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Utilizing the Advancement of Kasaj Topological Spaces for Engineering Breakthroughs

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Abstract: - The primary aim of this article is to present and explore the properties of Kasaj pre-neighborhoods, Kasaj pre-interior, Kasaj pre-closure, and Kasaj pre-limit points and later on to apply the idea in the various engineering domain. We derive significant results by leveraging the concepts of Kasaj pre-open sets, Kasaj pre-closed sets, and Kasaj pre-derived sets. This study delves into the properties and characterizations of pre-open and pre-closed sets within the framework of Kasaj topological space. Furthermore, we examine the interrelationships between the characterizations of Kasaj pre-open sets, Kasaj pre-closed sets, and Kasaj pre-derived sets.

Keywords: Kasaj topological space, Kasaj pre-neighborhoods, Kasaj pre-interior, Kasaj pre-closure, Kasaj pre-limit points.

I. INTRODUCTION

[a] Topological spaces plays significance role in Control Theory, Robotics and Path Planning, Network Theory, Signal Processing, Optimization Problems, Material Science, Quantum Computing and Electrical Engineering and serves many applications related to the mentioned various engineering fields.

[b] Levine's introduction of generalized closed sets in 1970 laid the groundwork for significant advancements in topology. Building on this foundation, Lellis Thivagar expanded the theoretical landscape by introducing nano topology. This approach employs approximations and boundary regions of a subset within a universe, defined via an equivalence relation, to characterize nano-closed sets, nano-interior, and nano-closure.

Subsequent explorations by numerous scholars have delved into weaker forms of nano open sets, including nanoα-open sets, nano semi-open sets, nano pre-open sets, and nano β-open sets, thereby enriching and adding complexity to the existing theoretical framework. In 2019, Chandrasekar pioneered the concept of micro topology, an extension of nano topology, emphasizing micro pre-open and semi-open sets. Later, Chandrasekar and Swathi introduced micro α-open sets. In 2018, SathishmohanP. and colleagues introduced the concept of nano pre-limit points, marking another significant contribution. Continuing this trajectory of innovation, K. G. Rachchh and S. I. Ghanchi introduced Kasaj topology in 2020, a further extension of micro topology. They also identified new categories of weakly open sets, including Kasaj-pre-open sets, Kasaj-semi-open sets, and Kasaj-regular-open sets, exploring their properties and interrelationships (See [8], [9], and [10]). Recently, the exploration of Kasaj topology has been further advanced with the introduction of Kasaj-Alpha-open sets and Kasaj-Beta-open sets, adding to the robust body of work in this evolving field.

II. LITERATURE REVIEW

The purpose of this paper is to introduce and examine the properties of Kasaj pre-neighborhoods, Kasaj pre-interior, Kasaj pre-closure, and Kasaj pre-limit points by leveraging the framework of Kasaj pre-open sets and Kasaj pre-derived sets.

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Definition 1.1 [3]: Let $(U,\Re)$ denote an approximation space, where $\chi \subseteq U$. Define the collection $\Im_{\Re}(x)$ as $\{x, \emptyset, \mathcal{J}_{\Re}(x), \mathcal{U}_{\Re}(x), \mathcal{B}_{\Re}(x)\}$, and assume it satisfies the following axioms:

1. U and $\emptyset$ are elements of $\Im_{\Re}(x)$.

2. The union of any subcollection of elements within $\Im_{\Re}(x)$ is also an element of $\Im_{\Re}(x)$.

3. The intersection of any finite subcollection of elements within $\Im_{\Re}(x)$ is also an element of $\Im_{\Re}(x)$.

Under these conditions, the pair $(U,\Im_{\Re}(x))$ is defined as *a nano topological space*. The elements of $\Im_{\Re}(x)$ are referred to as *nano open sets*.

Definition 1.2 [8]: Let $(U,\Im_{\Re}(x))$ be a nano topological space. The *Kasaj topology*, denoted as $KS_{\Re}(x)$, is defined by the set:

$KS_{\Re}(x) = \{(K\cap S) \cup (K'\cap S'): K, K' \in \Im_{\Re}(x), \text{ fixed } S, S' \notin \Im_{\Re}(x), S \cup S' = U\}$

Here, K and K' are elements of $\Im_{\Re}(x)$, while S and S' are fixed elements not belonging to $\Im_{\Re}(x)$, such that their union covers the entire set U.

Definition 1.3 [8]: The Kasaj topology $KS_{\Re}(x)$ adheres to the following axioms:

1. The sets U and $\emptyset$ are elements of $KS_{\Re}(x)$.

2. The union of any subcollection of elements in $KS_{\Re}(x)$ remains in $KS_{\Re}(x)$.

3. The intersection of any finite subcollection of elements in $KS_{\Re}(x)$ remains in $KS_{\Re}(x)$.

Given these properties, the structure $(U, \Im_{\Re}(x), KS_{\Re}(x))$ is defined as *a Kasaj topological space*. The elements of $KS_{\Re}(x)$ are referred to as *Kasaj-open (KS-open) sets*. The complement of a Kasaj-open set is known as *a Kasaj-closed (KS-closed) set*, and the collection of such complements is denoted by KSCL(x).

Definition 1.4[8] *The Kasaj closure* and *Kasaj interior* of a set $\wp$ are denoted by $KS_{cl}(\wp)$ and $KS_{int}(\wp)$, respectively. They are defined by

$KS_{cl}(\wp) = \cap\{\mathcal{O}: \wp \subseteq \mathcal{O}, \mathcal{O} \text{ is KS - closed}\}$

$KS_{int}(\wp) = \cup\{\mathcal{O}: \mathcal{O} \subseteq \wp, \mathcal{O} \text{ is KS - open}\}$.

Remark 1.5[8]

1. $KS_{int}(\wp)$ is the largest KS-open set contained in $\wp$.
2. $KS_{cl}(\wp)$ is the smallest KS-closed set containing $\wp$.

Definition 1.6 [8] Consider a Kasaj topological space $(U, \Im_{\Re}(x), KS_{\Re}(x))$, where $A \subseteq U$. The set A is termed *Kasaj pre-open*[8] if $A \subseteq KS_{cl}(KS_{int}(A))$. Here, $KS_{cl}(A)$ represents the KS-closure of A, and $KS_{int}(A)$ denotes the Kasaj interior of a set A. The complement of a Kasaj pre-open set is referred to as a *Kasaj pre-closed set* and is symbolized by $KSCL(U, \chi)$ or KSCL(x).

Definition 1.7 A point $x \in U$ is defined as *a limit point of a set A* if every neighborhood of x contains at least one point of A distinct from x itself.

Definition 1.8 The collection of all limit points of a set A is known as *the derived set of A*, denoted by D(A).

III. NEW DEVELOPMENT IN KASAJ TOPOLOGICAL SPACES

In this article, we conduct a comprehensive study of Kasaj pre-neighborhoods. Initially, we explore various neighborhoods associated with pre-open and pre-closed sets within the context of Kasaj topological space. Utilizing the concept of Kasaj pre-open sets, we then investigate the properties of the Kasaj pre-interior and analyze its relationship with the Kasaj-interior. Similarly, we examine the Kasaj pre-closure in relation to the

Kasaj pre-closed sets, establishing the connection between the Kasaj-closure and the Kasaj pre-closure. Additionally, we delve into the properties of Kasaj pre-limit points, leveraging the framework of Kasaj pre-derived sets to elucidate their characteristics.

(A) Kasaj pre-neighbourhoods :

In this section, we introduce and analyze the concepts of Kasaj pre-neighborhoods, Kasaj pre-interior, and Kasaj pre-closure within the framework of Kasaj topological spaces. Additionally, we derive and examine some fundamental properties associated with these notions.

Definition 2.1 A subset $P_x \subseteq U$ is termed *a Kasaj pre-neighborhood of a point $x \in U$* if there exists a set $A \in$ KSPO(Ա, χ) such that $x \in A \subseteq P_x$. Consequently, a point x is referred to as a Kasaj pre-neighborhood point of the set A.

Definition 2.2 The collection of all Kasaj pre-neighborhoods of a point x of the set U is referred to as *the Kasaj pre-neighborhood family of x,* and it is denoted by KSP-nbd(x).

Example 2.3 Let Ա = {ϒ, Ω, Ψ, Φ, Γ} with Ա/ℜ = {{ϒ}, {Ω, Ψ, Φ }, {Γ}} and χ = {Φ, Γ} ⊆Ա. Then $\Im_{\Re}$(χ) = {∅, Ա, {Γ}, {Ω, Ψ, Φ}, {Ω, Ψ, Φ, Γ}}. If we consider S = {ϒ, Ω, Γ} and S' = {Ψ, Φ}, then

- $KS_{\Re}$(x) = {∅, { Ω }, { Γ }, {Ψ, Φ}, {Ω, Γ}, {ϒ, Ω, Γ}, {Ψ, Φ, Γ}, {Ω, Ψ, Φ}, {Ω, Ψ, Φ, Γ}, Ա}.
- KSPO(Ա, χ) = {∅, { Ω }, { Γ }, {Ψ}, {Φ}, {Ψ, Φ}, {Ω, Γ}, {Ω, Ψ}, {Ω, Φ}, {Ψ, Γ}, {Φ, Γ}, {Ω, Ψ, Γ},{Ω, Φ, Γ}, {ϒ, Ω, Γ}, {Ψ, Φ, Γ}, {Ω, Ψ, Φ}, {ϒ, Ω, Ψ, Γ}, {ϒ, Ω, Φ, Γ} {Ω, Ψ, Φ, Γ}, Ա }
- KSP-nbd(ϒ) = { Ա, {ϒ,Ω,Γ}, {ϒ,Ω,Ψ,Γ}, {ϒ,Ω,Φ,Γ }}

KSP-nbd(Ω) = {A ∈ P(Ա) : Ω ∈ A}

KSP-nbd(Ψ) = {A ∈ P(Ա) : Ψ ∈ A}

KSP-nbd(Φ) = {A ∈ P(Ա) : Φ ∈ A}

KSP-nbd(Γ) = {A ∈ P(Ա) : Γ ∈ A}

Lemma 2.4 [Theorem 4.9(1), 8] If $P_\alpha \in$KSPO(Ա, χ), then $\cup\{P_{\alpha\,:\,\alpha \in I}\}\in$KSPO(Ա, χ), where I is an index set.

Definition 2.5 The *Kasaj-pre-interior of a set A,* denoted by $KSP_{int}(A)$, is defined as the union of all Kasaj pre-open sets that are subsets of A.

Example 2.6 In Example 2.3, if we take A = {ϒ}, then $KSP_{int}(A) = KSP_{int}(\{ϒ\}) = \emptyset$.

Lemma 2.7 [Theorem 4.9(2), 8] If $P_\alpha \in$KSPCL(Ա, χ), then $\cap_{\alpha \in I} P_\alpha \in$KSPCL(Ա, χ), where I is an index set.

Definition 2.8 The *Kasaj-pre-closure of a set A,* denoted by $KSP_{cl}(A)$, is defined as the intersection of all Kasaj pre-closed sets that are subsets of A.

Lemma 2.9 Let A,B ⊆ U. Then the following statements are hold :

(a) $A \subseteq KSP_{cl}(A)$.

(b) If $A \subseteq B$, then $KSP_{cl}(A) \subseteq KSP_{cl}(B)$.

(c) $KSP_{cl}(KSP_{cl}(A)) = KSP_{cl}(A)$.

(d) $KSP_{cl}(A) \in$ KSPCL(Ա, χ).

Lemma 2.10 Let A ⊆ U. Then the following statements are hold :

(a) $KSP_{cl}(A^c) \subseteq [KSP_{int}(A)]^c$.

(b) $KSP_{int}(A^c) \subseteq [KSP_{cl}(A)]^c$.

Theorem 2.11 A subset A of a space Ʉ is considered Kasaj-pre-open (i.e., an element of KSPO(Ʉ, χ)) if and only if A serves as a Kasaj pre-neighborhood for each point in A.

Proof: Let A ∈ KSPO(Ʉ, χ). Then by the definition, it is obviously clear that A is a Kasaj pre-neighborhood of each of its points, as x ∈ A ⊆A for all x ∈A and A ∈ KSPO(Ʉ, χ).

Conversely, assume that A is a Kasaj pre-neighborhood for each point in A. Then for each x ∈A, there exists a P_x ∈ KSPO(Ʉ, χ) such that P_x⊆ A. Then we can take A = ∪{ P_x : x ∈A }. Since each P_x ∈ KSPO(Ʉ, χ), it follows that A ∈ KSPO(Ʉ, χ) by [Theorem 4.9(1), 8].

Lemma 2.12 A subset A of a space Ʉ. A point x ∈Ʉ is in the Kasaj pre-interior of A if and only if there is a P∈KSPO(Ʉ, χ) such that P⊆ A.

Proof: Let x∈ KSP_{int}(A). Then by the definition of Kasaj pre-interior, there exists P∈KSPO(Ʉ, χ) such that x∈ P and P⊆ A. Therefore there is P∈KSPO(Ʉ, χ) such that P⊆ A.

Conversely, assume that P∈KSPO(Ʉ, χ) such that P⊆ A. Then x∈ P ⊆KSP_{int}(A). Therefore x∈ KSP_{int}(A).

Definition 2.13 A point x∈ Ʉ is called a *Kasaj pre-interior point of A⊆Ʉ*, if x∈ KSP_{int}(A).

Remark 2.14

(a) Let Ʉ be a universal space and A⊆Ʉ and x∈ Ʉ. Then x is a Kasaj pre-interior point of A if and only if A is a Kasaj pre-neighborhood of x.

(b) Since $KS_{\mathfrak{R}}$(x)⊆ KSPO(Ʉ, χ), it follows that KS_{int}(A) ⊆KSP_{int}(A).

Theorem 2.15 Let Ʉ be a universal space and A⊆Ʉ. Then KSP_{int}(A) ∈KSPO(Ʉ, χ) is the largest Kasaj pre-open subset of Ʉ contained in A.

Proof: To establish that KSP_{int}(A) ∈ KSPO(Ʉ, χ) is the largest Kasaj pre-open subset of Ʉ contained in A, we must demonstrate that there is no P ∈KSPO(Ʉ, χ) for which KSP_{int}(A) ⊆ P ⊆ A. Assume, for contradiction, that such a subset P ∈ KSPO(Ʉ, χ) exists such that KSP_{int}(A) ⊆ P ⊆ A. Consider any point x ∈ P. According to Remark 2.15, since x ∈ P ⊆ A, it follows that A serves as a Kasajppre-neighborhood for x. This implies that x is a Kasaj pre-interior point of A. By Lemma 2.13, it follows that x ∈KSP_{int}(A) since x ∈Ʉ implies x ∈KSP_{int}(A). Consequently, we conclude that P = KSP_{int}(A), and thus, KSP_{int}(A)∈KSPO(Ʉ, χ).Therefore, KSP_{int}(A) is the largest Kasaj pre-open set contained in A.

Theorem 2.16 In a Kasaj topological space (Ʉ, $\mathfrak{I}_{\mathfrak{R}}$(x), $KS_{\mathfrak{R}}$(x)), A subset A ⊆Ʉ is a Kasaj pre-open if and only if A = KSP_{int}(A).

Proof: Let A = KSP_{int}(A). Since KSP_{int}(A) is a Kasaj pre-open set by hypothesis, A is also Kasaj pre-open. Assume A is Kasaj pre-open. Therefore, A is a Kasaj pre-open set contained in A. According to Theorem 3.2, KSP_{int}(A) is the largest Kasaj pre-open set contained in A. Consequently, A ⊆KSP_{int}(A). However, KSP_{int}(A)⊆ A always holds. Hence, we conclude that A = KSP_{int}(A).

Theorem 2.17 If A ⊆ B, then KSP_{int}(A) ⊆ KSP_{int}(B).

Proof: By applying the similar arguments as we applied in above theorem we have the proof.

Remark 2.18 In example 2.3, we can see that KSP_{int}({ϒ, Φ}) = ∅ = KSP_{int}({ϒ, Ψ}).

This shows that the converse of above theorem need not true in general.

Lemma 2.19 Let A,B ⊆Ʉ. Then

(a) KSP_{int}(A) ∪ KSP_{int}(B)⊆KSP_{int}(A ∪B).

(b) KSP_{int}(A ∩B)⊆KSP_{int}(A) ∩ KSP_{int}(B).

Proof: It follows by Lemma 3.4.

Lemma 2.20 Let A,B ⊆Ʉ. Then

(a) $KSP_{cl}(A) \cup KSP_{cl}(B) \subseteq KSP_{cl}(A \cup B)$.

(b) $KSP_{cl}(A \cap B) \subseteq KSP_{cl}(A) \cap KSP_{cl}(B)$.

Proof:

(a) Since A ⊆A∪B and B ⊆A∪B. Then by Lemma 2.9(2), we obtain that $KSP_{cl}(A) \subseteq KSP_{cl}(A\cup B)$ and $KSP_{cl}(B) \subseteq KSP_{cl}(A\cup B)$. It follows that $KSP_{cl}(A) \cup KSP_{cl}(B) \subseteq KSP_{cl}(A\cup B)$.

(b) Since $KSP_{cl}(A \cap B) \subseteq KSP_{cl}(A)$ and $KSP_{cl}(A \cap B) \subseteq KSP_{cl}(B)$ by Lemma 2.9(2). Therefore it follows that $KSP_{cl}(A \cap B) \subseteq KSP_{cl}(A) \cap KSP_{cl}(B)$

(B). Kasaj pre-limit point:

In this section, we introduce the concept of the Kasaj pre-limit point for a subset of an approximation space Ʉ and derive several foundational properties associated with it.

Definition 3.1 A point x ∈Ʉ is defined as a *Kasaj pre-limit point of A* if for every P ∈ KSPO(Ʉ, χ), there exists P ∩ (A – {x}) ≠∅. The collection of all Kasaj pre-limit points of A is termed the *Kasaj pre-derived set of A*, denoted as $KSP_d(A)$.

Remark 3.2 Given that $KS_{\mathfrak{R}}(x) \subseteq$ KSPO(Ʉ, χ) as established by Lemma [Theorem 4.2, 8], it logically follows that every Kasaj pre-limit point of A is indeed a Kasaj limit point of A. Consequently, $KSP_d(A) \subseteq KS_d(A)$, where $KS_d(A)$ denotes the Kasaj derived set of A. However, it should be noted that this inclusion does not hold in general.

Lemma 3.3 Let A ⊆Ʉ. Then A ∈ KSPCL(Ʉ, χ) if and only if it contains the set of its Kasaj pre-limit points.

Proof: Consider a set A ∈ KSPCL(Ʉ, χ). By the definition of a Kasaj pre-closed set, the complement A^c is an element of KSPO(Ʉ, χ). Hence, we can state that A ∈ KSPCL(Ʉ, χ) if and only if every point in A^c possesses a Kasaj pre-neighbourhood that is entirely contained within A^c. Equivalently, this means that no point in A^c is a Kasaj pre-limit point of A. Therefore, A contains all its Kasaj pre-limit points.

Lemma 3.4 Let A, B⊆Ʉ with A⊆B. Then $KSP_d(A) \subseteq KSP_d(B)$.

Proof: Let x ∈Ʉ be a Kasaj pre-limit point of a set A. By the definition of a Kasaj pre-limit point, there exists a set P ∈ KSPO(Ʉ, χ) such that P ∩ (A\{x}) ≠∅. This implies that P ∩ (B\{x}) ≠∅, thereby indicating that x is also a Kasaj pre-limit point of B. Consequently, the Kasaj pre-derived set of A is a subset of or equal to the Kasaj pre-derived set of B, i.e., $KSP_d(A) \subseteq KSP_d(B)$.

Theorem 3.5 Let A, B⊆Ʉ. Then we have the following properties :

(i) $KSP_d(\emptyset) = \emptyset$.

(ii) If $x \in KSP_d(A)$, then $x \in KSP_d(A\setminus\{x\})$.

(iii) $KSP_d(A) \cup KSP_d(B) \subseteq KSP_d(A \cup B)$.

(iv) $KSP_d(A \cap B) \subseteq KSP_d(A) \cap KSP_d(B)$.

Proof:

(i) It is clearly apparent.

(ii) Consider a point $x \in KSP_d(A)$. By definition, this implies that x is a Kasaj pre-limit point of A. i.e., every Kasaj pre-neighbourhood of x contains at least one point of A distinct from x. Given this, it follows that each Kasaj pre-neighbourhood of x must include at least one point of A\{x} other than x itself. Consequently, we deduce that $x \in KSP_d(A\setminus\{x\})$.

(iii) It follows by Lemma 3.4.

(iv) It follows by Lemma 3.4.

Lemma 3.6 Let Ꮜ be a space and B⊆Ꮜ. Then $A \cup KSP_d(A) \in KSPCL(Ꮜ, \chi)$.

Proof: Assume $x \notin A \cup KSP_d(A)$. This condition implies that x is neither an element of A nor an element of $KSP_d(A)$. Given that $x \notin KSP_d(A)$, there exists a Kasaj pre-open set P_x containing x such that P_x includes no point of A other than x. Since $x \notin A$, it follows that P_x contains no points of A, thus $P_x \subseteq A^c$.Furthermore, since P_x is a Kasaj pre-neighbourhood of each of its points, and P_x contains no points of A, none of the points in P_x can be a Kasaj pre-limit point of A. Consequently, no point in P_x can belong to $KSP_d(A)$, implying $P_x \subseteq [KSP_d(A)]^c$.Therefore, we have $P_x \subseteq A^c \cap [KSP_d(A)]^c = [A \cup KSP_d(A)]^c$. Since $x \in P_x \subseteq A^c \cap [KSP_d(A)]^c$, it follows that $[A \cup KSP_d(A)]^c \in KSPO(Ꮜ, \chi)$. Hence, $A \cup KSP_d(A) \in KSPCL(Ꮜ, \chi)$.

Theorem 3.7 Let A⊆Ꮜ. Then $KSP_{int}(A) = A \setminus KSP_d(A^c)$.

Proof: Consider $x \in A \setminus KSP_d(A^c)$. This implies that x belongs to A but not to $KSP_d(A^c)$. Since $x \notin KSP_d(A^c)$, there exists a set $P \in KSPO(Ꮜ, \chi)$ such that $P \cap (A^c) = \emptyset$. Consequently, x is in P and P is contained in A, which leads to $x \in KSP_{int}(A)$. Thus, we establish that $A \setminus KSP_d(A^c) \subseteq KSP_{int}(A)$.

For the converse, assume $x \in KSP_{int}(A)$. By definition, $x \notin KSP_d(A^c)$. Given that $KSP_{int}(A)$ is a Kasaj pre-neighborhood of x, it follows that $KSP_{int}(A) \cap A^c = \emptyset$. Therefore, $KSP_{int}(A) \subseteq A$. This implies that $x \in A$. Hence, we conclude that $KSP_{int}(A) \subseteq A \setminus KSP_d(A^c)$.

IV. CONCLUSION

In this paper, we have introduced the concepts of Kasaj pre-neighbourhoods and Kasaj pre-limit points**,** building upon the framework of Kasaj pre-open sets and Kasaj pre-derived sets. Through our exploration, we have elucidated various properties associated with these notions. Our goal is to extend this foundational work in future research, with an emphasis on investigating potential practical applications and further theoretical implications in various associated engineering fields.

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