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Optimal Design of Mhd Conduction Pump by Simulated Annealing Method

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Abstract: - Optimization algorithms are a vital tool in many fields, from machine learning and data science to engineering and finance. They allow us to find the best solution to a given problem by searching through a space of possible solutions and selecting the one that maximizes or minimizes a particular objective function. Simulated Annealing (SA) is one of the simplest and best-known metaheuristic methods for addressing the difficult black box global optimization problems. The design of the pump is considered as an optimization problem where the objective function is the minimum of the MHD pump mass with both geometrical and electromagnetic contraints type. The obtained optimization results using the finite volume method with Matlab software show the performances of the used stochastic simulated annealing method.

Keywords: Optimization algorithms, Simulated Annealing (SA), MHD Pump; Maxwell equations thermal equation.

I. Introduction

The SA algorithm was proposed by Kirkpatrick et al. (1983) and Cerny (1985) independently. SA is based on the analogy between the way in which the crystalline structure of a metal achieves near global minimum energy states during the process of annealing and the way in which a function may reach minimum during a statistical search of the design space. The objective function corresponds to the energy state and moving to any new set of design variables corresponds to a change of the energy state. Although the method has been basically developed for discrete problems, it can be used in continuous problems in the same way as GAs are used, [1].

SA algorithm is one of the most preferred heuristic methods for solving the optimization problems. Kirkpatrick et al. introduced SA by inspiring the annealing procedure of the metal working. Annealing procedure defines the optimal molecular arrangements of metal particles where the potential energy of the mass is minimized and refers cooling the metals gradually after subjected to high heat. In general manner, SA algorithm adopts an iterative movement according to the variable temperature parameter which imitates the annealing transaction of the metals.

A simple optimization algorithm compares iteratively the outputs of the objective functions running with current and neighboring point in the domain so that, if the neighboring point generates better result than the current one, then it is saved as base solution for the next iteration. Otherwise, the algorithm terminates the procedure without searching the wider domain for better results. So that, the algorithm is prone to be getting trapped in local minima or maxima. Instead, SA algorithm proposes an effective solution to this problem as incorporating two iterative loops which are the cooling procedure, [2]. This article is concerned the optimization procedure based on the Simulated Annealing (SA) method uses a fitness function as the minimum of the mass of conduction magneto hydrodynamic pump MHD. The Hydrodynamic and thermel model are carried out by the finite volume method. The optimized results of the performance characteristics of the conduction pump are obtained

Magnetohydrodynamics or simply (MHD) is the field of science that studies the movement of conductive fluids subjected to electromagnetic forces. This phenomenon brings together concepts of fluid dynamics and electromagnetism. Formally, MHD is concerned with the mutual interactions between fluid flows and magnetic fields. Electrically conducting and non-magnetic fluids must be used, which limits the applications to liquid metals, hot ionized gases (plasmas) and electrolytes.

Over the years, MHD has been applied to a wide spectrum of technological devices, directed, for example, to electromagnetic propulsion or to biological studies.

Application arises in astronomy and geophysics as well as in connection with numerous engineering problems, such as liquid metal cooling of nuclear reactors, electromagnetic casting of metals, MHD power generation and propulsion [3].

The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic fields generate the Lorentz force, and allow the pumping of liquid metal [3,4]. Magnetohydrodynamics is widely applied in various domains, such as metallurgical industry, to transport or the liquid metals in fusion and the marine propulsion [5,6]. The advantage of these pumps, which ensure the energy transformation, is the absence of moving parts.

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The interaction of moving conducting fluids with electric and magnetic fields allows for a rich variety of phenomena associated with electro-fluid-mechanical energy conversion [7,8].
The schematic of the MHD pump is shown in (fig.1). The basic principle is to apply an electric current across a channel filled with electrically conducting liquids and a dc magnetic field orthogonal to the currents via permanent magnets.

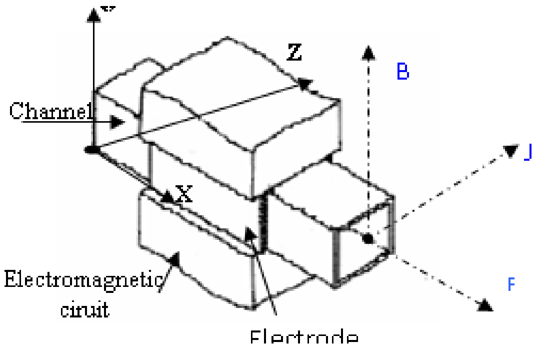

Figure. 1. Scheme of a DC MHD pump [5].

The properties of the mercury are given respectively in tables 1

Parameter	Mercury solution
Density ρ	$13.6*10^{3}(kg/m^{3})$
Electric conductivity σ	$1.06*10^{6}(S.m^{-1})$
Viscosity µ	$0.11*10^{-6}(m^{2}/s)$

II. OPTIMIZATION PROBLEM AND THE SIMULATED ANNEALING METHOD

Simulated annealing introduces the concept of a temperature parameter, which controls the probability of accepting worse solutions. During the iterative process, the algorithm generates a neighboring solution and evaluates its quality based on a cost or objective function.
If the new solution is better than the current solution, it is accepted as the new current solution. However, if the new solution is worse, it may still be accepted with a certain probability. The probability of accepting a worse solution is determined by a formula that depends on the temperature and the difference in cost between the new and current solutions. The formula is designed in such a way that the probability of accepting a worse solution is higher when the temperature is higher. As the algorithm progresses, the temperature gradually decreases. As iterations proceed, it becomes less likely to transition to a worse solution, and the process stabilizes.
Simulated annealing continues iterating and exploring the search space until either a stopping criterion is met (such as reaching a maximum number of iterations or reaching the final temperature) or no further improvements are observed. The best solution obtained during the iterations is an approximation of the optimal solution to the problem
In the formulation of the optimization problem, it is necessary to define the objective function and the constraints conditions. In this case, we have considered the mass of the conduction MHD pump as the objective function to be optimized whereas geometrical, electrical and electromagnetic conditions are inequalities constraints. The resolution of the design problem to determine the vector X will be equivalent to the resolution of the optimization problem (P).

$$(P)\begin{cases} \text{Objective function= Min mass (X)} \\ B(X)\leq 1.7T \\ X_{Lower}\leq X\leq X_{uper} \\ X=(X_1;X_2;X_3;X_4;X_5;X_6;X_7;X_8) \end{cases} \qquad (1)$$

where:
X_1 : channel's length;
X_2 : channel's width ;
X_3 : inductor's length;
X_4 : inductor's width;

X_5 : coil's length;
X_6 : coil's width;
X_7 : electrode's length;
X_8 : electrode's width.
The analogies between a physical system and simulated annealing are grouped in the Table 2
Rules for mathematical symbols and equations

Physical system	Optimization problem
Free energy	Objective function
Coordinates of the particles	Parameters of the problem
State of low energy	Optimal configuration
Temperature	Control parameter

In this algorithm, a new configuration is obtained from a small perturbation subjected to the current configuration. This new configuration is accepted with a probability p = 1 when the energy difference ΔE between it and the current configuration is less than zero. In the case where $\Delta E > 0$, the probability of acceptance p is given by an equation based on the Boltzmann law,[3]

$$P=e^{\frac{-\Delta E}{T}} \quad (2)$$

Where T is the temperature (control parameter). So, accepting an increase in the objective function, will allow the algorithm to come out of a hollow containing a local optimum; what qualifies this method as a global exploration method. If the temperature is lowered slow enough and well controlled in the simulated annealing method, the objective function will evolve towards a global optimal solution. Otherwise it will evolve to a local minimum if temperature is lowered suddenly (quenching). The process continues as long as the energy of the system decreases. When the value of the objective function does not change (the energy remains stationary), the process moves to another temperature level (the decrease of T is done according to a impose decay law) until it convergence to the final temperature where the system becomes frozen,[3].
The most common law of the variation of temperature is; given by the relation:

$$T_{K+1} = \lambda T_K \quad (3)$$

where T_k is the previous temperature at the step k and λ is the reduction factor ($0 < \lambda < 1$). To change the temperature level, one can simply specify a number of transformations, accepted or not, at the end of which the temperature is lowered. A high initial temperature is also chosen. This choice is then totally arbitrary and will depend on the decay law used.
As all metaheuristics approaches, the simulated annealing method can be applied in many optimization problems, such as in packet routing in networks, segmentation of images, the problem of the traveling salesman and the problem of the backpack
Figure 2 shows the flowchart of the implemented simulated annealing method.

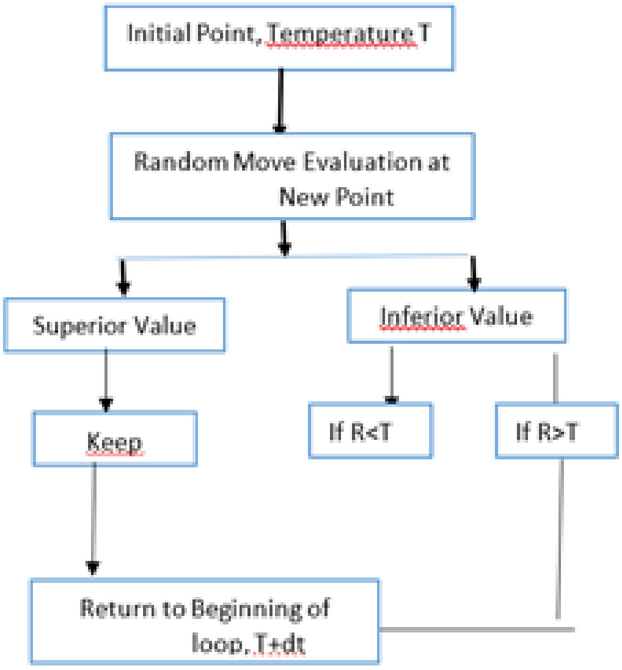

Figure. 2. Flowchart of simulated annealing method

III. MATHEMATICAL ANALYSIS OF PROBLEMS

a. ELECTROMAGNETIC PROBLEM

The schematic structure of the pump is shown in figure (1). In the pump, the electromagnetic forces are obtained from the Lorentz forces induced by interaction between the applied electrical currents and the magnetic fields, [4,6]. The electromagnetic model of the MHD pump is as follows:

$$\overrightarrow{rot}\left(\frac{1}{\mu}\overrightarrow{rot}\vec{A}\right) = \vec{J}_{ex} + \vec{J}_a + \sigma(V.\frac{\partial \vec{A}}{\partial x}) \quad (4)$$

The magnetic induction and the electromagnetic force are given by:

$$\vec{B} = \overrightarrow{rot}\vec{A}$$

$$\vec{F} = \vec{J} \wedge \vec{B} \quad (5)$$

Following the two-dimensional (2D) developments in Cartesian coordinates, where the current density and the magnetic vector potential are perpendicular to the longitudinal section of the MHD pump, the equation becomes:

$$\frac{-1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)=J_{ex}+J_a+\sigma(V_x\frac{\partial A}{\partial x}) \quad (6)$$

b. THERMAL PROBLEM

The thermal phenomena are studied only in the channel of the MHD pump. So, the governing thermal equation is given by

$$\rho C_p\left(\frac{\partial T}{\partial t}\right) = div(Kgrad(T) + P_s \quad (7)$$

Where ρ is the density of the fluid, Cp the specific heat, K the thermal conductivity, T the temperature and Ps the thermal source (electric power density) induced by eddy current such as:

$$P_s = \frac{1}{2\sigma} J_i^2 \quad (9)$$

After developments in Cartesian coordinates, replacing the source term Ps, we obtain:

$$\rho C_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial t}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{1}{2\sigma}J_i^2 \quad (10)$$

c. HYDRODYNAMIC PROBLEMS

The MHD flow of an incompressible, viscous and electrically conducting fluid in a transient state condition is governed by the Navier-Stokes equations [8]:

$$\frac{\partial\vec{V}}{\partial t}+(\nabla.\vec{V})\vec{V}=-\frac{1}{\rho}\nabla p+\nu\wedge\vec{V}+\frac{\vec{F}}{\rho} \quad (11)$$

$$div\vec{V} = 0 \quad (12)$$

Where **p** the is the pressure of the fluid, **υ** the kinemactic viscosity of the fluid, **F** the electromagnetic thrust and **ρ** the fluid density, [12,14].

The development of the equation of the flow in Carte- sian coordinates gives, [11,15]

$$\frac{\partial V_x}{\partial t}+V_x.\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial V_y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 V_x}{\partial^2 x^2}+\frac{\partial^2 V_x}{\partial^2 y^2}\right]+\frac{1}{\rho}F_x$$

$$\frac{\partial V_y}{\partial t}+V_x.\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial V_y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\frac{\partial^2 V_y}{\partial^2 x^2}+\frac{\partial^2 V_y}{\partial^2 y^2}\right]+\frac{1}{\rho}F_y$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial V_y}=0 \quad (13)$$

The real difficulty is the calculation of the velocity lies in the unknown pressure. To overcome this difficulty is to relax the incompressibility constraint in an appropriate way. So, the elimination of pressure from the equations leads to a velocity-stream function

The velocity vector is defined by:

$$\zeta=\overrightarrow{rot}\ V \quad (14)$$

The stream function is given in 2D Cartesian coordinates as:

$$\frac{\partial\psi}{\partial y}=V_x\ ;\quad \frac{\partial\psi}{\partial x}=V_y \quad (15)$$

Where **Vx** and **Vy** the components of the velocity **V**.

We eliminate the pressure from the equation (15) and we use the two new dependent variables ξ and Ψ to obtain the following equation:

$$\frac{\partial\zeta}{\partial t}+V_y\frac{\partial\zeta}{\partial y}+V_x\frac{\partial\zeta}{\partial x}=\nu\left[\frac{\partial^2\zeta}{\partial^2 x^2}+\frac{\partial^2\zeta}{\partial^2 y^2}\right]+\frac{1}{\rho}\left(\frac{\partial F_x}{\partial y}+\frac{\partial F_y}{\partial y}\right) \quad (16)$$

After substituting equation (11) into equation (12) we obtain an equation involving the new dependent variables **ξ** and **ψ** such as:

$$-\zeta=\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2} \quad (17)$$

IV. NUMERICAL METHOD AND RESULTS

There are several methods for the determination of the electromagnetic fields and the velocity; the choice of the method depends on the type of problem, [11, 12].
In our work, we thus choose the finite volume method; its principle consists on subdividing the field of study (Ω) in a number of elements. Each element contains four nodes of the grid. A finite volume surrounds each node of the grid. [13, 14].

The method consists of discretising differential equations by integration on finite volumes surrounding the nodes of the grid.In this method, each principal node P is surrounded by four nodes N, S, E and W located respectively at North, South, Est and West (Figure.2) We integrate the electromagnetic thermal and hydrodynamic equations in the finite volume method delimited by the surfaces E, W, N and S, [15]. Finally we obtain the algebraic equation which is written as:

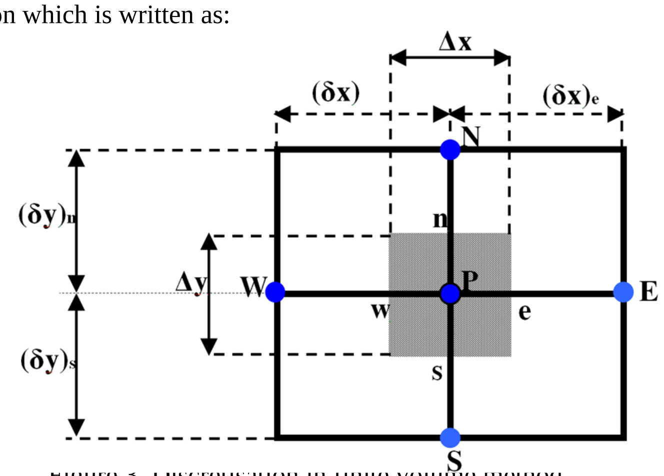

Figure 3. Discretisation in finite volume method.

$$\int_w^e\int_s^n\left[\frac{1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)\right]dx\,dy=\int_w^e\int_s^n\left(J_{ex}+J_a+\sigma V_x\frac{\partial A}{\partial x}\right)dx\,dy \tag{18}$$

$$\rho C_p\int_t\int_s^n\int_w^e \frac{\partial T}{\partial t}dxdydt=\int_t\int_s^n\int_w^e\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dxdydt+\int_t\int_s^n\int_w^e\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)dxdydt$$
$$+\int_t\int_s^n\int_w^e P_s\,dxdydt$$

After integration, the final algebraic equation will be:

$$a_p A_p = a_e A_e + a_w A_w + a_n A_n + a_s A_s + d_p \tag{19}$$

$$a_E=\frac{\Delta y}{\mu_e(\delta x)_e},\quad a_W=\frac{\Delta y}{\mu_w(\delta x)_w},\quad a_N=\frac{\Delta x}{\mu_n(\delta y)_n},\quad a_S=\frac{\Delta x}{\mu_s(\delta y)_s},$$

$$c_P T_P = c_E T_E + c_w T_W + c_N T_N + c_S T_S + d_P \tag{20}$$

$$c_E=\frac{K\Delta t\Delta y}{(\delta x)_e},\quad c_W=\frac{K\Delta y\Delta t}{(\delta x)_w},\quad c_N=\frac{K\Delta t\Delta x}{(\delta y)_n},\quad c_S=\frac{K\Delta t\Delta x}{(\delta y)_s},$$

We use the same steps for the hydrodynamic problem:

$$\int_s^n\int_w^e\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}\right)dxdy=-\int_s^n\left(\int_w^e \zeta dxdy\right. \tag{21}$$

$$b_P\zeta_P = a_e\zeta_e + b_w\zeta_w + b_n\zeta_n + b_s\zeta_s + b_0\zeta_0 + d_P \tag{19}$$

The resolution of the electromagnetic, thermic and the hydrodynamic equations makes it possible to determine the magnetic potential vector, magnetic induction ($\vec{A},\vec{B}$) the Electromagnetic force F, temperature and the velocity in the channel of the conduction pump.

V. APPLICATION AND RESULTS

Considering the constraints in a stochastic optimization method are often obtained by using a function of penalties [4], according to which the function to be minimized becomes equal to:

$$W(X) = f(X) + r\sum_{i=1}^{m} \max\,[0, g_i(X)]^2 \qquad (22)$$

Where f(X) objective function without constraints; gi(X) function's constraints; r: penalty coefficient.
Tables 2 show the solution vector and the pump performances.

Parameters	before optimization	After optimization
X1 [m]	0.2	0.195
X2 [m]	0.2	0.195
X3 [m]	0.07	0.06
X4 [m]	0.3	0.28
X5 [m]	0.025	0.021
X6 [m]	0.15	0.111
X7 [m]	0.05	0.08
X8 [m]	0.1	0.12
Iron mass (Kg)	4.1212	3.3070
Coil's masse (Kg)	1.6725	0.51937
Electrode's masse (Kg)	0.0520	0.0262
Mercury's masse (Kg)	3.2496	3.1568
Pump's masse (Kg)	9.1474	7.54

The figures (4) and (5) represent respectively the equipotential lines and the distribution of the magnetic vector potential in the MHD pump.

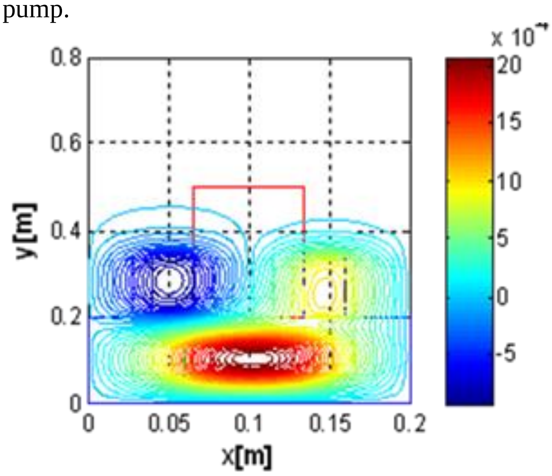

Figure. 4 – Equipotential lines in a DC MHD pump

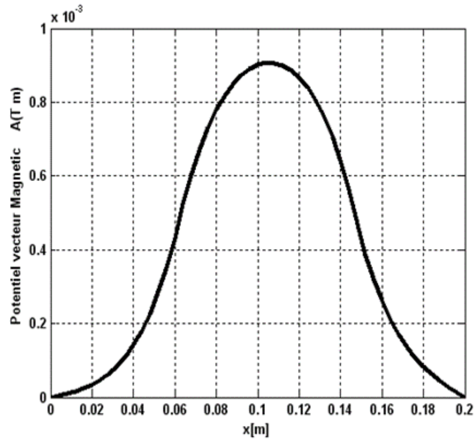

Figure. 5 – Magnetic vector potential in a Dc MHD pump

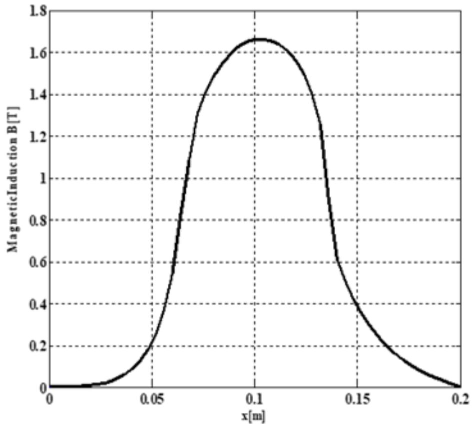

Figure. 6 – Magnetic induction in the MHD pump

The figure (6) represents the magnetic induction in the channel. It is shown that, the magnetic induction reaches its maximum value at the inductor and in the medium of the channel.

This figure (7) represents the electromagnetic force in the channel; it is note that, the maximum value in the medium of the channel of the MHD pump.

The figure (8) represents the velocity in the channel of the MHD pump. It is noticed that the velocity of the fluid flow passes by a transitory mode then is stabilized like all electric machine and the steady state is obtained approximately after ten seconds. The results obtained are almost identical qualitatively to those obtained by [6, 14].

The figure (9) shows the electric power density in the channel. The maximum induced power reaches $2.157*10^6$ W/m^3 . The pace obtained is directly related to that of the eddy current density. This characteristic of the heat source is used in the numerical calculation of the temperature.

The figure (10) shows the distribution of the temperature in the channel of the MHD pump. It is noticed that the temperature passes by a transitory mode then is stabilized.

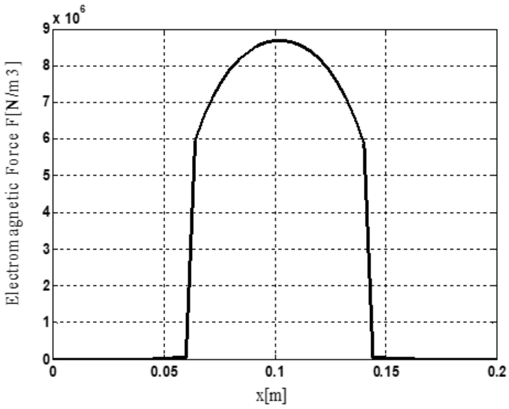

Figure. 7 – Electromagnetic force in The MHD pump

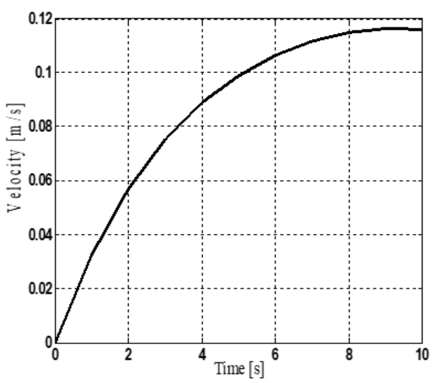

Figure. 8 –Velocity in the channel of the MHD

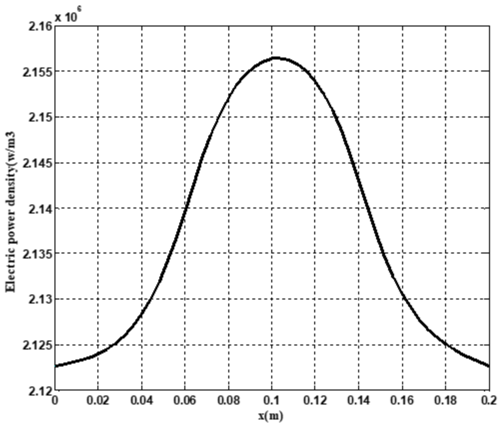

Figure 9. The electric power density in the channel

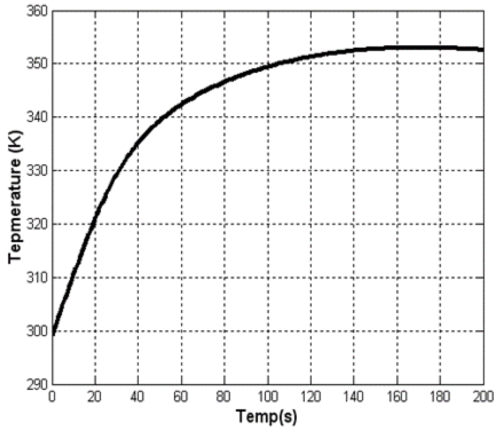

Figure. 10 – The temperature in the channel of the MHD pump

VI. CONCLUSION

In this paper is concerned the optimization procedure based on the Simulated Annealing (SA) method uses a fitness function as the minimum of the mass of conduction magneto hydrodynamic pump MHD. The Hydrodynamic and thermel model are carried out by the finite volume method. The optimized results of the performance characteristics of the conduction pump are obtained

The results of velocity obtained are almost identical qualitatively to those obtained by Majid Ghassemi and P.J. Wang.

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