[1]Dr. D. D. Pandya
[2]Dr. T. K. Pathak
[3]D. N. Panchotia
[4]Dr. H. M. Makadia
[5]Shyam Kotecha
[6]Keyur Khatsuriya
[7]Hem Bhatt
[8]Pratik Lakhani

Application of Scopious Stark Domination in Optimization

Journal of Electrical Systems

Abstract: - Optimal Scopious stark domination plays important role in optimization applicable to many real world and engineering field problem . Due to its characterization as the optimization tool, it is widely used in social network theory, network design, sensor network, resource allocation, fault tolerance in distributed system, control system, transport network and telecommunication network and work assignment problem. By applying the concept of Scopious stark domination, engineers can design more efficient, robust, and fault-tolerant systems across various domains. Converting technical situation into mathematical modeling, network can be assigned appropriate graph on which results of edge addition, edge removal can be applied through flow chart or programming for optimization.

Keywords: Scopious stark dominating set, minimum Scopious stark dominating set, Scopious stark domination number, Scopious stark dominating plus number, Scopious stark dominating minus number, critical graphs.

I. Introduction

Stark domination in graph was introduced by Cockayne, Dawes and Hedetniemi [1]. To overcome the limitation of stark domination for the graph having isolated vertices, Dr. D. K. Thakkar defined a new concept named as Scopious stark domination[2] and discussed results regarding critical graphs for this variant[3]. Many authors have studied the concept and applied for the vertex removal and addition to understand behavior of this variant w.r.t critical graphs.[4-6]. Sometimes for the optimization, only single edge addition does not work, so to optimize the graph numbers of edges must be added or removed. This concept was introduced first by Fink[7]. In this paper concept has been discussed in details with theorems and appropriate illustrations to understand it easily. Applications have been discussed in which optimization through flowchart or programming can help to stake holders.

II. Definitions

Definitions 2.1 : Scopious stark dominating set

If ψ is stark dominating set in the graph $\Omega - \Theta$ and Θ is the set of all isolated vertices of graph Ω then $\psi_1 = \psi \cup \Theta$ represents scopious stark dominating set of the graph Ω.

Definitions 2.2 : minimal Scopious stark dominating set

A Scopious stark dominating set ψ is called minimal Scopious stark dominating set if it is not having proper subset which is also Scopious stark dominating set.

Definitions 2.3 : minimum Scopious stark dominating set

A Scopious stark dominating set with minimum cardinality is minimum Scopious stark dominating set.

[1],[2,3,4] *Assistant Professor, Department of Science and Humanities, Government Engineering College, Rajkot, Gujarat, India.

[5] *Assistant Professor, Computer Engineering Department, Government Engineering College, Rajkot, Gujarat, India.

[6,7, 8] * Assistant Professor, Shantilal Shah Engineering College, Gujarat Technological University, Ahmedabad, Gujarat, India.

Corresponding Author : Dr. D. D. Pandya, ddpandya@gecrajkot.ac.in

Definitions 2.4 : Scopious stark dominating number

Cardinality of the minimum Scopious stark dominating set of the graph Ω is called Scopious stark domination number expressed as $\gamma^{s\,s}(\Omega)$ for graph Ω.

Definitions 2.5 : Scopious stark dominating plus number

Minimum number of edges whose removal increase Scopious stark dominating number is called Scopious stark dominating plus number.

Definitions 2.6 : Scopious stark dominating minus number

Minimum number of edges whose addition decrease Scopious stark dominating number is called Scopious stark dominating plus number.

III. EDGE REMOVAL IN SCOPIOUS STARK DOMINATION

Theorem 3.1: For every vertex v^1 in subset ψ of $\alpha(\Omega)$, ψ is minimal Scopious stark dominating set if v^1 is an isolated vertex in Ω or stark Private neighbourhood of v^1 is non empty in ψ

Proof : $\Leftarrow$ Assuming that conditions hold and still ψ is not minimal Scopious stark dominating set. So there exist a vertex v^2 in ψ such that $\psi_1 = \psi - \{v^2\}$ is Scopious Stark dominating set. Hence v^2 is adjacent to at least one vertex in $\psi_1 = \psi - \{v^2\}$ which contradicts condition that v^2 is isolated. Now if $\psi_1 = \psi - \{v^2\}$ is Scopious stark dominating set then every vertex is adjacent to at least one vertex in $\psi_1 = \psi - \{v^2\}$ which contradicts the private neighbor hood concept. So if conditions are satisfied then ψ is minimal Scopious stark dominating set.

$\Rightarrow$ Let ψ be minimal Scopious stark dominating set. So for every vertex v^2 in ψ, $\psi - \{v^2\}$ is not Scopious Stark dominating set means some vertex v^3 will not be dominated by $\psi - \{v^2\}$. Now considering the case $v^3 = v^2$ proves v^2 to be isolated . If v^3 is not Scopious dominated by $\psi - \{v^2\}$ but by ψ proves v^3 to be in private neighbor hood of v^2 which proves the theorem.

Lemma 3.2 : $\Delta(\Omega)$ will be less than or equal to $\delta + 1$ if removal of δ edges from the graph Ω produces the sub graph Θ with $\Delta(\Theta)$ is 1.

Proof : Suppose removal of δ edges from the graph Ω produces the sub graph Θ with $\Delta(\Theta)$ is 1. Now if we remove one more edge from Θ then Θ_1 is the new graph. $\gamma^{s\,s}(\Theta_1) > \gamma^{s\,s}(\Theta) > \gamma^{s\,s}(\Omega)$ proves the result $\Delta(\Omega)$ will be less than or equal to $\delta + 1$.

Theorem 3.3 : For any graph Ω, $\Delta(\Omega) \le \theta - 1$ where θ is the sum of the degree of vertices having distance exactly 2.

Proof : Assuming v^1 and v^2 to be two non adjacent vertices having distance exactly two and a common adjacent vertices of these two is v^3. Let Θ be the graph removing the edge adjacent to v^1 and v^2 except v^1v^3 and v^2v^3. Let Θ_1 be the new graph obtained from Θ removing the edge v^2v^3. Suppose ζ be a minimum Scopious start dominating set of the graph Θ_1 which implies v^2 will be in ζ as v^2 is isolated in Θ_1. Now at least one of v^1 and v^3 must belong to ζ as v^1 will not be adjacent to any vertex of ζ. Actually v^3 must be member of ζ and $\rho = \zeta - \{v^2\}$ is Scorpious stark dominating set of the graph Θ .So, $\gamma^{s\,s}(\Theta) \le |\rho| < |\zeta| = \gamma^{s\,s}(\Theta_1)$ which gives $\Delta(\Theta) = 1$ and hence by the lemma, $\Delta(\Omega) \le \theta - 2 + 1 = \theta - 1$

Theorem 3.4 : $\Delta(\Omega) \leq \theta$ **where** θ **is the degree of vertex** v^1 **such that** $\gamma^{ss}(\Omega - v^1) > \gamma^{ss}(\Omega)$

Proof : For the graph Ω, let $\gamma^{ss}(\Omega - v^1) > \gamma^{ss}(\Omega)$. Considering the graph $\Omega - Ev^1$ where Ev^1 denotes set of edges incident with v^1 and ζ be minimum Scopious stark dominating set of the graph $\Omega - Ev^1$. Here v^1 being isolated vertex must be in ζ. So if we define $\zeta_1 = \zeta - v^1$ then ζ_1 is Scopious stark dominating set in $\Omega - v^1$ gives the relation $\gamma^{ss}(\Omega - v^1) < \gamma^{ss}(\Omega - Ev^1)$ and by statement $\gamma^{ss}(\Omega - v^1) > \gamma^{ss}(\Omega)$ proves the statement, $\Delta(\Omega) \leq \theta$.

Theorem 3.5 : $\gamma^{s}(\Omega + v^1 v2) < \gamma^{s}(\Omega)$ **for two non adjacent vertices** v^1 **and** v^2 **in graph** Ω **iff there exist a subset** ζ **of** $\alpha(\Omega)$ **with cardinality less than** $\gamma^{s}(\Omega)$ **and these conditions hold.**

(a) **If** $v^1, v^2 \in \zeta$ **then** $N(v^1) \cap \zeta = \phi$ **or** $N(v^2) \cap \zeta = \phi$

(b) **If** $v^1 \in \zeta$, $v^2 \notin \zeta$ **then** $N(v^2) \subset \alpha(\Omega) - \zeta$

(c) **If** $v^1 \notin \zeta$, $v^2 \notin \zeta$ **then** ζ **is stark dominating set in** $\Omega - \{v^1\}$ **and** $\Omega - \{v^2\}$ **respectively.**

Illustration 3.6 : To represent upper bound of $\Delta(\Omega)$ **,** $\Delta(\Omega) \leq \theta - 1$ **where** θ **is the sum of the degree of vertices having distance exactly 2.**

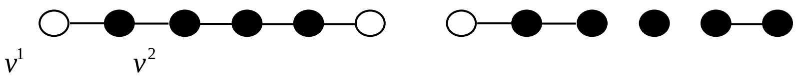


Figure 3.6.1

Illustration 3.7 : represents sharp upper bound, $\Delta(\Omega) = \theta - 1$ **where** θ **is the sum of the degree of vertices having distance exactly 2.**

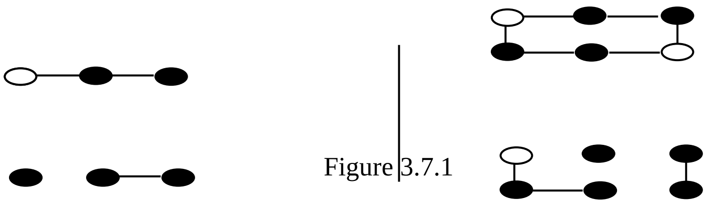


Figure 3.7.1

Illustration 3.8 : represents sharp upper bound, $\Delta(\Omega) = \theta - 1$ **where** θ **is the sum of the degree of vertices having distance exactly 2. here** $\Delta(\Omega) = \theta - 1$ **= 6 – 1 = 5**

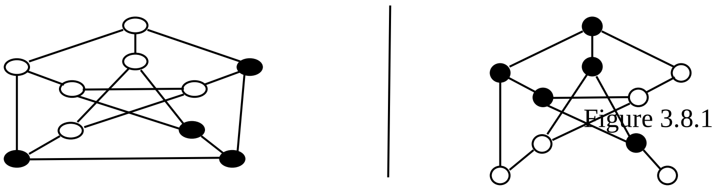


Figure 3.8.1

Illustration 3.9 : Following table denotes Scopious Stark domination number and Scopious stark domination minus number for complete grid graphs in general.

Complete grid graphs G	$\gamma^{SS}(\Omega)$	$\Delta((\Omega))$

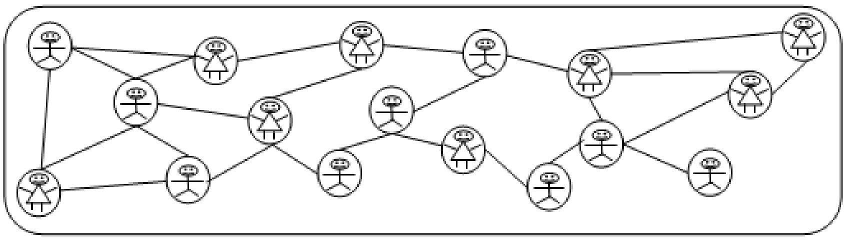
$P_2 \times P_n$	$2\left\lceil \frac{n}{3} \right\rceil, \quad n \geq 1$	$1, \quad n \equiv 0 \pmod 3$ $2, \quad n \equiv 1 \pmod 3$ $3, \quad n \equiv 2 \pmod 3 \quad (n \geq 2)$
$P_3 \times P_n$	$n, \quad n \geq 2$	1
$P_4 \times P_n$	$2\left\lceil \frac{2n+1}{3} \right\rceil, \quad n \equiv 1 \pmod 3$ $4\left\lceil \frac{n}{3} \right\rceil, \quad \text{otherwise}$	$1, \quad n \equiv 0 \pmod 3$ $2, \quad n \equiv 1 \pmod 3$ $3, \quad n \equiv 2 \pmod 3 \quad (n \geq 2)$
$P_5 \times P_n$	$n+2\left\lceil \frac{n+2}{3} \right\rceil, \quad n \equiv 1 \pmod 3$ $5\left\lceil \frac{n}{3} \right\rceil, \quad \text{otherwise}$	$1, \quad n \equiv 0 \pmod 3$ $2, \quad n \equiv 1 \pmod 3$ $3, \quad n \equiv 2 \pmod 3 \quad (n \geq 5)$

Table 3.9.1

IV. APPLICATION OF SCOPIOUS STARK DOMINATION

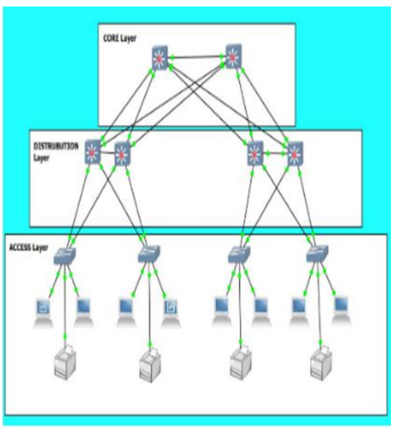
By applying the concept of total domination, engineers can design more efficient, robust, and fault-tolerant systems across various domains

Application 4.1 Social Network Theory



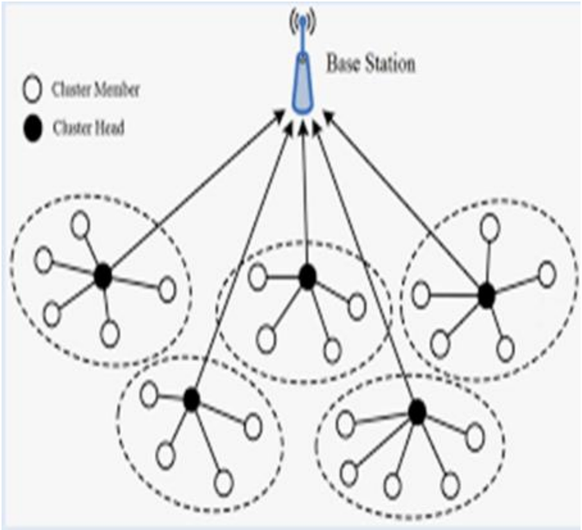
In her Ph. D dissertation, Kelleher presented research on dominating sets in social network graphs.[6]. In social network theory one studies relationships that exist among members of a group. The people in such a group are called actors. These relationships are typically defined in terms of one or more dichotomous properties, that is, a property that for any two actors unambiguously either holds or does not hold. Given such a property, one can construct a social network graph, in which thee vertices represent the actors and an edge between two vertices indicates that the property in question holds between the corresponding actors. Bondage number and reinforcement number come into picture for the optimization.

Application 4.2 : Optimization in network design



In the design and optimization of networks (e.g., telecommunications, computer networks, power grids), Stark domination can be used to ensure robustness and fault tolerance. Ensuring that every node (vertex) in the network is adjacent to a node in the dominating set can help in efficient monitoring, control, and data collection, and can ensure that the network remains connected even if some nodes fail. Our results for the Scopious stark plus domination number may help to optimize the problem converting situation into mathematical modeling and applying flow chart or programming.

Application 4.3 : Sensor Networks

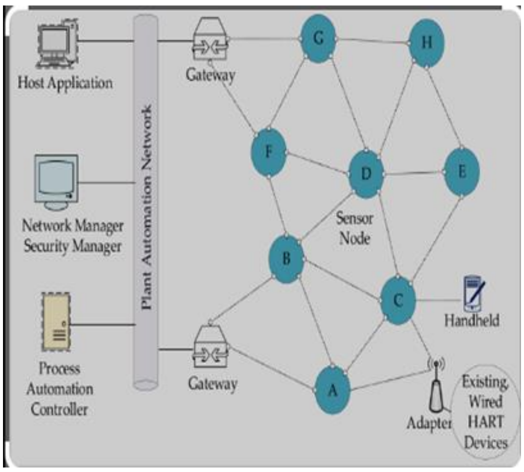


In sensor networks, Stark domination can be used to optimize the placement of sensors. By ensuring that every location in the monitored area is within the range of at least one sensor, one can guarantee complete coverage. This is crucial for applications like environmental monitoring, surveillance, and smart cities. PMU Placement for the full observability in electric network can be optimized using algorithm or programming for edge addition concept of Scopious stark domination.

Application 4.4: Resource Allocation

In resource allocation problems, such as allocating resources in a manufacturing plant or distributing emergency response units in a city, Stark domination can ensure that every required point has access to resources from at least one source. This can optimize response times and ensure efficient resource utilization. Bondage number and reinforcement number come into the picture for desired number.

Application 4.5 : Fault tolerance in distributed systems



In distributed systems, ensuring that every node has at least one adjacent node in the dominating set can help in maintaining system operations even if some nodes fail. This concept is used to design fault-tolerant systems where backup nodes can take over the responsibilities of failed ones. Scopious stark domination takes more care w.r.t stark domination which includes isolated vertices also. Vertices in dominating set are also connected with each other which helps system even if some nodes fail.

Application 4.6: Control systems

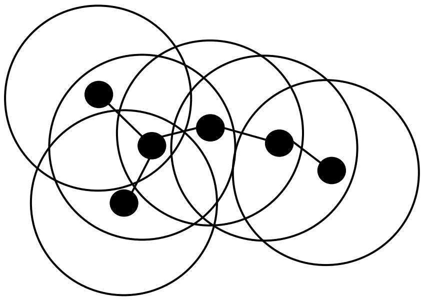
In control systems, particularly those involving distributed control or multi-agent systems, total domination can be applied to ensure that every subsystem or agent is within the control or communication range of at least one controller or leader. This helps in maintaining coordination and control over the entire system.

Application 4.7: Transportation Network

Railway network is the example of the largest physical graph in which stations are vertices and rail tracks are edges. Similarly bus routing in state , BRTS bus network in the city, virtual graph of social relation, work assignment problems, allocation of electric poles, water connection or broad casting tower require to minimize it for the cost cutting as well as ease of work. Programming for such network graphs by converting technical situation into mathematical modeling will give easiest solution.

Application 4.8: Telecommunications Networks

In designing telecommunications networks, Point shelter set can be used to ensure that every point (or vertex) in the network is covered by transmitters or receivers. This ensures reliable connectivity and efficient use of resources. Our results regarding edge addition, edge removal may helpful to optimize the problem with flowchart or programming. Sometimes if we want to optimize and removal of single edge is not sufficient than negative number or positive number will come into the picture.



REFERENCES

[1] E.J. Cockayne, R.M. Dawes and S.T. Hedetniemi, Total domination in graphs. *Networks* 10 (1980), 211–219

[2] D. K. Thakkar and J. C. Bosamiya,"Graph critical with respect to variants of domination" Ph.D thesis, Saurashtra university ,2011.

[3] "Domination related parameters and critical graphs", Ph.D Thesis, Saurashtra University, D. D. Pandya and D. K. Thakkar ,2015.

[4] Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater. "Fundamental of Dominatin in Graphs". Marcel Dekker Inc. New york.

[5] Tim Nieberg, "Independent and Dominating Sets in Wireless Communication Graphs" Ph. D thesis, Netherland, 2006.

[6] L. L. Kelleher. "Domination in Graphs and its application to Social Network Theory . PhD thesis, Northeastern University. 1985.

[7] J. F. Fink, M.S. Jacobson, L.F.Kinch and J.Roberts, "The bondage number of graph", Discrete Math. 86(1990)47-57

[8] S.Ao. "Independent domination critical graphs", Master Thesis, University of Victoria, Victoria BC, Canada.

[9] D. K. Thakkar and J. V. Changela, "Mathematical Modelling"Ph.D thesis, Saurashtra university 2011.