical modeling

In this research, three hub levels are considered: W-level hub, which is the lowest capacity level and is responsible for providing basic services of medical equipment. C-level hub, that provides additional services for medical equipment services. And finally, H-level hub, which provides the most advanced medical services, and also provides c-level hub services. The study area consists of a number of districts. Districts are also a subset of a larger area called region, which are covered by hubs. Each demand point must be covered by a w-level hub and a c-level hub as well as an h-level hub, but h-level hubs can also provide c-level hub services; Therefore, if a demand point is covered by the h-level hub, there is no need for that demand point to be covered by the c-level hub.

In each region, there must be at least one W-level hub. .

In one region, it is not possible to have both a c-level hub and an h-level hub at the same time.

No more than one H-level hub can be placed in one area.

Demand is considered as points.

Figure 1 shows the different coverage situations of finding a demand point based on the assumptions. In the first case, the demand point is assigned to all three existing hub levels. In this case, the demand of each level is assigned to the hub of the same level. In the second case, the demand point is assigned to two hubs at levels w and h. In this case, as described in the assumptions, level c demand point is covered by h-level hub. Because based on the assumptions of h-level hub, it can also provide c-level hub services.

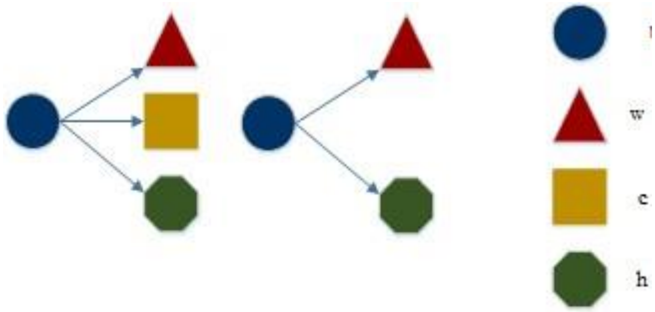

Figure 1. Different cases of coverage of a demand point

IV. PROPOSED MODEL

The first model

In this proposed model, it is assumed that there are a number of hubs at different levels in the area under investigation and the amount of demand coverage by these hubs should be checked and if there is a change in the capacity or location of the hubs, in which the cost is also justified, the changes are made in the location and capacity of existing hubs. Finally, a new structure of hubs is presented, in which the location and capacity of some hubs have changed, but on the other hand, the amount of demand coverage has increased. In this section, the mathematical model of the research is presented. In this issue, we are looking for the optimization of existing hubs with the aim of reaching a higher level of demand coverage. For this purpose, in our review, taking into account economic efficiency, we will address these measures: establishment of hub, removal of hub, increase and decrease of facilitation capacity.

Collections, indices, parameters and decision variables are introduced in this section.

Collections & Indices

N Set of grid points

B The set of areas in the model

R The set of regions in the model

S A set of uncertainty scenarios

F_r The set of nodes in the region r

Objective functions & limitations

1 $$\text{Max} \sum_i h_i z_i^0$$

2 $$\text{Min} \sum_{l,j,t} F^l_{j,t} x^l_{j,t} + \sum_{l,j} r^l_j(\varphi)$$

3 $$\text{Max} \sum_{l,j,t} G^l_{j,t} x^l_{j,t} \quad \text{s.t.}$$

4 $$\sum_{j \in N_b} a^{l,s}_{i,j} \geq z^s_i \quad \forall i \in N_b, b \in B, l \in \{w,h\}, s$$

5 $$\sum_{j \in F_r} (a^{c,s}_{i,j} + a^{h,s}_{i,j}) \geq z^s_i \quad \forall i \in F_r, r \in R, s$$

i Index showing demand points $i \in N$

j Index showing candidate points for establishing hubs $j \in N$

l Hub surface index $l \in \{w, h, c\}$

t Index related to hub capacity level

b Index related to the region

$N_b$nodes in the region $N_b \in N, b$

r Area index

s Index for each scenario

Decision variables

h_i Point i demand

$F^l_{j,t}$ The fixed cost of establishing a hub at the level l at the point j with the level of capacity t

$G^l_{j,t}$ The green level of the hub is at the level l at the point j with the capacity level t

$level_t$The amount of capacity related to the surface t

$Q^l_{j,t}$ 1 If the hub already exists on the level l at the point j and capacity level t; 0 otherwise

P^l The number of hubs in the level l that should be at the end

$d_{i,j}$The distance between two points i and j

R_{Max} Maximum acceptable coverage radius

p_sAcceptable level of deviation from the optimal amount of each scenario in the main model

Decision variables

ρ^*_sThe optimal amount of covered demand in each scenario

r^*_sThe optimal cost in each scenario

$z^s_i \in \{0,1\}$ 1 if the point i demand is covered; 0 otherwise

$x^l_{j,t} \in \{0,1\}$ 1 If the level i hub at the point j in the capacity level t does not exist before; 0 otherwise

$$\sum_{j\in N_b,t} y^w_{j,t} \ge 1 \quad \forall b \in B \tag{6}$$

$$\sum_{j\in N_b,t} y^c_{j,t} + \sum_{j\in N_b,t} y^h_{j,t} \le 1 \quad \forall b \in B \tag{7}$$

$$\sum_{j\in F_r,t} y^h_{j,t} \le 1 \quad \forall r \in R \tag{8}$$

$$\sum_{l,t} y^l_{j,t} \le 1 \quad \forall j \tag{9}$$

$$\sum_i a^{l,s}_{i,j} h_i \le \left(\sum_t level_t y^l_{j,t}\right) \quad \forall j, l, s \tag{10}$$

$$r^l_j \ge \sum_s d_{i,j} a^l_{i,j} \quad \forall i, l, j \tag{11}$$

$$r^l_j \le R_{Max} \sum_t y^l_{j,t} \quad \forall l, j \tag{12}$$

$$\sum_{j,t} y^l_{j,t} \le P^l \quad \forall l \in L \tag{13}$$

$$x^l_{j,t} = |y^l_{j,t} - Q^l_{j,t}| \quad \forall l, j, t \tag{14}$$

$$\sum_i h_i z^s_i \ge (1 - p_s)\rho^*_s \quad \forall s \in S/0 \tag{15}$$

$$\sum_{l,j,t} F^l_{j,t} x^l_{j,t} + \sum_{l,j} r^l_j(\varphi) \le (1 + p_s) r^*_s \quad \forall s \in S/0 \tag{16}$$

r_j The hub coverage radius available at the point j

$a^{l,s}_{i,j} \in \{0,1\}$ 1 if the demand point i is assigned to the hub j at the level l in the scenario s; 0 otherwise

$y^{l}_{j,t} \in \{0,1\}$ 1 If there is a level hub l at the capacity point j and level t; 0 otherwise

The objective function (1) shows the amount of covered demand. The objective function (2) shows the cost caused by changes in the structure of the hubs. In this way, in case of locating a new hub or changing the capacity of a hub or moving a hub, the binary variable is set to one and the cost of this change is calculated. In the second part of the equation, the cost of covering the radius of each hub is calculated as a function of the radius. The objective function (3) performs the maximization of the level of greenness of locating the hubs. Each demand point is covered under the condition that it is assigned to at least one hub in the level and to one hub in the level in its own area. This condition is considered in relation (4). According to the assumptions of the model, the hub at the level can eliminate the demand point from being assigned to the hub at the level. Therefore, in relation (5), the assumption is applied that by assigning a demand point to a surface hub in another area, it is not necessary to assign that point to a surface hub. Relation (6) states the requirement to establish at least one level hub in each region. The assumption of prohibiting the establishment of two hubs at the same level and in the same region is considered in relation (7). Relation (8) restricts the establishment of more than one hub in a level in a region. Relation (9) limits the establishment of more than one hub at one point. Relationship (10) is the limitation of the availability and capacity of the hubs, which prevents allocating more than the optimal capacity level for the hub, and if the hub is not available in the desired scenario; it prevents the allocation of demand to that hub in that scenario. Relations (11) and (12) determine the coverage radius of each of the hubs. Relationship (13) shows the allowed number of network changes at each level. Relation (14) detects that the hub has changed or remained unchanged during positioning. Relations (15) and (16) also determine that the value of the objective function does not deviate from the optimal value in each scenario by more than the allowed percentage.

The second proposed model

In this proposed model, it is assumed that there are a number of hubs at different levels in the investigated area and the amount of demand coverage by these hubs should be checked and if there is a change in the capacity or location of the facility, in which the cost is also justified, the changes are made in the location and capacity of existing hubs. Finally, a new structure of hubs is presented, in which the location and capacity of some hubs have changed, but on the other hand, the amount of demand coverage has increased.

Collections, indices, parameters and decision variables are introduced in this section.

Collections & Indices

N Set of grid points

B The set of areas in the model

R The set of regions in the model

S A set of available scenarios

F_r The set of nodes in the region r

i Index showing demand points $i \in N$

j Index showing candidate points

Limitations

17 $$Max \sum_{i,s} p^s(h_i z^s_i)$$

18 $$Min \sum_{l,j,t} FC^l_{j,t} x^l_{j,t} + \sum_j r_j(\varphi)$$

19 $$Max \sum_{l,j,t} G^l_{j,t} x^l_{j,t}$$

20 $$\sum_{j \in N_b,t} a^{l,s}_{i,j} \geq z^s_i$$
$$\forall i \in N_b, b \in B, l \in \{w,h\}, s \in S$$

21 $$\sum_{j \in F_r,t} (a^{c,s}_{i,j} + a^{h,s}_{i,j}) \geq z^s_i$$
$$\forall i \in F_r, r \in R, s \in S$$

22 $$\sum_{j \in N_b,t} y^{w}_{j,t} \geq 1$$
$$\forall b \in B$$

for establishment of facilitation $j \in N$

l The index related to the level of facilitation $l \in \{w, h, c\}$

t Index related to hub capacity level

b Index related to the region

$N_b$nodes in the region $N_b \in N$, b

r Area index

s Index for each scenario

Parameters

h_i Point i demand

$FC^l_{j,t}$ The fixed cost of establishing a hub at the level l at the point j with the level of capacity t

$G^l_{j,t}$ The green level of the hub is at the level l at the point j with the capacity level t

$level_t$ The amount of capacity related to the surface t

$Q^l_{j,t}$ 1 If the hub already exists on the level l at the point j and capacity level t; 0 otherwise

P^l The number of hubs in the level l that should be at the end

$d_{i,j}$ The distance between two points i and j

R_{Max} Maximum acceptable coverage radius

p^s The probability of the scenario s occurring

u^s_j 1 if the existing hub at the point j in the scenario s is available; 0 if the existing hub at the point j in the scenario s is disturbed and out of reach

Decision variables

z^s_i if the point i demand is covered; 0 otherwise

$x^l_{j,t}$ 1 If the level i hub at the point j in the capacity level t does not exist before; 0 otherwise

r_j The hub coverage radius available at the point j

$a^l_{i,j}$ 1 if the demand point i is assigned to the hub j at the level l; 0 otherwise

23 $$\sum_{j\in N_b,t} y^c_{j,t} + \sum_{j\in N_b,t} y^h_{j,t} \le 1 \quad \forall b \in B$$

24 $$\sum_{j\in F_r,t} y^h_{j,t} \le 1 \quad \forall r \in R$$

25 $$\sum_{l,t} y^l_{j,t} \le 1 \quad \forall j$$

26 $$\sum_{i} a^{l,s}_{i,j} h_i \le \left(\sum_{t} level_t y^l_{j,t}\right) u^s_j \quad \forall j, l, s \in S$$

27 $$r_j \ge d_{i,j} a^{l,s}_{i,j} \quad \forall i, l, j, s \in S$$

28 $$r_j \le R_{Max} \sum_{l,t} y^l_{j,t} \quad \forall j$$

29 $$\sum_{j,t} y^l_{j,t} \le P^l \quad \forall l \in L$$

30 $$x^l_{j,t} = |y^l_{j,t} - Q^l_{j,t}| \quad \forall l, j, t$$

31 $$z^s_i, x^l_{j,t}, a^{l,s}_{i,j}, y^l_{j,t} \in \{0,1\} \quad \forall i, j, l, t, s$$

32 $$r_j \ge 0 \quad \forall j$$

$y_{j,t}^{l}$1 If there is a level hub l at the capacity point j and level t; 0 otherwise

The objective function (17) shows the amount of covered demand. The objective function (18) shows the cost caused by changes in the structure of the hubs. In this way, in case of establishing a new hub or changing the capacity of a hub or moving a hub, the binary variable $x_{j,t}^{l}$ takes the value of one and the cost of this change is calculated. In the second part of the equation, the cost of covering the radius of each hub is calculated as a function of the radius. Each demand point is covered under the condition that it is assigned to at least one hub in the level w and to one hub in the level h in its own area; this condition is considered in equations (19). The third objective function (20) is to maximize the level of greenness of the hubs. According to the assumptions of the model, the hub at the level c can eliminate the demand point from being assigned to the hub at the level h, therefore in equations (21), the assumption is applied that by assigning a demand point to a surface hub h in another area, the assignment of that point c is not required to facilitate the surface. Constraint category (22) states the requirement to establish at least one surface hub w in each region. The assumption of prohibiting the establishment of two hubs at the same time has been considered in the levels h and c in equations (23). Equations (24) limit the establishment of more than one hub on the surface h in one area. Equations (25) limit the establishment of more than one hub at one point. Equation (26) is the limitation of the hub capacity and prevents the allocation of more than the optimal capacity level for the hub. Constraints (27) and (28) determine the coverage radius of each hub. Constraints (29) show the allowed number of final network hubs at each level. Constraint (30) recognizes that the hub has changed or remained unchanged during the location-relocation process.

Stabilization of the mathematical model

Considering that the presented model is a non-linear model. In this part, uncertainty in demand will be added to the model with the help of robust planning and Bertsimas and Sim's approach. Therefore, the first objective function and constraints No. 10 and 26 will be modified as Bertsimas model. Investigations in this research show that the customer demand parameter is one of the important parameters whose values may exceed the nominal values. Therefore, considering this parameter in uncertain conditions can bring the proposed model closer to the reality of the problem. To consider the uncertainty in the demand, as mentioned, the robust planning and Bertsimas and Sim's approach will be used. The robust optimization method looks for optimal or near-optimal solutions that are justified with a high probability. Bertsimas and Sim's approach is one of the four main approaches to consider uncertainty in robust planning. In this section, we will briefly mention this approach. For this purpose, we consider the following linear programming model:

$$
\begin{aligned}
& Min \sum_{j} c_j x_j \\
& s.t \\
& \mathrm{Ax} \le \mathrm{b}
\end{aligned}
$$

In this model, we assume that only the coefficients on the right side of the constraints, i.e., matrix A, have uncertain values, and the terms of this matrix a_{ij} fluctuate in the range $[\tilde{a}_{ij} - \hat{a}_{ij}, \tilde{a}_{ij} + \hat{a}_{ij}]$, which $\tilde{a}_{ij}$ and $\hat{a}_{ij}$ are the nominal values and the maximum deviation of the parameter a_{ij}, respectively. The robust model proposed by Bertsimas and Sim is as follows.

$$
\begin{aligned}
& Min \sum_{j} c_j x_j \\
& s.t.\ \sum_{j} a_{ij} x_j + z_i \Gamma_i + \sum_{j \in j_i} \mu_{ij} \le b_i \qquad \forall \mathrm{i} \\
& \qquad z_i + \mu_{ij} \ge \hat{a}_{ij} x_{ij} \qquad \forall \mathrm{i,j} \\
& \qquad z_i, \mu_{ij} \ge 0 \qquad \forall \mathrm{i,j}
\end{aligned}
$$

In these relations z_i and μ_{ij} auxiliary variables are twofold and the parameter Γ_i called the uncertainty budget shows the level of conservatism that is chosen according to the importance of the constraint and the risk-taking of the decision maker.

Solution Method

Solving the First Proposed Model

In general multi-objective problems, there is no single optimal solution that simultaneously optimizes all objective functions. Therefore, the concept of optimality is replaced by Pareto optimality or efficiency. Pareto-optimal (efficient, non-defeated) solutions are solutions that cannot be improved in one objective function without worsening performance in at least one of the other objective functions. A Pareto set is a set of Pareto optimal solutions. According to Huang and Massoud (2016), three main methods for solving multi-objective optimization problems are known based on the stage of application of the decision maker's preferences: prior methods, posterior or productive methods, and interactive methods. In the previous methods, the decision maker presents his preferences before the trends. This method has the problem of quantifying the preferences that are determined by the decision maker. The latter method is based on the optimization of all objective functions simultaneously. In this method, the Pareto set is created first. Then, at the end of the search process, the decision maker selects the Pareto set that is preferred. In the interactive method, the decision maker is involved in the calculation process instead of expressing his preferences in the dialog process. In this method, after several iterations, the process usually approaches a more favorable solution. The decision maker sequentially directs the search to the preferred solution with its answers. In order to solve this two-objective model, the exact epsilon constraint solving method is used. In this method, one of the objective functions is kept as the objective function and the rest of the objective functions are converted into unequal constraints according to the minimization or maximization of the objective function as follows.

$$\max ax \quad \rightarrow \quad ax \geq \varepsilon \tag{33}$$

$$\min ax \quad \rightarrow \quad ax \leq \varepsilon \tag{34}$$

In this model, the first limitation, which is the maximization of the covered demand, is kept as the objective function, but the second objective function, which is the minimization of the cost and the amount of changes, is considered as a limitation and is changed as follows:

$$\sum_{l,j,t} F^l_{j,t} x^l_{j,t} + \sum_j r_j(\varphi) \leq \varepsilon \tag{35}$$

In this method, first, an initial value for the epsilon parameter (ε) is considered, and the epsilon value (ε) is improved during repetition in solving the model in order to achieve the optimal solution. In the epsilon method, the limitation of the model must be linear, but the model presented in this article is a non-linear mixed model, so we will explain the linearization process of the model in the following.

Linearization

In the model presented in this article, the relations (3-14) and (3-30) are non-linear, and if these equations are linearized, we get a mixed integer linear model (MIP). In the case of this non-linear limitation, the presence of an absolute value sign has caused the model to become non-linear, and due to the presence of a variable equivalent to the absolute value in the objective function, linearization is performed as follows:

First, we introduce two binary variables $x'^l_{j,t}$ and $x''^l_{j,t}$. In the following, we rewrite relations (3-14) and (3-30) as follows:

$$y^l_{j,t} - Q^l_{j,t} = x'^l_{j,t} - x''^l_{j,t} \tag{36}$$

$$x^l_{j,t} = x'^l_{j,t} + x''^l_{j,t} \tag{37}$$

In this way, the model presented in this article becomes a mixed integer linear model.

Freeing binary variables

The existence of binary variables can increase the time to solve the model. In the second model presented in this thesis, there are 4 categories of binary variables; among these 4 categories of binary variables, two categories of variables can be released and the time of solving the model can be reduced. The category of variables $x'^l_{j,t}$, $x''^l_{j,t}$ and z^i_s can be freed and converted into positive variables. By changing the mentioned variables, no change is made in the final solution of the model and the model is solved in a shorter time than before. The final model after linearization and freeing is as follows:

$$Max \sum_{i,s} p^s(h_i z^s_i) \tag{38}$$

$$Min \sum_{l,j,t} F^l_{j,t}(x'^l_{j,t} + x''^l_{j,t}) + \sum_j r_j(\varphi) \tag{39}$$

$y^l_{j,t} - Q^l_{j,t} = x'^l_{j,t} - x''^l_{j,t}$ $\forall l, j, t$ (40)

$a^{l,s}_{i,j}, y^l_{j,t} \in \{0,1\}$ $\forall i, j, l, t, s$ (41)

$z^s_i, x'^l_{j,t}, x''^l_{j,t}, r_j \geq 0$ $\forall i, j, l, t, s$ (42)

Solution of the second proposed model

The second proposed model is a mixed integer two-objective model based on the scenario. To solve this proposed model, we first convert the three-objective model into an equivalent single-objective model by using the Torabi-Hassini method; then, we solve the single-objective model by using Benders decomposition algorithm.

Torabi-Hassini method

This method was presented by Torabi and Hassini (2008) and is called the TH method. The efficiency of the answers produced by this method can be proven. If we consider the following general model:

$Max[f_1(x), f_2(x)]$ (43)

s.t. (44)

$x \in F_x$ (45)

The general shape of the model in the TH method is as follows:

$Max\ \gamma\beta_0 + (1-\gamma)\sum_k w_k\mu_k(x)$ (46)

$s.t.$

$\mu_k(x) \geq \beta_0;\ \forall k$ (47)

$x \in F_x$ (48)

$\beta_0 \in [0,1]$ (49)

In this model, γ is considered as a compromise coefficient between goals. β_0 represents the minimum satisfaction level of the objective functions ($\beta_0 = \min_k\{\mu_k(x)\}$). The values $w_k$represent the weight of the objective functions. The values $\mu_k(x)$ that represent the level of satisfaction of each of the objective functions are also calculated in the following order:

$$\mu_1 = \begin{cases} 1 & If\ z_1 \leq z^{PIS}_{1\beta} \\ \frac{z^{NIS}_{1\beta} - z_1}{z^{NIS}_{1\beta} - z^{PIS}_{1\beta}} & If\ z^{PIS}_{1\beta} \leq z_1 \leq z^{NIS}_{1\beta} \\ 0 & If\ z^{NIS}_{1\beta} \leq z_1 \end{cases} \quad (50)$$

$$\mu_2 = \begin{cases} 1 & If\ z_2 \leq z^{PIS}_{2\beta} \\ \frac{z^{NIS}_{2\beta} - z_2}{z^{NIS}_{2\beta} - z^{PIS}_{2\beta}} & If\ z^{PIS}_{2\beta} \leq z_2 \leq z^{NIS}_{2\beta} \\ 0 & If\ z^{NIS}_{2\beta} \leq z_2 \end{cases} \quad (51)$$

To determine the values $z^{PIS}_{i\beta}$ and $z^{NIS}_{i\beta}$ which, respectively, the optimistic and pessimistic $i$values of the faithful function; we do the following: To obtain the optimistic value of a target function, we solve the model for the same target function and consider the value obtained as the optimistic value. When we solve the model on the basis of a target function; the value obtained for the other objective function can be considered as the pessimistic value of that target function. Note the following equations:

$Z^{NIS}_{1\beta} = Z_1(x^{PIS}_{2\beta})$ (52)

$Z^{NIS}_{2\beta} = Z_2(x^{PIS}_{1\beta})$ (53)

Benders Decomposition Method

Benders Analysis Algorithm is a mixed integer-programming problem that has been used in various fields. This algorithm is a large-scale resource distribution method proposed by Benders in 1962 and has been used in networking, transportation, location, and so on. The method can be used independently and in combination with other methods such as Lagrange release and branch and boundary algorithm in problem solving. In short, it seeks to redefine the problem under consideration by applying concepts such as dualism and the use of primary-bilateral relationships.

$Min\, z = c^T x + b^T y$ (54)

$s.t. Ax \geq d,$ (55)

$Bx + Dy \geq h,$ (56)

$x \in \mathrm{X}, y \geq 0.$ (57)

Model 1

Benders decomposition method divides the variables of Model 1-3 into two groups (x, y), so that in each repetition of the algorithm, problems include x or y (and not both). The components y are assumed to be continuous, but x can be continuous or discrete. Also, there are two groups of constraints, the first group being solely related to variables x and the second group being related to variables x and y. Of course, the first group's constraints may not be present in the problem, in which case we can consider the appropriate boundary on the variable x and use it as the first group constraints.

Model 3-1 may be computationally complex with both groups of variables x and y . While considering the problem with variables x or y easier. For example, if x is the vector of integer variables, this classification leads to a linear programming model in terms of y and an integer programming model in terms of x, or there may be a problem with specific structures such as the transport problem structure that facilitates it.

In Model 3-1, we consider x as a complex variable that makes it difficult to solve the problem. For example, x can be correct or correct, or a variable that disrupts the particular structure of the problem, and given the constant value for it, the problem of a particular structure, such as the structure of the transport problem, is divided into several smaller problems. x also includes limitations such as the accuracy of some entities. We rewrite Model 3-1 as Model 3-2.

$Min\, z = c^T x + \Phi(x)$ (58)

$s.t. Ax \geq d,$ (59)

$x \in \mathrm{X}$ (60)

Model 2, that $\Phi(x)$ is defined by model 3-3 and is called the Benders Subsidiary.

$\Phi(x) = Min\, b^T y$ (61)

$Dy \geq h - Bx,$ (62)

$y \geq 0.$ (63)

Model 3

For a fixed value for x model 3-3 is a linear programming problem, in fact the Model 3-3 can be considered a parametric problem in which the parameter x is on the right side of the constraints. According to the dual theorem, the 3-3 model can be written as a 3-4 model, in which the vector πcorresponds to the dual variables of the 3-4 model.

$Max\, \pi^T(h - Bx)$ (64)

$\pi^T D \geq b,$ (65)

$\pi \geq 0.$ (66)

Model 4

Without losing the overall, suppose the problem with the Model 3-3 is per x, in fact, taking into account the deficiency and surplus variables, the Model 3-3 can be written in a way that can be violated, and this error in the target function is fined with a positive coefficient of large enough. Also suppose the 3-3 model has a finite answer per X, because if the model is infinite, the 3-1 model will also be endless. Given these two assumptions on the 3-3 model, according to the dual theorem, the Model 3-4 will have a finite answer per x.

We show the range of the Model 3-4 with 𝔽, which is independent of choice x and the maximum occurs at one of the top points of 𝔽. Without losing the totality, 𝔽 can be assumed to be bound, because if 𝔽 is infinite, assuming that (π) is a columnar matrix m of the component $\pi^T = [\pi_1 \ \pi_2 \ \ldots \ \pi_m]$, adding the following limitation, we cut the barrier to the barrier, in which M is considered a large positive number.

$\sum_{i=1}^{m} \pi_i \leq M$ (67)

So, suppose $\{Q_1, Q_2, \dots, Q_k\}$ are the top points of the area $\mathbb{F}$, in which case the 3-4 and the 3-4 are equivalent.

$$Max\ \pi^T(h - Bx) \quad (68)$$

$$\pi \in \{Q_1, Q_2, \dots, Q_k\} \quad (69)$$

Model 5

Thus the Model 3-6 will be equivalent to the original problem (Model 3-1).

$$Min\ z = c^T x + (max\ \pi^T(h - Bx)) \quad (70)$$

$$s.t. Ax \geq d, \quad (71)$$

$$\pi \in \{Q_1, Q_2, \dots, Q_k\}, \quad (72)$$

$$x \in \mathrm{X} \quad (73)$$

Model 3-6

We rewrite Model 3-6 as Model 3-7 and describe the Benders algorithm accordingly.

$$Min\ z = c^T x + \varphi \quad (74)$$

$$\pi^T(h - Bx) \leq \varphi, \pi \in \{Q_1, Q_2, \dots, Q_k\}, \quad (75)$$

$$Ax \geq d, \quad (76)$$

$$x \in X, \varphi \in R \quad (77)$$

Model-7

In the above problem, Φ is a free variable in terms of sign. By solving this problem, the optimal solution to the initial problem, X^* can be obtained. The question may be raised whether it is necessary to identify all the vertices $\{Q_1, Q_2, \dots, Q_k\}$ of the region F? No, to determine the optimal solution of the problem, an iteration method is used, and in each iteration, one of the vertices is identified and the corresponding constraint is added to the problem. The general idea is to determine an appropriate upper bound ($z_{UB} = c^T x + b^T y$) and lower bound ($z_{LB} = c^T x + \varphi$) on the optimal value of the objective function z* from model 1. So that the $z_{LB} \leq z^* \leq z_{UB}$ upper and lower bounds are changed successively, so that each new bound is obtained by identifying a new vertex point of F and finally, at the vertex point for which $z_{LB} = z_{UB}$ the optimal solution will be reached. Therefore, this process will end after identifying a maximum of K vertices from F region. According to the mentioned materials, we will describe the Benders analysis algorithm in chapter 4.

Non dominated sorting genetic algorithm (NSGA-II)

Genetic algorithm with non-defeated sorting is one of the most popular and widely used optimization algorithms in the field of multi-objective optimization. This algorithm was introduced by Deb in 2002. Along with all the functions that NSGA-II has, it can be considered as the model for the formation of many multi-objective optimization algorithms. This algorithm and its unique way of dealing with multi-objective optimization problems have been used many times by different people to create newer multi-objective optimization algorithms. Undoubtedly, this algorithm is one of the most basic members of the evolutionary multi-objective optimization algorithm collection, which can be called the second generation of such methods. In order to implement the NSGA-II algorithm, first the initial parent population P is created. The population is sorted based on the sorting algorithm and its Pareto front rank is assigned to each individual. Now the multiple optimization problems are transformed into a simple problem of minimization of Pareto front utility function. Binary tournament selection, fertilization and mutation operators are used to create a population of Q children with N children. From this generation onwards, the way of working will be different due to the application of the process of elitism. In the process of elitism, a mixed population of parents and children is first formed. Then the combined population is sorted based on the swarm comparison operator and its N best individuals are considered as the next generation population Pt+1. Then, by using N population Pt+1 and using selection, fertilization and mutation operators, N population Qt+1 is made. In this algorithm, the diversity of the population in each generation will be guaranteed by applying the swarm comparison operator when choosing the binary tournament, where there is no need for sharing parameters. Therefore, it will not have the weakness of other methods such as NSGA. Also, the crowding distance is calculated in the space of utility functions, which of course can also be calculated with the space of parameters. Another point is that in the construction of the population of each generation, the selection method of a+b is used instead of (a,b), which will increase the stability of the method and ensure that the good people of the previous generation are not removed in the new generation.

Particle swarm optimization algorithm

Particle swarm optimization algorithm is a successful technique in artificial intelligence. Consider a group of insects or a group of fish. If one member of the group finds a good path to advance (for example, to reach food, safe place, etc.) the other members of the group will be able to follow that path after this member. This phenomenon is modeled using particles that have their own location and speed. In this algorithm, group members exchange information with each other about the best location found up to the current stage. Also, the best location found up to the current stage among a neighborhood of members is also exchanged among the members of this neighborhood. In other words, the best answer found for all members of the group is known, also the best answer found in a neighborhood of members is known only among the members of this neighborhood. This information is used to update the position and speed of the members in the stages. Particle swarm optimization algorithm is simpler than genetic algorithms and ant colony optimization. Also, the population size of the particle swarm optimization algorithm is smaller than the genetic algorithm. Therefore, initializing the population using this algorithm is simpler than other intelligent optimization algorithms. The particle swarm optimization algorithm is easy to implement and has been used in solving many discrete and continuous nonlinear optimization problems. The interesting point is that this algorithm uses only basic mathematical operators and provides good results in constant, noisy and continuously changing environments. Due to its advantages such as simple concept, easy implementation and fast convergence, today the particle swarm optimization algorithm has attracted a lot of attention and has wide applications in various fields.

The mechanism of particle swarm optimization algorithm

In the particle swarm optimization algorithm, each solution is only one particle in the search space and is called a member. All particles have a merit value which is evaluated by the merit function to be optimized. In addition, each particle i has a position in the dimensional space D of the problem, in which t is represented by a vector in the form of the following relationship:

$$X_i^t = (x^t{}_{i1}, x^t{}_{i2}, \ldots, x^t{}_{iD})$$

Also, this particle has a speed that is represented by the following vector in the repetition of t:

$$V_i^t = (v^t{}_{i1}, v^t{}_{i2}, \ldots, v^t{}_{iD})$$

And this particle has a memory of its best previous position in each repetition which is represented in the following relationship by the vector P:

$$P_i^t = (p^t{}_{i1}, p^t{}_{i2}, \ldots, p^t{}_{iD})$$

In each repetition of the search, each member is updated considering the two best values. The first one is related to the best solution that the particle has experienced so far (the merit value of the best solution is also stored). This value is called the best P or Pbest. The second best followed by the particle swarm optimization algorithm is the best position ever achieved in the population. This optimal value is general and is called Gbest. When a member considers a part of the population as the topology of its neighbors, the best value is a local best and is called Lbest. After the two best values are found, the position and velocity of each member are updated by the following formulas:

$$V_i(t+1) = wV_i(t) + c_1 r_{1,i}(t)(P_i(t) - X_i(t)) + c_2 r_{2,i}(t)(P_g(t) - X_i(t))$$

$$X_i(t+1) = X_i(t) + V_i(t+1)$$

In the above formulas, t represents the repetition number and variables c_1, c_2 are learning factors that control the displacement of a particle in one repetition. r_1, r_2 are two uniform random numbers in the interval [0,1]. w is an algebraic weight that is initialized in the interval [0,1]. A larger algebraic weight facilitates a general exploration and a smaller algebraic weight facilitates a local exploration.

V. Computational results

In Figure 2, the values of the model solving time, the first objective function and the second objective function obtained for solving the model in the mode of applying the Benders algorithm and the mode of solving with the GAMS solver are displayed. As can be seen in the table, for solving problems with low dimensions, the time to solve the problem does not differ much with the use of Benders decomposition algorithm. Even in some cases, the time to solve using the Benders algorithm is longer than the time to solve with the GAMS solver. However, with the increase in the dimensions of the problems, the solving time with the GAMS solver increases significantly and the time difference of the solution increases exponentially. Since the values obtained for the objective functions from the Benders decomposition algorithm have very little difference with the values obtained from the GAMS solver, it can be said that the Benders decomposition algorithm has a suitable efficiency for solving this problem. As can be seen; the solving time with GAMS solver increases exponentially; while with the Benders analysis algorithm, the increase in solving time is very small and this shows the efficiency of this algorithm.

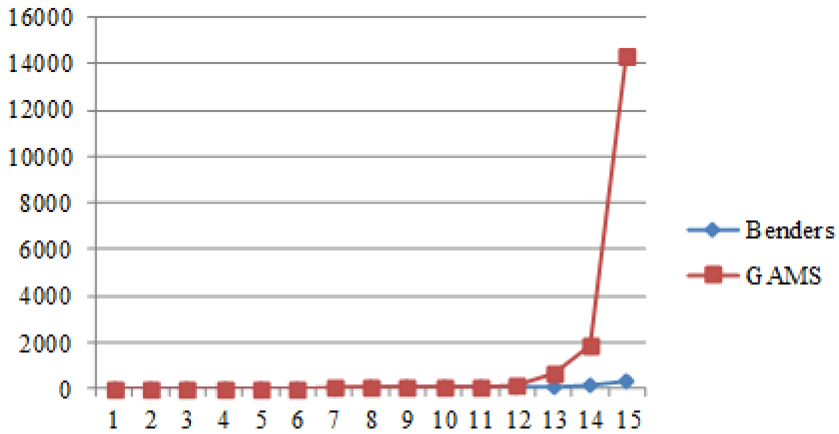

Figure 2. Time to solve sample problems in two modes of solving with GAMS solver and solving with Benders decomposition algorithm

Solving sample problems in larger sizes with MOPSO and NSGA II algorithms

In this section, in order to solve sample problems in larger sizes, 15 sample problems are designed based on random data based on uniform distribution. Also, from each sample problem, 5 problems of the same size have been designed in the defined data range, and the averages of each index have been evaluated and compared as a basis for comparison. Then, the T-Test statistical test was used for the significance of the difference between the averages of each index and each index was evaluated. Finally, it has been used to determine the most efficient algorithm for solving the virtual water problem. The sizes designed to solve the problem were randomly generated with MATLAB software (Table 2).

Table 2. Size of sample problems in larger size

Sample problem	i	j	b
1	4	4	6
2	6	4	6
3	6	4	7
4	10	5	8
5	12	10	9
6	15	12	11
7	15	14	12
8	15	15	12
9	16	16	13
10	16	16	14
11	17	16	15
12	18	17	15
13	19	17	16
14	19	17	16
15	20	20	20

To solve each sample problem, in order to prevent the generation of wrong random data, 5 other problems in the same production and with the problem with the same data generated by NSGA II and MOPSO algorithms were solved and the average of the calculation results was used as the basis of evaluation and comparison. Tables 3 and 4 respectively show the average objective functions and comparison indices of meta-heuristic algorithms for each sample problem.

Table 3. Average objective functions and comparison indices in solving with NSGA II algorithm

Sample problem	The first & third objective functions	The second objective function	The number of Kara	The answer of expansion index	The most indexing	Computational time interval
1	633806.72	62601.70	9	270273.91	0.37	36.46
2	778692.87	78381.54	19	585593.25	0.77	108.00
3	881581.31	85446.74	20	479316.63	0.70	170.30
4	1033814.58	90080.84	14	850298.87	0.57	242.53
5	1674913.50	103382.09	14	1129077.89	0.41	335.50
6	2369557.62	114251.75	22	1508175.51	0.55	434.40
7	2500890.63	125554.34	23	1797128.03	0.53	545.77
8	3416474.10	132080.49	18	2739770.14	0.57	669.07
9	4301935.98	140272.83	21	2529228.60	0.40	819.60
10	4860023.44	159821.22	23	3529017.44	0.75	959.67
11	5040590.70	163061.03	23	3087180.76	0.74	1040.13
12	8540218.42	178342.69	23	4883033.12	0.69	1326.00
13	8887924.17	182872.57	30	3839628.23	0.66	1528.37
14	10361985.83	193154.36	21	456422.62	0.77	1802.27
15	12608666.41	207290.80	24	5383709.71	0.87	2640.00

Table 4. Average objective functions and comparison indices in solving with MOPSO algorithm

Sample problem	The first & third objective functions	The second objective function	The number of Kara	The answer of expansion index	The most indexing	Computational time interval
1	635858.69	60567.72	8	109850.13	0.46	34.40
2	776699.89	71074.60	14	329845.53	0.62	39.07
3	871134.25	89693.95	8	370471.43	0.23	51.66
4	1046187.49	94437.00	16	463108.57	0.59	95.93
5	1653146.41	107002.93	18	817523.73	0.35	131.20
6	2353344.22	115415.91	23	15261236.70	0.49	280.50
7	2450251.68	123910.36	16	208648.76	0.55	349.16
8	3434001.90	136349.08	31	2559860.14	0.75	494.70
9	4334688.39	148225.98	28	3694417.30	0.64	723.16
10	4817592.14	151730.43	19	2215230.18	0.59	980.40
11	5020566.34	165792.57	12	2437807.90	0.76	1328.75
12	8500502.39	175673.57	25	3887334.58	0.44	1834.56
13	8759033.18	187113.32	12	3757576.19	0.72	2337.30
14	10251098.76	192138.59	12	4593286.90	0.66	2983.04
15	12554017.27	207281.68	17	5138916.08	0.51	3957.90

Tables 3 and 4 show the averages of the objective functions and comparison indices of meta-heuristic algorithms in each sample problem with NSGA II and MOPSO algorithms. To compare the results obtained, T-Test was used at the 95% confidence level to compare the significant difference between the averages of each index. Therefore, if the value of the P test statistic obtained for each index is less than 0.05; the null hypothesis is rejected and shows that there is a significant difference between the averages of that index, and if the value of the P test statistic is greater than 0.95; alternative hypothesis is rejected and indicates no significant difference in the averages of that index.

Examining the T-Test on the averages of the first and third objective function

Table 5 shows the output of the T-test on the averages of the first target function. Figs. 3 and 4 also show the comparative averages of the first objective function in each sample problem, as well as a box diagram to reject or accept the null hypothesis in the T-test.

Table 5. T-Test output results on the averages of the first objective function

Algorithm	The number of samples	Average	Standard deviation	Confidence interval 95%	T test statistics	P test statistics
NSGA II	15	5626072	3852039	4041*53686	2.49	0.026
MOPSO	15	4497208	3821250			

According to Table 5 and taking into account the P test statistic, it can be seen that there is a significant difference between the averages of the first objective function obtained by solving with NSGA II and MOPSO algorithms. With these interpretations, according to the minimization mode of the first objective function, it can be concluded that in this index, the MOPSO algorithm has obtained better results than the NSGA II algorithm. According to the observations of Figure 3, it can be seen that the MOPSO algorithm has obtained better results than the NSGA II algorithm among the sample problems (12) to (15). This shows that the efficiency of the MOPSO algorithm in obtaining the results of the first objective function will be higher in very large dimensions. According to the box diagram in Figure 4, it can be stated that because the null hypothesis is not included in the obtained interval, there is a significant difference between the averages of the first objective function obtained from NSGA II and MOPSO algorithms.

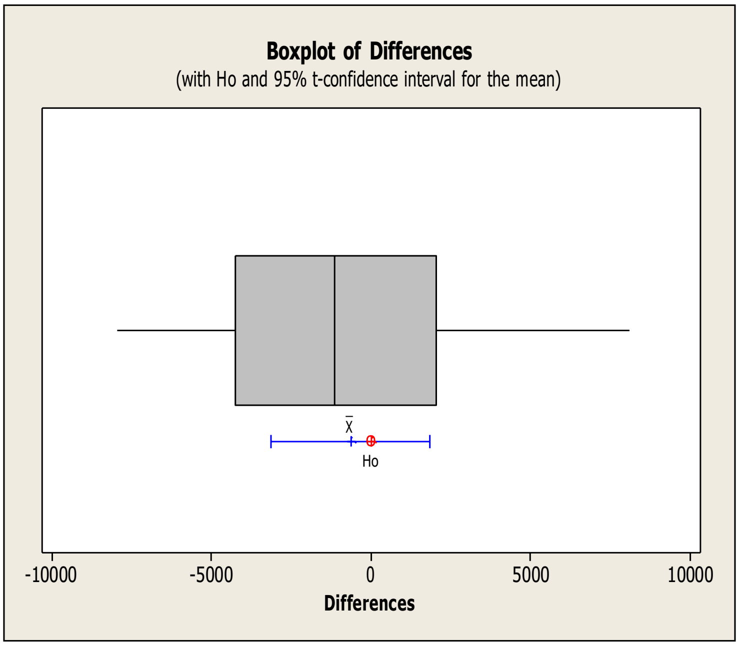

Figure 3. Box plot to confirm or reject the null hypothesis for the means of the first objective function

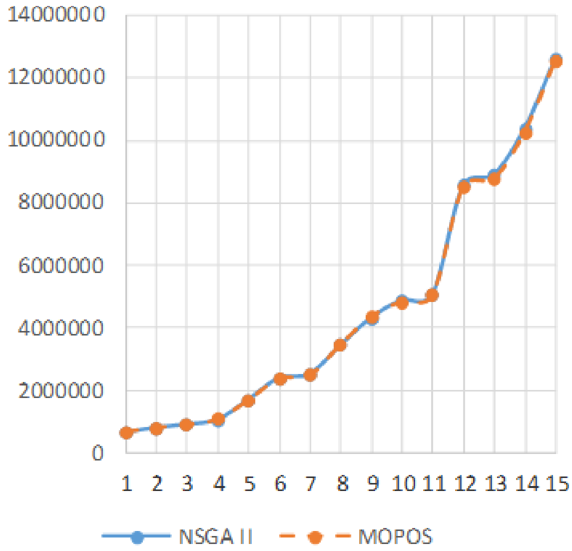

Figure 4. Comparison of the averages of the first objective function in example problems with meta-heuristic algorithms

Examining the T-Test on the averages of the second objective function

Table 6 shows the output results of the T-Test on the averages of the second objective function. Figs. 5 and 6 also show the comparative graph of the averages of the second objective function in each sample problem, as well as the box plot for rejecting or accepting the null hypothesis in the T-Test.

Table 6. T-Test output results on the averages of the second objective function

Algorithm	The number of samples	Average	Standard deviation	Confidence interval 95%	T test statistics	P test statistics
NSGA II	15	134440	45240	-3156*1848	0.56	0.584
MOPSO	15	135094	45418			

According to the P test statistic value of 0.584 obtained from Table 6, it can be concluded that there is no significant difference between the averages of the second objective function.

Figure 5 shows the comparison chart of the averages of the second objective function in different sample problems. It can be seen that the results obtained in the sample problems do not differ from each other. Therefore, it is not possible to easily comment on the efficiency of the algorithms in obtaining the results of the second objective function. Figure 6 is also added to the results of Table 6, and considering the placement of the null hypothesis in the 95% confidence interval, it can be concluded that there is no significant difference between the averages of the second objective function obtained by NSGA II and MOPSO algorithms.

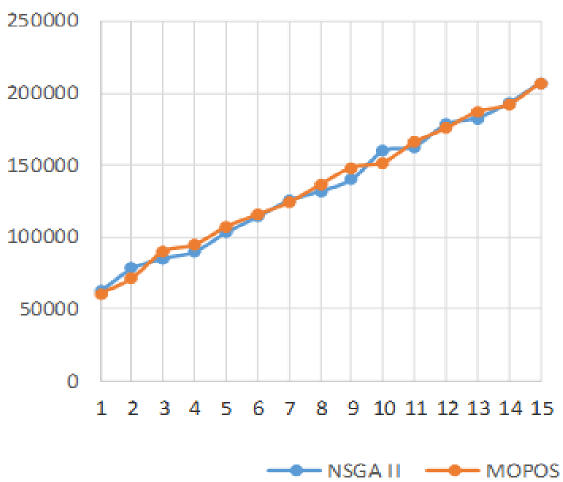

Figure 5. Comparison of averages of the second objective function in example problems with meta-heuristic algorithms

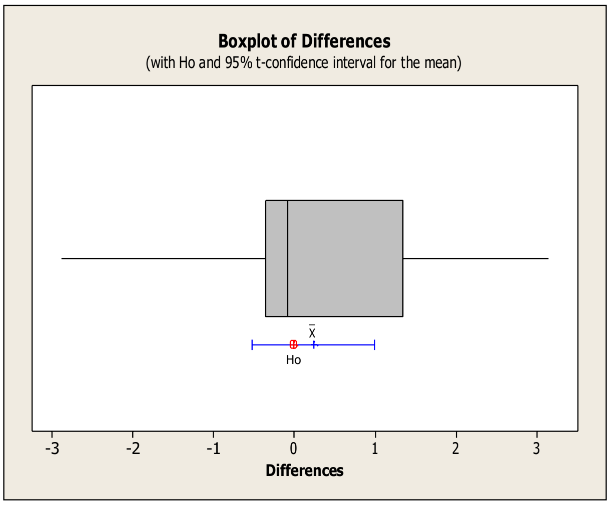

Figure 6. Box plot to confirm or reject the null hypothesis for the means of the second objective function

Examining the T-Test on average computing time

Table 7 shows the output results of the T-Test on the average computing time. In addition, Figs. 7 and 8 show the comparative graph of average computing time in each sample problem, as well as the box graph for rejecting or accepting the null hypothesis in the T-Test.

Table 7. The output results of the T-Test on average computing time

Algorithm	The number of samples	Average	Standard deviation	Confidence interval 95%	T test statistics	P test statistics
NSGA II	15	844	730	-483*88	1.48	0.160
MOPSO	15	1041	1220			

According to the value of the P-test statistic obtained from the comparisons of the T-Test on the computing time averages, it can be acknowledged that there is no significant difference between the computing time averages obtained from solving sample problems with NSGA II and MOPSO algorithms.

Comparison of computing time averages in example problems with meta-heuristic algorithms

In Figure 7, we can see that the computational time increases exponentially with the increase in the size of the sample problems, which is a reason for the NP-Hardness of the problem. However, the MOPSO algorithm is better than the NSGA II algorithm in terms of computing time for medium-sized problems, but with the increase in the size of the problem, the computing time obtained by this algorithm has greatly increased. Finally, the diagram in Figure 8 shows that the null hypothesis is placed in the 95% confidence interval, which is a reason for rejecting alternative hypothesis. Therefore, it can be concluded that there is no significant difference between the average computing time obtained by NSGA II and MOPSO algorithms.

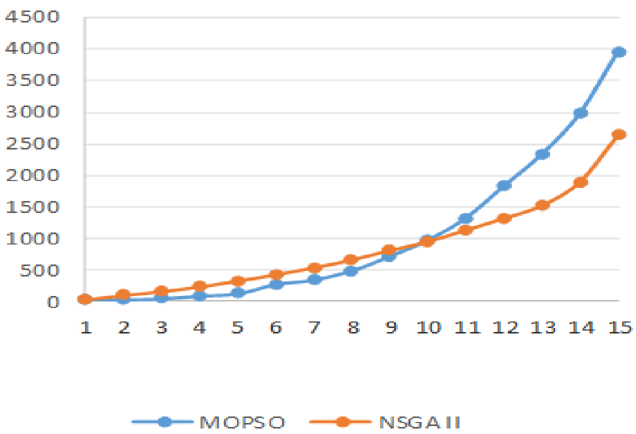

Figure 7. Comparison of computing time averages in example problems with meta-heuristic algorithms

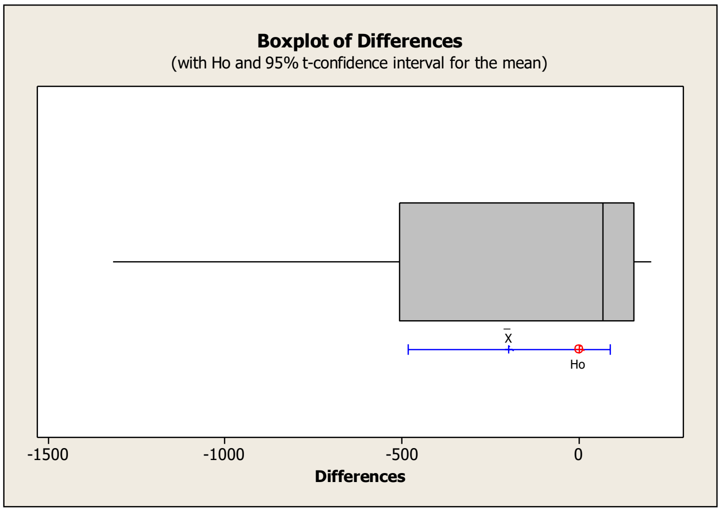

Figure 8. Figure box plot to confirm or reject the null hypothesis for computing time averages

VI. CONCLUSIONS

According to the presented mathematical model, based on the approach of Benders method, the presented mathematical model scenarios were discussed and analyzed, and in the evaluation, a comprehensive analysis of the performance of the optimization scenarios was presented and it was shown which scenarios are better than other optimization scenarios. And then due to the NP-hardness of the mathematical model, NSGAII and MOPSO algorithms were developed on the mathematical model and in the analysis it was shown that the meta-heuristic algorithm MOPSO has a better performance in medium and small dimensions in the optimization of the mathematical model.

Since the proposed model of this research examines the existing facilities in a health network and provides suggestions to improve the demand coverage, as well as the point that the improvement suggestions include making changes in the existing facilities or even removing a facility and these changes also require high costs; therefore, these suggestions can be attractive and implementable if by doing them the coverage of demand has increased to an acceptable extent. As stated in the presentation of the study sample; by solving this model, it is possible to significantly increase the demand coverage by paying an acceptable cost. Facilities are considered as hub locations; it is possible that there are different relationships between different levels of facilities, and some of these relationships are considered in this research. Other types of these relationships can also be considered.

One of the challenges in the issue of location of facilities is to consider facilities in different places in the same way and with similar services and capacity. This is despite the fact that the characteristics of the located points, the limitations, the type and amount of needs in each area are not necessarily similar to each other and these differences make the same facilities not efficient enough. In this research, the facilities can be different from various aspects. The facilities are considered hierarchically and at three different levels. The facility capacity is not fixed and the capacity of each facility can be determined from three different levels by solving the model. The coverage radius of each facility is also different and is considered as a decision variable. Therefore, according to the conditions at each point of the network, facilities can have different characteristics.

There are suggestions for future research in order to develop and improve the research as follow:

- Considering the fact that distance as a factor to improve the quality of demand coverage alone is not enough. Therefore, considering limitations such as time, chain disturbances, along with paying attention to the distance criterion, can bring the model closer to the real world.
- Due to the fact that the facilities considered in this research are health facilities and also due to the importance of time to handle the demand in the health field, it can be useful to consider the queuing theory in this model.
- Considering that the important goals in improving a network of health facilities are not limited to considering the cost and the amount of covered demand, it is appropriate that other goals of maximizing the quality of receiving service or reducing the concentration of demand coverage by a facility are also considered.
- Considering that there are different methods to deal with uncertainty; it is possible to implement other methods and compare the resulting answers and choose the best method, such as feasibility planning or fuzzy development.

- When facilities are considered as hub locations; it is possible that there are different relationships between different levels of facilities, and some of these relationships are considered in this research. Other types of these relationships can also be considered.
- Considering the backup facility, in such a way that every time a disruption occurs, a backup facility is considered for the disrupted facility.
- Considering the limits and goals that make the situation of demand coverage not worse in any region or area after the implementation of the model. Currently, there may be a facility at level w in an area, but due to the non-establishment of facilities of other levels and the continued lack of full coverage of that area, the existing facility will also be closed or moved, which will worsen the conditions of the said area.

VII. References

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