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Advancing Coherent Direction-of-Arrival Estimation in Coprime Sensor Arrays

Abstract: - Direction-of-Arrival (DOA) estimation plays a pivotal role in accurate spatial signal processing within sensor array systems. This paper introduces MUSICAP, a novel approach designed to advance DOA estimation specifically for coprime sensor arrays, focusing on coherent signal sources. MUSICAP utilizes an enhanced Capon beamformer tailored to amplify the detection capability of coherent signals originating from desired directions in coprime sensor array configurations. The customized beamformer generates highly focused sensor beams, refining the detection process significantly. Furthermore, a dedicated Beamformer module further enhances these sensor beams, aligning them closely with those produced by the modified Capon method. Through the integration of beams derived from both methodologies with the MUSIC algorithm, MUSICAP achieves precise DOA estimation, particularly effective for fully coherent signals. Simulation results illustrate the effectiveness of this approach in enhancing DOA estimation accuracy, surpassing conventional implementations of the MUSIC algorithm and other subspace methods in coprime sensor array contexts.

Keywords: Direction-of-Arrival (DOA) , MUSIC, Capon, MUSICAP, Coprime sensor array

I. INTRODUCTION

Direction-of-Arrival (DOA) estimation stands as a cornerstone in sensor array systems [1], crucial for accurately processing spatial signals across diverse applications such as radar, wireless communications, and array signal processing [2][3][4]. Precise DOA estimation facilitates pinpointing the spatial positions of signal sources, enabling targeted signal reception, interference mitigation, and overall system enhancement. However, accurately estimating DOA for coherent signals presents formidable challenges in sensor array setups. Coherent signals, characterized by overlapping frequencies and closely spaced sources, often introduce ambiguity, complicating the accurate differentiation of individual sources.

Conventional methods like MUSIC [2], ESPRIT [3][4], and other subspace techniques such as ESPRIT-Like [5], IESPRIT-Like [6], and MMUSIC [6a] encounter limitations when resolving fully coherent signals due to their reliance on spectral analysis or eigenstructure decomposition. These standalone approaches typically fail to provide sufficient resolution and struggle with distinguishing closely spaced sources.

Recent advancements address these challenges from various perspectives. Xiang et al. [7] propose a deep sparse prior technique utilizing deep learning for enhanced signal resolution in coherent DOA estimation. However, these methods often demand substantial computational resources and extensive datasets, limiting their real-time applicability. Zhang et al. [8] introduce atomic norm minimization for DOA estimation in coprime arrays, achieving improved resolution but facing challenges under noisy or sparse data conditions. Zheng et al. [9] present augmented covariance matrix reconstruction to enhance accuracy in spatially correlated signals, yet its sensitivity to noise and model mismatches affects practical robustness. Yao et al. [10] propose an online recursive least squares-based method addressing unknown source statistics but requiring high computational resources and facing tracking errors in dynamic environments. Li et al. [11] offer a joint DOA and range estimation method suitable for scenarios with varying source numbers but may suffer accuracy issues in noisy or closely spaced source environments. Fang et al. [12] introduce an improved sparse representation method for DOA estimation in complex noise scenarios, although sensitivity to varying noise characteristics remains a concern. Zhang et al. [13] propose a joint estimation approach for Direction of Departure (DOD) and DOA in coprime Multiple Input Multiple Output (MIMO) radar systems, achieving enhanced accuracy at the cost of increased computational complexity.

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In this study, we propose a novel approach integrating a modified Capon beamformer with the MUSIC algorithm to address these challenges effectively. The modified Capon beamformer enhances the detection of coherent signal sources by forming beams in desired directions, complementing the spectral resolution capabilities of the MUSIC algorithm. Leveraging the unique structure of coprime sensor arrays, the newly developed MUSICAP method strategically combines the strengths of both techniques, enabling improved discrimination and precise DOA estimation, particularly in scenarios involving highly coherent signals. This two-stage process overcomes the limitations of standalone methods, providing a robust solution to the challenge of fully coherent signal resolution.

II. ARRAY SIGNAL MODEL

Consider a coprime array comprising two subarrays: Subarray 1 consists of M sensors spaced at intervals N d, and Subarray 2 consists of N sensors spaced at intervals Md. Assume M and N are coprime positive integers, with d representing the minimum unit inter-sensor spacing as shown in Fig.1. Assuming $M < N$, align the rightmost sensor of both subarrays as the reference location. The sensor locations in the coprime array can be represented as:

$$S = \{mNd - M(N-1)d \mid 0 \le m \le M-1\} \cup \{nMd - M(N-1)d \mid 0 \le n \le N-1\} \quad (1)$$

Hence, the total number of sensors is $|S| = M+N-1$. Define $l = [-l_{|S|}, -l_{|S|-1}, \ldots, -l_1]^T$ as the sensor locations with $-l_1 = 0$, where $-l_i \in S$, $i = 1, 2, \ldots, |S|$.

Suppose there are K far-field narrowband coherent signals arriving from distinct directions $\theta = [\theta_1, \theta_2, \ldots, \theta_K]^T$.
Consequently, the received data vector by the coprime array at time t is modeled as:

$$\begin{aligned} \mathbf{x}(t) &= s(t)\sum_{k=1}^{K} \alpha_S a_s(\theta_k) + \mathbf{n}_s(t) \\ &= s(t)\mathbf{A}s\alpha + \mathbf{n}_s(t) \end{aligned} \quad (2)$$

Here, $a_S(\theta_k) = [e^{-l_{|S|}\bar{\theta}_k}, e^{-l_{|S|-1}\bar{\theta}_k}, \ldots, e^{-l_1\bar{\theta}_k}]^T$ represents the steering vector with the normalized DOA $\bar{\theta}_k = j2\pi \sin\theta_k/\lambda$, where λ denotes the signal wavelength. $\mathbf{A} = [a_S(\theta_1), a_S(\theta_2), \ldots, a_S(\theta_K)]$ is the manifold matrix. $s(t)$ stands for the reference signal waveform, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]^T$ represents the nonzero complex-valued fading coefficient vector, and $\mathbf{n}_S(t)$ is a random additive white noise vector following a complex Gaussian distribution $\boldsymbol{C}_N(0, \sigma^2_n \boldsymbol{I})$, uncorrelated to signals, with σ^2_n representing the noise power

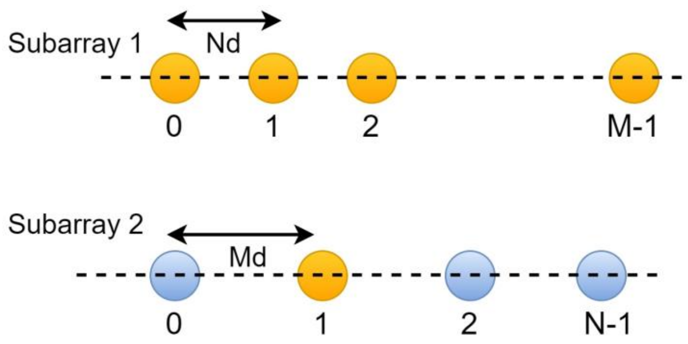

Fig. 1: Co-prime Sensor array.

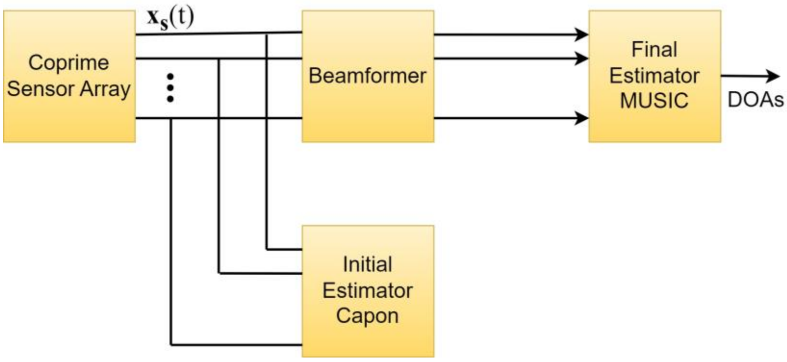

Fig. 2: Block diagram of the proposed method

III. A NEW MUSICAP METHOD FOR RESOLUTION OF FULLY COHERENT SIGNALS

In Stage 1 of this approach, we introduce the modified Capon beamformer, designed to enhance DOA estimation by prioritizing coherent signal sources through the creation of highly focused beams. This beamforming technique aims to improve the signal-to-noise ratio and spatial resolution, crucial for accurately identifying the directions of incoming signals in sensor array systems.

Moving to Stage 2, our methodology integrates the focused beams generated by the modified Capon beamformer with the outputs from the beamforming process using the MUSIC algorithm. This integration forms the core of the MUSICAP method, harnessing the complementary strengths of both techniques. By combining the spatial focusing capabilities of the modified Capon beamformer with the spectral analysis power of the MUSIC algorithm, MUSICAP achieves enhanced resolution and robustness in resolving coherent sources.

Named after its constituent techniques, MUSICAP proves particularly advantageous in wireless communication scenarios where precise DOA estimation is critical for mitigating interference, optimizing signal reception, and enhancing overall system performance. This integrated approach not only improves the accuracy of DOA estimation but also extends its applicability to environments characterized by closely spaced or overlapping signal sources. MUSICAP represents a novel and effective methodology that leverages advanced beamforming and spectral analysis techniques to address the challenges posed by coherent signal sources in sensor array systems, offering significant improvements in resolution and reliability for various wireless communication applications.

Stage 1:

Stage 1 of the MUSICAP method initiates with the deployment of the modified Capon beamformer, specifically crafted to tackle the intricate challenge of DOA estimation for coherent signals within coprime arrays. This beamformer assumes a pivotal role by leveraging its distinctive capabilities to generate focused beams aimed at coherent signals in predefined directions. Unlike conventional beamformers, the modified Capon beamformer excels in enhancing resolution by precisely steering beams towards coherent sources, thereby accentuating their presence amidst received signals. Through this focused beam formation aligned with the directions of coherent signals, the modified Capon beamformer sets the stage for subsequent phases to concentrate on these targeted directions. To proceed, we compute the sample covariance matrix **R** following the formulation provided in Equation (2):

$$\mathbf{R} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_{\mathrm{s}}(t)\mathbf{x}_{\mathrm{s}}^{\mathbf{H}}(t) \qquad (3)$$

Where T represents the snapshot index.

The main aim of this algorithm is to reduce the power received from incoming signals in all directions, while ensuring that there is no attenuation in the "look direction." The algorithm adheres to the following constraint:

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}_s(\theta)}{\mathbf{a}_s^H(\theta)\mathbf{R}^{-1}\mathbf{a}_s(\theta)} \tag{4}$$

Here, **w** represents the weight that requires determination. The optimal weight is derived using the Lagrange optimization method, as detailed in [15] [16].

The steering vector for the modified Capon technique is formulated as follows:

$$\mathbf{A} = [g(\theta_1)\mathbf{a}_s(\theta_1)\ g(\theta_2)\mathbf{a}_s(\theta_2)\ .\ .\ .\ g(\theta_{M+N-1})\mathbf{a}_S(\theta_{M+N-1})^T \tag{5}$$

In the context of the modified Capon technique, the steering vector is defined considering the array gain in a specific direction θ_i, denoted as g(θ_i) . For coprime sensor arrays, the array gain g(θ_i) equals 1 for $i = 1, 2, \ldots, M+N-1$). Therefore, the values of a(θ_i) are significantly influenced by g(θ_i) which is derived from the array gain pattern. Since the gain pattern directly affects the array response in the "look direction," the constraint of the modified Capon algorithm can be expressed as follows:

$$\mathbf{w} = \min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}\mathbf{w} \quad subject\,to\ \mathbf{w}^H\mathbf{a}_s(\theta) = \mathbf{g}(\theta) \tag{6}$$

The Lagrange multiplier, denoted as λ, and the weight vector www, derived from applying the Lagrange optimization approach to the constraint described in Equation (8), are expressed as follows:

$$\lambda = \frac{\mathbf{g}^H(\theta)}{\mathbf{a}_s^H(\theta)\mathbf{R}^{-1}\mathbf{a}_s(\theta)} \tag{7}$$

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}_s(\theta)\mathbf{g}^H(\theta)}{\mathbf{a}_s^H(\theta)\mathbf{R}^{-1}\mathbf{a}_s(\theta)} \tag{8}$$

The power spectrum of the modified Capon algorithm is given by

$$P_{MCap} = \frac{\mathbf{a}_s(\theta)\mathbf{a}_s^H(\theta)}{\mathbf{a}_s^H(\theta)\mathbf{R}^{-1}\mathbf{a}_s(\theta)} \tag{9}$$

Stage 2:

In Stage 2 of the process, the beams generated by the modified Capon beamformer and the subsequent beamformer section are combined with the MUSIC algorithm to achieve accurate Direction-of-Arrival (DOA) estimation. The MUSIC algorithm further refines these beams, leveraging spectral properties to enhance the precision of DOA estimates. This multi-stage approach effectively resolves the complexities posed by fully coherent signals, a task that standalone methods often struggle to accomplish. The steps involved in Stage 2 are outlined as follows:

Covariance Matrix Calculation:

$$\mathbf{R} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_s(t)\mathbf{x}_s^{\mathbf{H}}(t) \tag{10}$$

where $\mathbf{x}_S$(t) represents the received signal vector at time t, according to the coprime array model.

Eigenvalue Decomposition:

$$\mathbf{R} = \mathbf{U\Lambda U}^H \tag{11}$$

In the context where **U** represents the matrix composed of the eigenvectors of **R**, and **Λ** denotes a diagonal matrix containing the eigenvalues of **R.**

Null Space Calculation: $\mathbf{Q}_N = [\mathbf{u}_{L+1}, \mathbf{u}_{L+2}, \ldots, \mathbf{u}_N]$ (12)

where, $[\mathbf{u}_{L+1}, \mathbf{u}_{L+2}, \ldots, \mathbf{u}_N]$ are the eigenvectors associated with the smallest eigenvalues, representing the noise subspace of the coprime array.

MUSIC Spectrum Calculation:

$$P_{MU} = \frac{1}{\left|\mathbf{a}_{\mathbf{s}}^{H}(\theta)\mathbf{Q}_N\mathbf{Q}_N^{H}\mathbf{a}_{\mathbf{s}}(\theta)\right|} \quad (13)$$

where $\mathbf{a}_S(\theta)$ denotes the steering vector corresponding to the direction θ in the coprime array model.

Peak Detection: Detect peaks in P_{MU} to identify the DOAs.

The MUSICAP method provides significant benefits for estimating the direction of arrival (DOA) of coherent signals in coprime arrays:

1. It combines the Capon beamformer and MUSIC algorithm to achieve high-resolution DOA estimation.
2. It effectively resolves fully coherent signals, surpassing individual techniques.
3. It adapts well to unknown numbers of sources, ensuring robust DOA estimation.
4. It discriminates closely spaced coherent sources, thereby enhancing accuracy.

IV. RESULTS AND DISCUSSION

In this study, we design a coprime array system using the coprime numbers $M = 3$ and $N = 5$. This configuration results in a coprime array consisting of a total of S = ($M+N-1$) = 7 physical sensors. These sensors are strategically placed at specific intervals: [0, 3d, 5d, 6d, 9d, 10d, 12d], where d represents the inter-sensor spacing. To evaluate the performance of our proposed coprime array system, we compare it with several existing methods including ESPRIT-Like [5], improved ESPRIT-Like (IESPRIT-Like) [6], modified MUSIC (MMUSIC) [7], and Cramér–Rao bound (CRB) [17]. Each comparison method is simulated using the coprime array to ensure a fair assessment of their respective capabilities in direction of arrival (DOA) estimation.

The root mean square error (RMSE) of the estimated DOAs is employed as the metric to quantify the performance of each method. RMSE is defined as

$$RMSE = \sqrt{\frac{1}{K \cdot M}\sum_{i=1}^{K}\sum_{j=1}^{M}\left(\hat{\theta}_i^{(j)} - \theta_i\right)^2} \quad (14)$$

Here, $\hat{\theta}_i^{(j)}$ represents the estimated direction of arrival (DOA) for the i-th source in the j-th instance of the Monte Carlo trial, where θ_i denotes the true DOA. The evaluation includes K sources and is conducted over M = 500 Monte Carlo iterations, providing a thorough analysis of the accuracy in estimating DOAs across various scenarios and conditions. This rigorous approach ensures robustness and reliability in assessing the performance of the DOA estimation methods under consideration.

Consider a scenario involving five distinct coherent signals originating from specific directions: $\theta_1 = 0°$, $\theta_2 = 20°$, $\theta_3 = 10°$, $\theta_4 = 30°$, and $\theta_5 = 50°$. In Figure 3, the root mean square error (RMSE) plotted against signal-to-noise ratio (SNR) spans from -10 dB to 40 dB, with the number of snapshots fixed at 400. The ESPRIT-Like technique exhibits poor performance due to its reliance on incomplete data extracted from the covariance matrix for direction of arrival (DOA) estimation. In contrast, our proposed method consistently outperforms other methods across the entire SNR spectrum. It demonstrates remarkable stability, maintaining low RMSE values even amidst sudden drops observed in the RMSE of other algorithms.

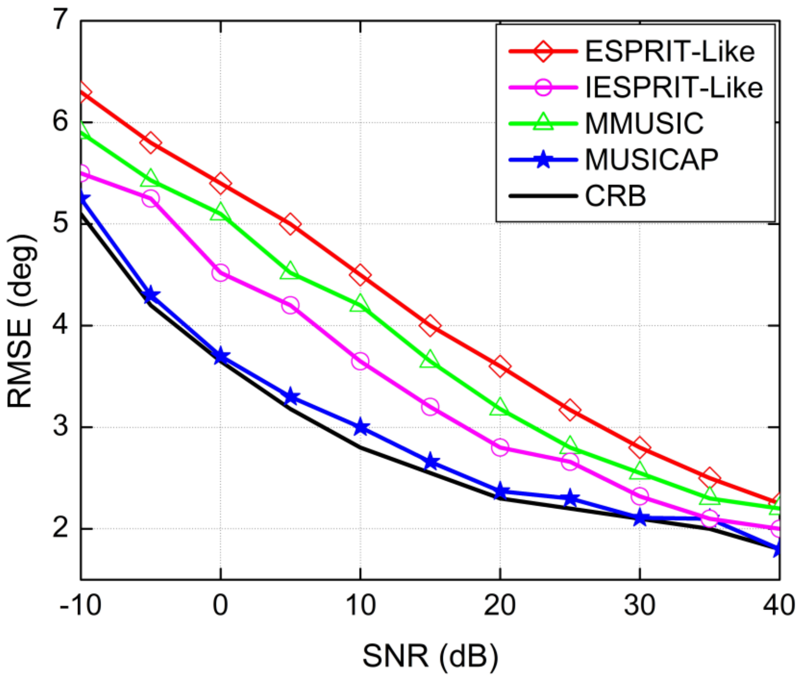

Fig. 3: Comparing RMSE performance at various SNR.

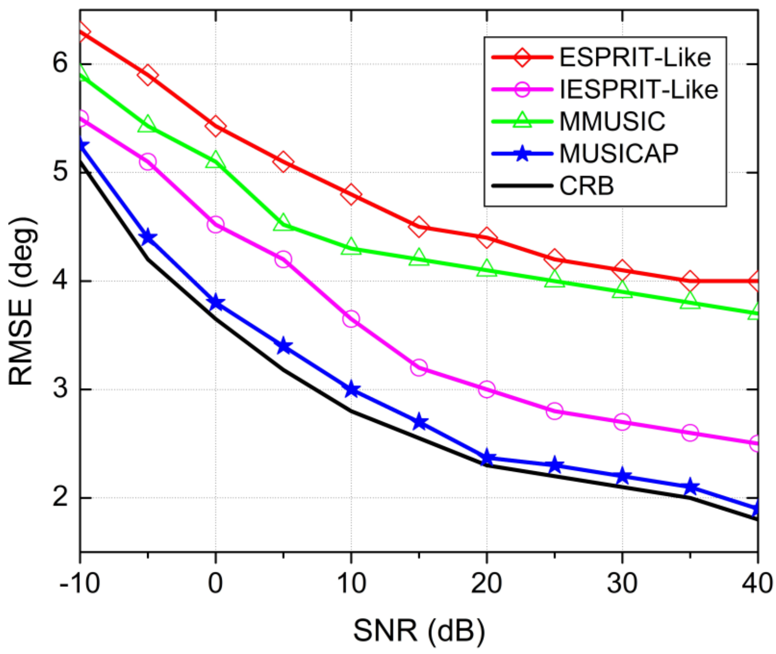

Fig. 4: Comparing RMSE performance at various SNR for closely spaced coherent targets.

Additionally, let's consider another scenario where five closely located coherent sources originate from nearby directions: $\theta_1 = 0°$, $\theta_2 = 5°$, $\theta_3 = 8°$, $\theta_4 = 10°$, and $\theta_5 = 12°$. In Figure 4, while keeping other parameters constant, the RMSE performance is depicted. The proposed MUSICAP method shows superior performance compared to previous approaches, exhibiting significantly improved RMSE behavior. Therefore, for scenarios with closely located coherent sources, our proposed method demonstrates notably enhanced performance compared to alternative methods.

In Figure 5, the relationship between root mean square error (RMSE) and the number of snapshots, ranging from 10 to 500 with a fixed signal-to-noise ratio (SNR) of 15 dB, is illustrated. It clearly demonstrates that our proposed algorithm achieves superior performance under these conditions. Now, consider a scenario involving two coherent signals with angular separations $\theta_1 = 0° - \Delta\theta/2$ and $\theta_2 = 0° + \Delta\theta/2$, where $\Delta\theta$ varies between 1° and 10°. In Figure 6, the RMSE plotted against angular separation is depicted, with a fixed SNR of 10 dB and

300 snapshots. Remarkably, the RMSE of our proposed algorithm consistently outperforms that of other algorithms across this range of angular separations. This highlights the robustness and effectiveness of our method in accurately resolving closely spaced coherent signals, even under challenging conditions of varying angular separations.

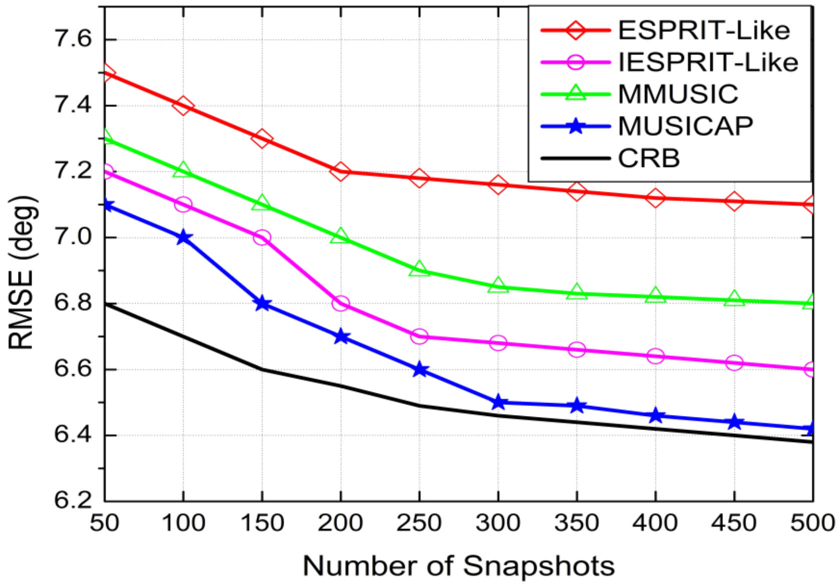

Fig. 5: Comparing RMSE performance at various Snapshots for closely spaced coherent targets.

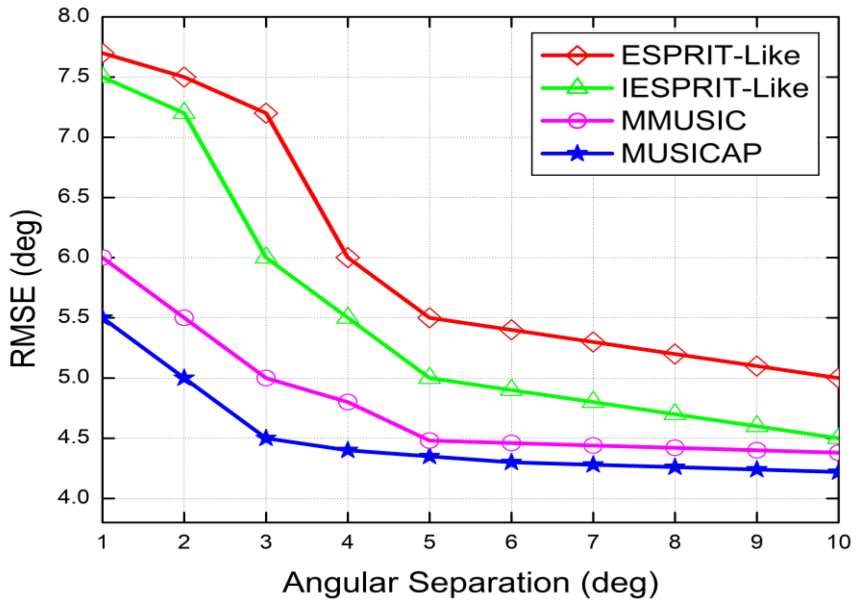

Fig. 6: Comparing RMSE performance at various Angular Separation for closely spaced coherent target.

V. CONCLUSION

In conclusion, MUSICAP represents a significant advancement in the field of DOA estimation within coprime sensor arrays. By leveraging a modified Capon beamformer and integrating outputs via the MUSIC algorithm, MUSICAP excels in resolving fully coherent signals, effectively overcoming the limitations of single-method approaches in wireless communication. Comparative evaluations against established methods such as CRB, MMUSIC, IESPRIT-Like, and ESPRIT-Like across diverse scenarios consistently demonstrate MUSICAP's superior performance in accurately pinpointing coherent signals. This superiority is particularly pronounced in scenarios featuring highly coherent sources and closely spaced signal sources, where MUSICAP achieves enhanced accuracy and stability, surpassing existing techniques. Moreover, its robust performance across varying snapshot numbers and angular separations underscores its versatility and reliability in real-world applications. The findings illustrated in Figures 3, 4, 5, and 6 highlight MUSICAP's efficiency and effectiveness in practical DOA estimation, confirming its potential as a comprehensive solution for resolving coherent signals in complex sensor array systems.

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