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Research on Flight Dynamic Price Adjustment Based on Two-Step Passengers Choice Behavior Evaluation

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Abstract: - Dynamic pricing represents an emerging trend in the domain of research and innovation within airline revenue management. A crucial aspect of dynamic pricing involves the modeling of passenger choice behavior, particularly the challenge of estimating the behaviors of unobservable passengers who choose not to purchase. Traditional research predominantly employs the Multinomial Logit (MNL) model and leverages algorithms such as Expectation Maximization (EM) or Markov Chain Monte Carlo (MCMC) algorithm to address this challenge. Nonetheless, these algorithms are difficult to directly apply in practice due to long computation times and the scarcity of literature considering competitive factors. In this paper, we initially undertake the task of quantifying competitive factors and integrating these into the factors impacting passenger choice behavior. Subsequently, we introduce a two-step non-homogeneous estimation method that decomposes the log-likelihood function into marginal and conditional components to evaluate the parameters of passenger choice behavior. Employing this strategy allows us to obtain the probability of passenger purchases and passenger arrival rate across diverse pre-sale periods. In conclusion, this study introduces a sophisticated dynamic price adjustment model, meticulously designed for the segmentation of cabin classes based on logical criteria. The empirical validation of this model lends credence to its efficacy, showing that the passenger choice model considering competitive factors results in alignment with real-world flight sales situations, particularly in estimating passenger choice behavior and assessing arrival rates. Remarkably, when juxtaposed with prevailing airline pricing strategies, our proposed dynamic pricing adjustment strategy demonstrates a significant elevation in average flight revenue.

Keywords: Passenger Choice Behavior, Revenue Management, Dynamic Pricing, Competition.

I. INTRODUCTION

Revenue management is a fundamental decision support technique used by airlines to navigate market competition and enhance sales revenue. Traditional revenue management techniques rely on a system of fare classes with varying booking restrictions. Underpinning these approaches is the assumption of passenger seat choice independence, where historical booking data is leveraged to predict future seat demand. Then, they optimize the allocation of seats across different fare classes according to the demand forecast in order to maximize revenue. In recent years, the advent of e-commerce has equipped passengers with online tools for comparing trips, enabling airlines to gather richer flight sales data and passenger behavior data. Technological advancements have catalyzed the transformation of conventional static pricing strategies, which emphasize quantity control, towards dynamic pricing strategies that prioritize price optimization. In contrast to static pricing, dynamic pricing operates under the assumption that prices of identical products will fluctuate over time. Therefore, it becomes a key issue to determine the optimal price of the same class in different pre-sale periods based on passengers' choice behavior. In the current research on dynamic pricing based on passenger choice behavior, there are problems such as insufficient consideration of factors affecting passenger choice behavior, insufficiently detailed division of route flight types, and long estimation time for parameters of passenger choice model, which cannot meet the needs of airlines.

This paper aims to mitigate the pressing need for dynamic pricing in the airline industry by integrating observable natural states more comprehensively. It thoroughly examines the characteristics of the non-homogeneous Poisson distribution related to potential demand from passengers who have arrived but not yet made purchases, without being observable. The proposition recommends integrating competitive factors into observable states and utilizing

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a two-step parameter estimation method to effectively estimate the parameters of the passenger choice behavior model and passenger arrival rates. Based on this premise, the study quantitatively characterizes the probabilities of passenger cabin selection and subsequently formulates a dynamic price adjustment model grounded in passenger choice behavior. In contrast to the predominant literature, which typically concentrates on the empirical analysis of a singular line [1-2], this research specifically examines representative business and tourism lines operated by a specific airline in China. Initially, the study scrutinizes passenger choice behavior and arrival rates across distinct scenarios. Subsequently, it elucidates the variations in dynamic price adjustment strategies under varying situations and observable conditions. Finally, it juxtaposes and scrutinizes revenue differentials between the pricing approach posited in this paper and the extant strategy adopted by the airline predicated on empirical rules.

II. Related Work

Fiig et al. [3] propose that dynamic pricing encompasses the dynamic computation of optimal prices, taking into account airline strategies, customer-specific data, and real-time pricing of alternative products. Kumar et al. [4] conceptualize dynamic pricing as a pricing tactic that substitutes the discrete opening and closing mechanisms of cabin classes with continuous price modifications at predefined temporal intervals. Wittman and Belobaba [5] encapsulate dynamic pricing within three frameworks that amalgamate historical and future perspectives: assortment optimization, dynamic price adjustment, and continuous pricing. Each framework allows airlines to adjust their prices based on changes in "observable natural states" during the pre-sale period, including seat availability at the time of pricing, time to departure, forecasted future demand, competitor behavior, and even individual passenger booking session behavior. The existing airline reservation system comprehensively accommodates the distribution model of "assortment optimization" and partially facilitates "dynamic price adjustment" through the integration of compatible plug-ins from other information systems. However, basically it is difficult to support the future model of "continuous pricing".

A notable disparity between dynamic pricing and static pricing lies in the incorporation of supplementary "observable natural states" beyond historical booking information. These observable natural states are commonly amalgamated into decision optimization models as probabilities of passenger choice, rectifying the independence assumption of conventional passenger behavior methodologies. Talluri and van Ryzin [6] initially apply the multinomial logit (MNL) passenger choice model to the allocation of single-leg flight seats. They utilize the expectation maximization (EM) algorithm for the estimation of the passenger arrival rate λ, which follows a homogeneous Poisson distribution, and the characteristic attribute parameters $\boldsymbol{\beta}$ based on historical data. Furthermore, they highlight the challenge of estimating unobservable passengers who have arrived but have not yet made purchases. Zhang and Cooper [7] develop a discrete-time markov random dynamic model grounded in passenger choice behavior, employing it for the seat allocation of parallel flights. Building upon a passenger choice model, Hopman et al. [8] posit that each passenger would opt for the currently lowest fare. Consequently, they formulate a dynamic programming model for the allocation of single-leg flights across multiple cabin classes. Feldman and Topaloglu [9] investigate the optimization of cabin classification considering passenger choice following a nested logit model. Zhou et al. [10] reveal substantial variations in preferences among distinct passenger segments using latent class models, which are utilized in the analysis of airline passenger choice. Bockelie and Belobaba [11] and Chiambaretto [12] study passenger choice behavior of ancillary services based on passenger segmentation.

Only a small fraction of the passenger choice models mentioned above have been utilized in the context of flight seat control [6-8]. However, the ultimate revenue optimization models can all be traced back to the assortment optimization framework defined by Wittman and Belobaba [5], which can be viewed as an extension of conventional static control methods. In China, domestic airlines, exemplified by the three major carriers, are currently adopting dynamic pricing control strategies grounded in empirical rules, following the introduction of foreign revenue management systems. These strategies primarily center on passenger preferences in a competitive market environment and, categorized by passenger attributes, place greater emphasis on the dynamic computation of real-time lowest fares and safeguarding inventory for high-priced cabins. While revenue determinations guided by empirical guidelines may enhance decision-making efficiency in the immediate pre-sale period, they do not ensure optimal flight revenue in the long run. Otero and Akhavan-Tabatabaei [1] and Chen et al. [2] respectively formulate dynamic pricing decision models for multiple cabin classes that integrate passenger arrival rate and passenger choice behavior, which can be based on the estimated passenger arrival rate and the probability of passengers choosing different cabin classes during different pre-sale periods, the prices of the predetermined three cabin classes can be adjusted within the specified price set, achieving the "dynamic price adjustment" concept as defined by Wittman and Belobaba [5]. Such an approach epitomizes the prevailing trajectory in airline dynamic pricing preceding the advent of "continuous pricing" in forthcoming industry developments.

Estimating the parameters of the passenger choice model poses a challenge due to the need to estimate parameters for unobservable passengers who have arrived but not yet made purchases [13]. Previous research often made the simplifying assumption that the passenger arrival rate follows a homogeneous Poisson distribution that remains constant throughout the entire pre-sale period [2,13-14]. However, in practice, the arrival process of passengers during different time intervals of the pre-sale period exhibits a high degree of randomness. Therefore, recent research has mainly utilized non-homogeneous Poisson distributions to characterize the passenger arrival process [15-16]. Otero and Akhavan-Tabatabaei [1] present an approach to estimate passenger choice probabilities resembling the willingness-to-pay (WTP) estimation technique introduced by Wittman and Belobaba [5] in the context of the "continuous pricing" model. The estimation relies on reservation data and employs phase-type (PH) distributions, and this approach primarily focuses on the influence of passenger arrival processes and ticket prices on passenger choice behavior. In contrast to most researchers, who commonly employ the EM algorithm to estimate unobservable passengers that have arrived but have not yet made purchases [14,17-18], Chen et al. [2] develop a MNL model to depict passenger choice substitution behavior. They utilize the markov chain monte carlo (MCMC) algorithm to estimate the attribute parameters within the model. However, the EM and MCMC algorithms both require a significant amount of computation time, making them impractical for use in airlines. Furthermore, these algorithms require the time of booking to be discretized into slices of a predetermined size by the modeler. The selected time slice must be sufficiently large to maintain the stability of the model calculations, yet also small enough to ignore the likelihood of two or more individuals arriving within the same time interval [13]. The decision of the time slice presents a challenging task for algorithm implementation. To address the aforementioned issues, Newman et al. [14] transform time slices into time windows, replacing discrete time slices with continuous time windows. They employ a two-step parameter estimation method, first estimating characteristic attribute parameters $\hat{\boldsymbol{\beta}}$ and then estimating arrival rate $\hat{\lambda}$. This approach not only improves computational speed but also resolves the issue of estimating unobservable characteristic attributes.

III. MODELING PASSENGER CHOICE BEHAVIOR

3.1 Basic Assumptions

This study focuses on the dynamic pricing adjustment of economy class across multiple cabin classes on single-leg flights. While airlines typically offer multiple cabin classes during the pre-sale period of flights, operators are primarily concerned with dynamically adjusting the prices of the lowest-tier cabin class and managing seat reservations for potential high-fare passengers booking closer to the departure date. As a result, in the context of dynamic pricing practices, multiple cabin classes are often consolidated into three logical classes: low, medium, and high, each logical cabin class contains multiple subclasses. Prices are adjusted distinctively within subclass in each logical class based on real-time flight sales dynamics, ensuring compatibility with decision outcomes and the existing reservation system.

Representing the logical cabin classes as a set $j \in \{1,2,\ldots,n\}$, with prices ranked from lowest to highest, and the logical cabin classes are divided into 3 classes, denoted as n=3 in the case of this paper . Divide the flight pre-sale period into T consecutive time windows of the same duration from near to far, denoted as $t \in \{1,2,\ldots,T\}$. At the moment when the flight pre-sale period commence at $t = T$, the flight seats are initialized and allocated to each logical cabin class, represented by the vector $\boldsymbol{W} = (w_1, w_2, w_3)$. Denote $\boldsymbol{Z}_t = (z_{t1}, z_{t2}, z_{t3})$, the vector of the number of seats booked in each logical class during the pre-sale period t . Let the number of passenger arrivals in each pre-sale period be C_t, and C_t can be described by a random variable with mean λ_t and obeying a Poisson distribution, then the arrival rate of passengers in each pre-sale period can be denoted as vector $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_T)$. Let $\boldsymbol{S}_{tj}$ denote the remaining seats in logical class *j* in pre-sale period *t*, then $\boldsymbol{S}_t = (S_{t1}, S_{t2}, S_{t3})$denotes the vector of remaining seats in each logical class in pre-sale period *t* .

The feasible price set of logical cabin class j is represented as a partially ordered set $\boldsymbol{f}_j =< f_j^1, f_j^2, ... f_j^k >$, arranged from low to high prices, where k represents the number of feasible prices comprising logical class j, and $f_j^k < f_{j+1}^1$. In this way, the price of each logical class can be denoted as the set $\boldsymbol{F} = \{\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3\}$.

The study presented in this paper is predicated on the following assumptions: (1) Exclusion of seat cancellations, no-shows, and overbooking. (2) The seat allocations for each logical cabin class are predetermined prior to the pre-sale period, and issues related to seat assignment during the sales process are not taken into account. (3) In instances where a passenger's preferred seat is unavailable or inaccessible, the passenger will refrain from purchasing seats in alternative classes. (4) The acquisition of group tickets is not within the scope of consideration.

3.2 Considering Competitive Factors in Passenger Choice Behavior Model

Discrete choice models, such as MNL, have found widespread application in diverse domains to capture decision-making processes, wherein the core objective lies in optimizing choices based on the principle of utility maximization. When examining air passenger choice behavior through the MNL model, it is crucial to acknowledge that different attributes considered in the model may exert distinct influences on passenger satisfaction, leading to heterogeneous utility outcomes. Key attributes commonly employed in the modeling of air passenger choice behavior encompass factors such as seat availability, fare structures, lead time of booking, cancellation fees, among others. Literature analysis rarely emphasizes the integration of competitive factors as essential attributes in the investigation. However, within the operational framework of airlines, competitive factors play a pivotal role in shaping passenger preferences. Consequently, this study introduces a methodology to quantify competitive factors and integrate them into the framework governing passenger choice behavior.

Airline operators typically take into account flight timing, the proportion of remaining capacity on the line, airline brand, and other factors when evaluating flight competitiveness. Among these, flight timing and airline brand are constant factors regardless of the pre-sale period *t*, whereas the remaining capacity on the line fluctuates continuously with *t*. Through extensive consultations with multiple airlines and diverse line operators, along with insights from operators and passenger interviews, this paper quantifies the competitive factors for flights during the pre-sale period t as outlined below:

$$p_t = 0.2 \cdot Brand\ Weight + 0.5 \cdot Timing\ Factor + 0.3 \cdot Proportion\ of\ Remaining\ Capacity \quad (1)$$

The brand weight is determined according to social ranking weights [20]. The timing factor is allocated a value of 10 for the early peak and midday peak periods. For each hour of deviation forwards or backwards, a deduction of 1 is applied. An additional deduction of 1 is made in the presence of competition from other airlines within the adjacent 1-hour timeframe. The remaining capacity proportion is computed as the ratio of the remaining economy class seats on the flight at the commencement of the pre-sale period t to the total remaining seats on the flights for that day, which is then multiplied by the quantified value of 10.

The comprehensive utility function for logical class *j* during the pre-sale period *t* is expressed as follows:

$$V_{tj} = u_{tj} + \varepsilon_{tj} \quad (2)$$

In the equation, u_{tj} represents the deterministic utility value for passengers choosing logical class j during the pre-sale period *t*, and ε_{tj} is the random utility error, a random variable with a mean of 0 and following a Gumbel distribution. Where $u_{tj} = \boldsymbol{\beta}^T \boldsymbol{X}_{tj}$, $\boldsymbol{X}_{tj}$ represents the feature attribute vectors of various considered factors for logical class *j* during the pre-sale period *t*, including the aforementioned flight competition attributes. $\boldsymbol{\beta}$ is a parameter vector of $\boldsymbol{X}_{tj}$ multiple feature attributes to be estimated. Assuming the utility value for not choosing any class is $u_{t0} = 0$, the probability of passengers choosing logical class *j* during the pre-sale period *t* is given by:

$$P_{tj}(\boldsymbol{\beta}) = \frac{exp(\boldsymbol{\beta}^T \boldsymbol{X}_{tj})}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti}) + 1} \quad (3)$$

The probability of passengers not purchasing any logical class is:

$$P_{t0}(\boldsymbol{\beta}) = \frac{1}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti}) + 1} \quad (4)$$

Since$\sum_{j=1}^{n} P_{tj}(\boldsymbol{\beta}) + P_{t0}(\boldsymbol{\beta}) = 1$, the total probability $\sum_{j=1}^{n} P_{tj}(\boldsymbol{\beta})$ that passengers choosing to purchase can be represented as:

$$P_{t|\otimes}(\boldsymbol{\beta}) = 1 - P_{t0}(\boldsymbol{\beta}) = \frac{\sum_{j=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{tj})}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti})+1} \quad (5)$$

The conditional probability of passengers choosing to purchase logical class *j* during the pre-sale period *t*, given all passenger purchases, can be further obtained as follows:

$$P_{tj|\otimes}(\boldsymbol{\beta}) = \frac{exp(\boldsymbol{\beta}^T \boldsymbol{X}_{tj})}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti})} \quad (6)$$

3.3 Model Parameter Estimation

Due to the unobservable nature of passengers who have arrived but have not yet made purchases, estimating the arrival rates of passengers solely based on observed data leads to bias. Therefore, in the estimation of model parameters, the likelihood function under incomplete data conditions needs to be selected. Building on the research by Vulcano et al. [20], this paper establishes the likelihood function under incomplete data conditions as follows:

$$\mathscr{L}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \prod_{t=1}^{T} \Big[\mathbb{P}(m_t \text{ passengers buy in pre-sale period } t \,|\boldsymbol{\beta}, \boldsymbol{\lambda}) \cdot \frac{m_t!}{\prod_{j=1}^{n} z_{tj}!} \prod_{j=1}^{n} \left(\frac{exp(\boldsymbol{\beta}^T \boldsymbol{X}_{tj})}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti})} \right)^{z_{tj}} \Big] \quad (7)$$

In the above equation, it is assumed that m_t passengers have chosen to purchase logical classes within the pre-sale period of *t*, where m_t is a random variable following a Poisson distribution with mean $\lambda_t \sum_{j=1}^{n} P_{tj}(\boldsymbol{\beta})$. Due to the equality $\sum_{j=1}^{n} P_{tj}(\boldsymbol{\beta}) = P_{t|\otimes}(\boldsymbol{\beta})$, that is,

$$\mathbb{P}(m_t \text{ passengers buy in pre-sale period } t|\boldsymbol{\beta}, \boldsymbol{\lambda}) = \frac{\left[\lambda_t P_{t|\otimes}(\boldsymbol{\beta})\right]^{m_t} exp\left[-\lambda_t P_{t|\otimes}(\boldsymbol{\beta})\right]}{m_t!}$$

In order to facilitate the determination of the parameters that maximize the probability, the equation is transformed into the log likelihood function as follows:

$$ln\mathscr{L}(\boldsymbol{\beta}, \boldsymbol{\lambda}) = \sum_{t=1}^{T} \Big[m_t \, log\left[\lambda_t P_{t|\otimes}(\boldsymbol{\beta})\right] - \lambda_t P_{t|\otimes}(\boldsymbol{\beta}) - \sum_{j=1}^{n} log\left(z_{tj}!\right) + \sum_{j=1}^{n} z_{tj} \, log\left[P_{tj|\otimes}(\boldsymbol{\beta})\right] \Big] \quad (8)$$

As equation (8) contains the following term, directly maximizing the log-likelihood function with respect to $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ can be difficult to solve.

$$\lambda_t \frac{\sum_{j=1}^{n} exp\left(\boldsymbol{\beta}^T \boldsymbol{X}_{tj}\right)}{\sum_{i=1}^{n} exp(\boldsymbol{\beta}^T \boldsymbol{X}_{ti}) + 1}$$

Newman et al. [14] propose a two-step method to decompose the log-likelihood function into marginal and conditional components for separate estimation. They demonstrate that this method can yield consistent estimates of arrival rate and parameters compared to directly estimating the log-likelihood function. Additionally, they find that the two-step method offers superior computational efficiency when compared to other estimation methods. As a result, this study utilizes this method for parameter estimation.

Step one: Consider only the last term in (8), expressed as:

$$ln\mathscr{L}_1 = \sum_{t=1}^{T} \sum_{j=1}^{n} z_{tj} log\left[P_{tj|\otimes}(\boldsymbol{\beta})\right] \quad (9)$$

The conditional probability in this part of the log likelihood function only involves the passenger purchase probability, and is only a function of $\boldsymbol{\beta}$. The passenger no-purchase behavior is excluded from the model, which means that only the data of passenger purchase behavior need to be considered in the estimated sample data, and there is no unobservable data of passenger no-purchase behavior. In (9), the second derivative of the log likelihood function of $\boldsymbol{\beta}$ is always negative (the proof process is similar to the derivation process of $\boldsymbol{\lambda}$ in the second step below), that is, it is a concave function in the global range and has a unique maximum value. Therefore, the parameter $\boldsymbol{\beta}$ can be estimated by the maximum likelihood estimation (MLE) method, and then the parameters $\widehat{\boldsymbol{\beta}}$ corresponding to each attribute can be obtained.

Step two: Given the premise that $\widehat{\boldsymbol{\beta}}$, maximize the log likelihood function with respect to the other parameters. McFadden et al. [21] demonstrates that by maximizing the log likelihood function of the MNL model containing only a subset of alternatives, consistent estimates can be obtained as in the MNL model with all alternatives included. That is, the parameters estimate $\widehat{\boldsymbol{\beta}}$ obtained from the MNL model excluding data of no-purchase passengers is consistent with the parameter estimate of the MNL model with complete data, meaning that under the known condition of parameter $\widehat{\boldsymbol{\beta}}$, it is only necessary to maximize the log likelihood function with respect to $\boldsymbol{\lambda}$, where $-\sum_{j=1}^{n} log(z_{tj}!)$ is a constant term that can be ignored during the solution process. The log likelihood function with respect to $\boldsymbol{\lambda}$ can be expressed as:

$$ln\mathscr{L}_2(\boldsymbol{\lambda}) = \sum_{t=1}^{T}\left[m_t\, log\left[\lambda_t P_{t|\otimes}(\widehat{\boldsymbol{\beta}})\right] - \lambda_t P_{t|\otimes}(\widehat{\boldsymbol{\beta}})\right] \quad (10)$$

Since $\widehat{\boldsymbol{\beta}}$ is known, it is easy to calculate $P_{t|\otimes}(\widehat{\boldsymbol{\beta}})$ from (4) and (5). Before estimating $\boldsymbol{\lambda}$ in the second step, the calculated $P_{t|\otimes}(\widehat{\boldsymbol{\beta}})$ is treated as a constant term in advance, and finally the log likelihood function (10) about $\boldsymbol{\lambda}$ is directly maximized, where the first derivative of (10) for $\boldsymbol{\lambda}$ is:

$$\frac{\partial ln\mathscr{L}_2(\lambda)}{\partial \lambda_t} = \frac{m_t}{\lambda_t} - P_{t|\otimes}(\widehat{\boldsymbol{\beta}}) \quad (11)$$

The second derivative is:

$$\frac{\partial^2 ln\mathscr{L}_2(\boldsymbol{\lambda})}{\partial^2 \lambda_t} = -\frac{m_t}{\lambda_t^2} \quad (12)$$

As the value of m_t during each pre-sale period is always non-negative, (12) is always negative. Therefore, (10) is a globally concave function. By finding the stationary point of (10), we can further obtain the passenger arrival rate $\hat{\lambda}_t$ for each pre-sale period as follows:

$$\hat{\lambda}_t = \frac{m_t}{P_{t|\otimes}(\widehat{\boldsymbol{\beta}})}, \quad t = 1, \ldots, T \quad (13)$$

The aforementioned two-step estimation method can be summarized simply as follows: first estimate the parameter vector $\widehat{\boldsymbol{\beta}}$ corresponding to each observable attribute, and then solve for the arrival rate $\hat{\lambda}_t$ for passengers within each advance pre-sale period. The first step in the two-step method involves maximizing a globally concave function with respect to $\boldsymbol{\beta}$,which can be solved using the MLE method. Once the parameters $\widehat{\boldsymbol{\beta}}$ are obtained, the second step directly involves solving for $\hat{\boldsymbol{\lambda}}$. This approach significantly reduces the iteration count compared to EM and MCMC algorithms, which jointly solve for the parameter vectors $\widehat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\lambda}}$, thus accelerating computation speed.

IV. Dynamic Price Adjustment Model

Define $V_t(\boldsymbol{S}_t)$ as the maximum expected revenue that can be obtained when the remaining seats in the three logical cabin classes within the pre-sale period *t* are in state $\boldsymbol{S}_t = (S_{t1}, S_{t2}, S_{t3})$. Let c_{tj} be the number of seats that may be sold in logical class *j* during the pre-sale period *t*, and $\boldsymbol{c}_t = (c_{t1}, c_{t2}, c_{t3})$ be the vector of possible sales seats in the three logical classes during the pre-sale period *t*.

The probability that a seat is sold during the pre-sale period *t* is $\sum_{j=1}^{n}\sum_{c_{tj}=0}^{S_{tj}} P\,(\boldsymbol{c}_t|\boldsymbol{\beta}, \boldsymbol{\lambda}_t)$,and the expected revenue obtained at this time can be expressed as $\sum_{j=1}^{n}\sum_{c_{tj}=0}^{S_{tj}} P\,(\boldsymbol{c}_t|\boldsymbol{\beta}, \lambda_t)\cdot\left[\boldsymbol{c}_t\cdot\boldsymbol{f}_j + V_{t+1}(\boldsymbol{S}_t - \boldsymbol{c}_t)\right]$, Let $P_0(\boldsymbol{c}_t|\boldsymbol{\beta}, \lambda_t)$ denote the probability that no seats are sold during the pre-sale period *t*. Due to $\sum_{j=1}^{n}\sum_{c_{tj}=0}^{S_{tj}} P\,(\boldsymbol{c}_t|\boldsymbol{\beta}, \lambda_t) + P_0(\boldsymbol{c}_t|\boldsymbol{\beta}, \lambda_t) = 1$, and considering the possibility of unsold seats being sold in the next pre-sale period, the expected revenue from no seats being sold at this time is equal to $P_0(\boldsymbol{c}_t|\boldsymbol{\beta}, \lambda_t)V_{t+1}(\boldsymbol{S}_t)$.

Building on the above, the dynamic pricing adjustment model for each logical class within the pre-sale period t based on the markov decision process (MDP) is as follows:

$$V_t(\boldsymbol{S}_t) = \max_{\boldsymbol{f}_j \in R_+} \left\{ \sum_{j=1}^{n} \sum_{c_{tj}=0}^{S_{tj}} P\left(\boldsymbol{c}_t | \boldsymbol{\beta}, \lambda_t\right) \cdot \left[\boldsymbol{c}_t \cdot \boldsymbol{f}_j + V_{t+1}(\boldsymbol{S}_t - \boldsymbol{c}_t)\right] + P_0(\boldsymbol{c}_t | \boldsymbol{\beta}, \lambda_t) V_{t+1}(\boldsymbol{S}_t) \right\} \quad (14)$$

The boundary conditions of the dynamic programming expression above are:

(1) If the remaining seats in a logical cabin class during the pre-sale period are zero, the cabin class will be automatically closed and not reopened in subsequent pre-sale periods. When all seats on the flight are sold out, $V_t(0) = 0,\ t = 1, \dots, T$ at this time.

(2) When the pre-sale period ends, denoted as t=0, any revenue outside the pre-sale period is zero, denoted as $V_0(\boldsymbol{S}_0) = 0, \forall \boldsymbol{S}_0 \in \left\{(S_{01}, S_{02}, S_{03}) \colon 0 \le S_{0j} \le w_j, \forall j \in \{1,2,3\}\right\}$, where $\boldsymbol{S}_0$ is the vector of remaining seats in each logical cabin class when the pre-sale period ends.

V. Model Demonstration and Analysis

5.1 Dataset selection

Different types of airline lines exhibit distinct sales characteristics. In this study, we examine the 2019 annual data from a prominent Chinese airline's representative business line and tourism line (referred to as A and B, respectively), which remained unaffected by the epidemic. Our analysis focuses on understanding passenger choice behavior and empirically validating a dynamic price adjustment model. The data analysis indicates that a significant portion of ticket purchases occur within approximately one month of the pre-sale period, with tourist line typically experiencing earlier ticket purchases compared to business line. Furthermore, the peak of ticket purchases in the lower class tends to precede the peak in the higher class for the same flight. Towards the end of the pre-sale period, the number of sales in the lower class declines, while sales in the higher class continue to increase to some extent. Additionally, flights from these two types of lines exhibit varying sales patterns on both regular and holiday days. Certainly, these characteristics are intricately intertwined with passenger demand on the one hand and, on the other hand, they are closely linked to the operational control strategies implemented by airline operators. It is essential to note that a reduced number of sales does not necessarily equate to decreased demand.

The paper initiates by conducting a comparative analysis of passenger choice behavior and arrival processes for flights on lines A and B across various flight dates. Subsequently, it elucidates the variations in dynamic price adjustment strategies for corresponding cabin classes under diverse pre-sale periods and available seat capacities. Lastly, a thorough comparative analysis is undertaken to examine the revenue differentials between the proposed dynamic price adjustment strategy in this study and the incumbent revenue strategy of the airline grounded in empirical rules.

5.2 Analysis of passenger choice behavior

5.2.1 The impact of observable states on passenger choice behavior

Due to the fact that the majority of ticket purchases occur within the final month of the pre-sale period, the model analysis excludes the sales status 30 days prior. Consequently, this paper uniformly partitions the pre-sale period into 30 consecutive time windows, denoted as T=30, with each time window representing one day. The cabin classes are categorized into three logical cabin classes. Within the dataset specified in Section 5.1, an analysis is performed to explore the influence of different observable states on passenger choice behavior for flights on lines A and B, distinguishing between regular days and holidays (comprising a total of four scenarios). Observable states encompass commonly utilized indicators, including booked seat count, seat prices, days booked in advance, cancellation and change fees, in addition to the novel factor of flight competition proposed in this study. Holidays encapsulate prominent holiday periods such as National Day, Labor Day, and the Spring Festival, whereas regular days denote dates that do not fall within holiday periods. By employing MLE to compute the parameters in (9), the outcomes are presented in Table 1. In Table 1, $\hat{\beta}_b^j$、 $\hat{\beta}_f^j$、 $\hat{\beta}_t^j$、 $\hat{\beta}_r^j$ and $\hat{\beta}_c$ represent the parameter estimation results of passenger characteristic attributes under the aforementioned five observable states. The superscript $j \in \{1,2,3\}$

for the first four attributes denotes the logical cabin classes, while $\hat{\beta}_c$ represents the competition factors mentioned above, unrelated to logical cabin classes.

The absolute value of the parameter $\hat{\boldsymbol{\beta}}$ reflects the sensitivity of passengers to each characteristic attribute. A larger absolute value of the parameter indicates a higher sensitivity of passengers to the respective attribute. Based on the parameter estimates in Table 1, the following conclusions can be drawn:

(1) Analysis of seat price: Observing the parameters $\hat{\beta}_f^j$ for price in Table 1, whether on regular days or holidays, the absolute values of the price parameters for tourist line B are generally higher than those for business line A. This suggests that leisure passengers exhibit higher price sensitivity to seat prices compared to business passengers. Furthermore, for both business line A and tourist line B, the absolute values of the price parameters during holidays tend to surpass those on regular days. This observation highlights the heightened price sensitivity of holiday passengers compared to passengers traveling on regular days. In all four scenarios, $\hat{\beta}_f^1$ and $\hat{\beta}_f^2$ are both negative values. This indicates that for low and medium logical classes, as the seat prices increase, passenger utility decreases. It is noteworthy that the parameter $\hat{\beta}_f^3$ for high logical class is positive, indicating that regardless of whether it is business line or tourist line, and whether it is a regular day or a holiday, there always exists a portion of high-price passengers who are insensitive to seat prices.

(2) Analysis of days booked in advance: Observing the parameters $\hat{\beta}_t^j$ for days booked in advance in Table 1, whether on regular days or holidays, the parameters $\hat{\beta}_t^1$ for low logical class in business line A are lower than that for tourist line B, while the absolute values of the parameters $|\hat{\beta}_t^2|$ and $|\hat{\beta}_t^3|$ for medium and high logical classes in business line A are higher than those for tourist line B. This indicates that leisure passengers tend to book low-priced cabins in advance, while business passengers are more inclined to purchase medium and high logical classes in the late pre-sale period. In all four scenarios, the parameters $\hat{\beta}_t^1$ for low logical class are positive, indicating that in the early pre-sale period ,when t is large, the airline will release low-priced cabins, and low-priced passengers will start to make purchases due to the influence of passengers' out-of-pocket trips. The early purchasing behavior of low logical class passengers is not significantly related to the nature of the line. The parameters $\hat{\beta}_t^2$ and $\hat{\beta}_t^3$ for medium and high logical classes are negative, indicating that as the pre-sale period approaches (t becomes smaller), the utility of medium and high logical classes passengers increases. That is, airlines tend to raise prices and limit the sales of low cabins as the advance pre-sale period approaches.

(3) Analysis of competition factor : Observing the parameters $\hat{\beta}_c$ for competition factors in Table 1, they are all positive in each scenario, indicating that as the increase in flight competitiveness, passenger utility values increase. The competition factor parameters in business line A are generally higher than those in tourist line B, with a significant difference on regular days and a relatively small difference on holidays. This suggests that business passengers are more concerned about factors such as airline brand and flight schedule, compared to leisure passengers.

Table 1: Estimation results of passenger choice behavior attribute parameters

	Business Line A		**Tourism Line B**	
	Regular Days	**Holidays**	**Regular Days**	**Holidays**
$\hat{\beta}_b^1$	0.0142	0.0118	0.0096	0.0079
$\hat{\beta}_b^2$	0.0255	0.0171	0.0085	0.0103
$\hat{\beta}_b^3$	0.0468	0.0243	0.0192	0.0199
$\hat{\beta}_f^1$	-0.0023	-0.0048	-0.0161	-0.0427
$\hat{\beta}_f^2$	-0.0015	-0.0029	-0.0144	-0.0214
$\hat{\beta}_f^3$	0.00028	0.00033	0.0005	0.00069
$\hat{\beta}_t^1$	0.0134	0.0118	0.0305	0.0431
$\hat{\beta}_t^2$	-0.0513	-0.0361	-0.0339	-0.0225
$\hat{\beta}_t^3$	-0.0753	-0.0272	-0.0483	-0.0097
$\hat{\beta}_r^1$	-0.0184	-0.0342	-0.0375	-0.0616
$\hat{\beta}_r^2$	-0. 0177	-0.0511	-0. 0228	-0.0493
$\hat{\beta}_r^3$	-0.0034	-0.0064	-0.0091	-0.0286
$\hat{\beta}_c$	0.6083	0.2907	0.1865	0.2624

5.2.2 Passenger Arrival Rates Evaluation

Using the two-step parameter estimation method described in Section 3.3, after obtaining the parameters $\hat{\boldsymbol{\beta}}$ for observable attributes, continue to calculate the passenger arrival rate $\hat{\lambda}_t$ within each pre-sale period *t*. The scenarios for business line A and tourist line B on regular days and holidays are labeled as A1, A2, B1, and B2. The passenger arrival rates for different scenarios within each pre-sale period are illustrated in Fig 1, with *t*=0 representing the flight's departure day.

From Fig 1, the business line on regular days (A1), the business line on holidays (A2), and the tourist line on regular days (B1) demonstrate a similar overall trend in passenger arrival rates. Specifically, during the initial days of the pre-sale period, the passenger arrival rates remain relatively low and gradually escalate as the flight's departure date nears. Notably, as the flight's departure date approaches, the growth in passenger arrival rates for Business line on regular days (A1) surpasses that of the other two scenarios significantly, and this upward trend persists. On the other hand, for the tourist line on regular days (B1), the passenger arrival rates are higher than those in scenarios A1 and A2 during the early pre-sale period, exhibit a significant increase in the middle pre-sale period, and demonstrate a certain degree of decrease in the late pre-sale period. This observation suggests that business passengers tend to purchase airline tickets during the near-term pre-sale period, while leisure passengers start purchasing tickets earlier in the pre-sale period. It is observed that the passenger arrival rates for the business line on holidays (A2) are significantly lower than those for the business line on regular days (A1) as the flight departure date approaches, indicating a noticeable decline in passengers traveling for business purposes during holidays.

Tourist line on holidays (B2) displays notable distinctions from the three scenarios discussed. During the early-middle pre-sale period, the passenger arrival rates exhibit a significant increase compared to the other scenarios. A slight decrease is observed in the middle pre-sale period, followed by relatively stable rates with minor fluctuations leading up to the flight departure date. This pattern corresponds with passengers' inclination to plan their trips well in advance for major holiday periods.

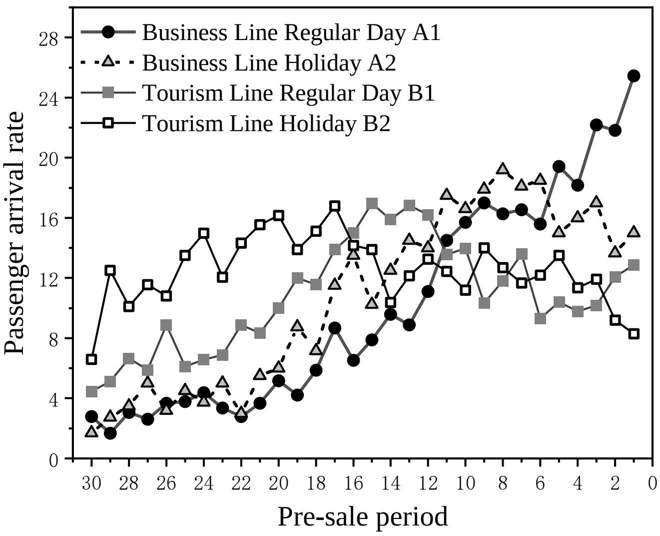

Fig 1: Estimated passenger arrival rates in scenarios A1, A2, B1, and B2

The simulated results of passenger arrival rates presented above align closely with the analysis conclusions regarding the impact of various observable state attributes on passenger choice behavior in Section 5.2.1. This alignment is in accordance with the prevailing sales control strategies commonly employed by airlines.

5.3 Dynamic Price Adjustment Strategy and Results Analysis

Building upon the established understanding of passenger choice behavior and arrival rates, this section undertakes simulated and empirical analysis for the dynamic price adjustment model introduced in section 4.

Firstly, analyze the impact of the pre-sale period on cabin price decisions, without considering the dynamic changes in seat availability during the sales process. Fix the number of remaining seats at 40, selecting pre-sale periods of 30 days and 20 days to represent early pre-sale periods, 10 days and 8 days for middle pre-sale periods, and 6 days and 4 days for late pre-sale periods. Also, fix the allocated number of available seats for low, medium, and high logical cabin classes at (20, 10, 10) and (10, 15, 15), respectively. In the first scenario, more seats are allocated to the low logical class, while in the second scenario, more seats are allocated to the medium and high logical classes. Taking a specific flight on business line A1 on a regular day (denoted as a1) as an example, with the feasibility price sets for low, medium, and high logical classes set at [370, 455], [780, 889], and [1100, 1250] respectively. The decision prices for each logical class within every stage of the pre-sale period for this flight are derived using (14), as illustrated in Table 2.

From Table 2, it is evident that the low logical class generally sustains a price of 370 yuan, offering a suggestion to raise the price to 455 yuan only when there is a limited seat allocation for the low logical class in the late pre-sale period (scenario 2). The medium logical class, when assigned more seats (scenario 2), adopts a strategy of lower pricing. however, in the middle and late pre-sale periods with reduced seat allocation (scenario 1), it provides suggestions for price escalation. The high logical class recommends low prices during the early pre-sale periods for both scenarios, high prices in the late pre-sale period for both scenarios, and the price suggestion in the middle pre-sale period shows a negative correlation with the allocated seat quantity.

Table 2: Impact of pre-sale period on optimal price decisions

Pre-sale period	Remaining seats in low class	Remaining seats in medium class	Remaining seats in high class	Low class decision price	Medium class decision price	High class decision price
4	20	10	10	370	889	1250
	10	15	15	455	780	1250
6	20	10	10	370	889	1250
	10	15	15	370	780	1250
8	20	10	10	370	889	1250
	10	15	15	370	780	1100
10	20	10	10	370	780	1250
	10	15	15	370	780	1100
20	20	10	10	370	780	1100
	10	15	15	370	780	1100
30	20	10	10	370	780	1100
	10	15	15	370	780	1100

Subsequently, a more in-depth analysis is carried out to evaluate the impact of fluctuations in the remaining number of cabin seats on pricing strategies. An experimental study is conducted to assess pricing decisions by manipulating the availability of seats in each logical cabin class of flight a1. Flight a1 comprises a total of 169 seats in the economy class, with initial seat distributions of 79, 38, and 52 seats for the low, medium, and high logical classes, respectively. The experiment entailed fixing the remaining seats for the high logical class within a specific pre-sale period and dynamically adjusting the remaining seats for the medium and low logical classes. The optimal pricing decisions for various remaining seats in the medium and low classes are presented in Table 3 (partial pricing decision results are displayed due to space limitations).

Table 3: Optimal price decisions for medium and low logical classes with fixed seat quantity in high logical class

Remaining seats in low class	Remaining seats in medium class							
	18	19	20	21	22	23	24-30	31
32	(455,889)	(455,889)	(455,889)	**(455,889)**	(455,780)	(455,780)	**(455,780)**	**(370,780)**
33	**(455,889)**	**(455,889)**	**(455,889)**	**(455,889)**	**(455,780)**	**(455,780)**	**(455,780)**	**(370,780)**
34	(370,889)	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)

35	(370,889)	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)
36	(370,889)	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)
37	(370,889)	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)
38-48	(370,889)	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)
49	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)	(370,780)
50	(370,889)	(370,889)	**(370,889)**	(370,780)	(370,780)	(370,780)	(370,780)	(370,780)

In Table 3, the horizontal and vertical variations represent fluctuations in seat availability for the medium and low logical classes, respectively. The prices in parentheses correspond to the optimal pricing decisions for the low and medium logical classes. The findings suggest that in cases where there are an ample number of remaining seats for both classes, the pricing strategies favor the lowest prices within their designated price sets. As the number of remaining seats diminishes, the pricing decisions escalate accordingly. In the scenario where the remaining seat capacity for the low logical class falls below or equals 48, coupled with a reduction in the available seats for the medium logical class to 21, the pricing for the medium logical class experiences an increment from 780 yuan to 889 yuan. The abundance of seat availability in one cabin significantly influences the optimal pricing strategies for other cabins. For instance, with the remaining seats for the low logical class at 33 and an increase in seat availability for the medium logical class to 31, the pricing for the low logical class sees a reduction from 455 yuan to 370 yuan. This circumstance arises due to the relatively substantial total remaining seat capacity, which introduces the risk of unsold seats, necessitating a reduction in prices to stimulate passenger demand. Table 3 displays threshold lines denoted in bold, indicating pivotal points for price adjustments in both cabin classes. These threshold lines serve as essential reference markers guiding the timing and manner in which the airline should modify cabin fares.

In summary, the comprehensive analysis reveals that, on the one hand, as the flight departure date approaches and the total remaining seat capacity diminishes, pricing decisions for cabin classes demonstrate an increasing trend. This strategic adjustment is made in anticipation of selling the remaining seats at elevated prices. Alternatively, during the initial phase of pre-sale period characterized by ample remaining seat capacity, pricing decisions for cabin classes lean towards a lower price. The primary objective at this juncture is to boost overall seat sales volume and augment the expected revenue for the flight. Thus, the dynamic pricing adjustment framework delineated in Section 4 harmonizes with the predominant sales control strategies embraced by commercial airlines.

Finally, utilizing the flight data for the six months post-2019 extracted from the dataset in Section 5.1, a comparative analysis is conducted on the average revenue variance between the dynamic pricing adjustment method advocated in this paper and the empirical rule-based method utilized by airline revenue management practitioners. The comparative study encompasses four distinct scenarios: business line on regular days(A1),business line on holidays(A2), tourism line on regular days(B1), and tourism line on holidays(B2).The results, as demonstrated in Fig 2, illustrate that the dynamic pricing adjustment strategy advocated in this paper yields average revenue increases of 15.03%, 9.51%, 12.47%, and 11.86% across the corresponding scenarios over the six-month timeframe when compared to the existing control strategy employed by airlines.

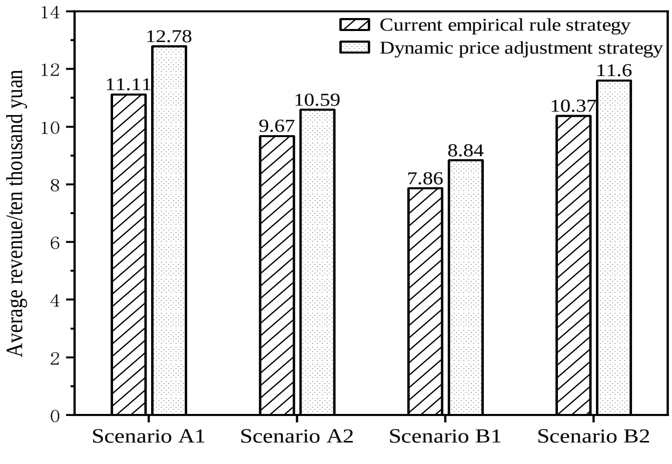

Fig 2: Comparison of average revenue for flights in four scenarios

VI. Conclusions

In response to the dynamic pricing requirements prevalent in airline industry practices, this study comprehensively quantifies competitive factors including flight timing, remaining capacity on the flight line, and airline brand. In addition to the conventional multinomial logit passenger choice model, this research integrates flight competition factors and underscores the importance of considering unobservable passengers who have arrived but have not yet made purchases. Establishing a non-homogeneous passenger choice model to characterize passenger cabin choice behavior. To mitigate challenges associated with lengthy computational times in model parameter estimation often encountered with commonly used EM and MCMC algorithms, a relatively novel two-step methodology is employed. Based on the estimation of passenger choice model parameters, this study further estimates passenger arrival rates in different stages of the pre-sale period. Subsequently, a dynamic price adjustment model is developed, which can be integrated into the current civil aviation reservation system. Empirical findings demonstrate that passengers display diverse choice behaviors and arrival patterns on lines of distinct characteristics and flight dates, influenced by factors including flight timing, seat availability, seat prices. These factors exert varying degrees of influence on the ultimate dynamic pricing decisions. The empirical findings further illustrate that the formulated dynamic pricing model, in contrast to airline operational practices grounded in empirical rules, yields a substantial average revenue enhancement. Taking into account the fluctuations in flight booking periods and seat availability, this study additionally presents rational price adjustment threshold boundaries to steer airline dynamic pricing strategies.

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