[1] M. G. Neethialagan

[2] S. Meenakshi

Secure Distance Matrix Domination Graphs

Abstract: - Let $D_{SDM}(G) = (V(D_{SDM}), E(D_{SDM}))$ become an undirected, fundamental graph. Predominance in graphical representations is a specific area of theory pertaining to graphs the fact that has been thoroughly explored. A subset $D_{SDM}(S)$ in the event every vertex within $D_{SDM}(S)$ has become either contained within $D_{SDM}(S)$ or closest to a separate the vertices in $D_{SDM}(S)$, subsequently the overall quantity of points that compose the graph's structure has been identified as the dominant set. This article determines novel domination outcomes in graphs known as secure distance matrix domination. A dominating set of $D_{SDM}(S)$ regarding $D_{SDM}(G)$ can be considered to have the attributes of a stable, dominant group of $D_{SDM}(G)$ once it has just one available. vertex $D_{SDM}(u) \in (D_{SDM}(V) \setminus D_{SDM}(S))$ with respect to $D_{SDM}(uv) \in (D_{SDM}(E)$ as well as $(D_{SDM}(S)) \setminus (D_{SDM}\{v\} \cup D_{SDM}\{u\})$ has become a dominant set have been D_{SDM} that includes each points $D_{SDM}(v) \in D_{SDM}(S)$. The issue is minimally secure determining a set that dominates of $D_{SDM}(G)$ using a minimum secure cardinality is the definition of dominance. A few secure distance matrix dominant set theorems are outlined.

Keywords: Domination, Dominating set, Secure distance matrix domination, Distance domination, complete graph.

1. Introduction

An essential subfield of graph theory is dominance. Investigating dominant initiates within graphs dates back to 1862, when Campbell [1] investigated the issue of figuring out how many queens are required to control a chessboard. The field of research of dominating sets in graphs came into being about 1960. The centre of graph theory study has been the theory of dominance. Within known as $G* = (V*, E*)$ a graph, Assume that the point set is $V*$ and the border set is $E*$. The investigation of being dominant establishes takes up a large amount of room through the field of graph theory. The dominance was first introduced as a graph theoretic concept by C. Berge and O. Ore [2].

The phrases "dominant set" and "domination number" were also created by O. Ore [2]. Place $D*$ is an extremely powerful set, or within close proximity of dominance $D_{SDM} N_d[D*] = D_{SDM}V*$ [3], As long as every vertex is present in $D_{SDM}V* - D_{SDM}D*$ has become located adjacent to any vertex within $D_{SDM} D*$. If and only if no edge connects any two of the vertices of a graph with the same number of vertices, V*(G*), it is referred to as the complement graph of a simple graph G* [4].

If every point in $D_{SDM}G*$ the fact that does not exist in $D_{SDM}D*$ is close to at least one of the vertices in $D_{SDM}D*$, and then $D_{SDM}G*$ represents the being dominant set. The lowest possible cardinality of a set that dominates in $D_{SDM}G*$ is equal to the dominance the number $D_{SDM}(\gamma*(G*))$. In the event there are no two vertices that are close together in set $D_{SDM}S* \subset D_{SDM}V*$, then set S is independent. A peak performance independent determined by $D_{SDM}G*$ has a pair of minimum and maximum cardinalities whose

---

[1]Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai- 600 117, India. Email: neethialagan15@gmail.com

[2] Associate Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Pallavaram, Chennai- 600 117, India. Email: meenakshikarthikeyan@yahoo.co.in

respective values equivalent the degree of independence number $D_{SDM}(\beta_0(G*))$ as well as a dominant number ascertained independently $D_{SDM}(i(G*))$.

The cardinality's value of a dominated set of $D_{SDM}G*$ which has the minimum is its controlling number as well, denoted by $D_{SDM}\gamma*(G*)$. The variable $D_{SDM}G*$ is the $D_{SDM}\gamma*(G*)$. This particular issue has an operational equivalent called the power source Control Decision Problems. It inquiries whether, considering a graph $D_{SDM}G*$ and an upside-down a number $D_{SDM}k*$, there currently is a being dominant collection of cardiac values greater than $D_{SDM}k*$.

Another may obtain an extensive overview of the available research on the a requirement dominance problem and a lot of its variations in length. Secure Supremacy Problem. Another of the significant variants of the minimal dominance problem is that which follows explanation of the problem.

The being dominant set $D_{SDM}S* \subseteq D_{SDM}V*$ of $D_{SDM}G*$ can be considered secure if there is a vertex $(D_{SDM}v*) \in (D_{SDM}S*)$ nearby to u that ensures that $(D_{SDM}S* \setminus D_{SDM}\{v*\}) \cup D_{SDM}\{u*\}$ is a set that dominates that includes every $(D_{SDM}u*) \in (D_{SDM}V* \setminus D_{SDM}S*)$. $D_{SDM}N_k(v*) \cup D_{SDM}(v*)$, or $\{D_{SDM}(u*)|D_{SDM}d(u*,v*) \leq D_{SDM}k*\}$ has become the description of the closed $D_{SDM}k*$- neighbourhood $D_{SDM}N_k(v*)$.

When $D_{SDM}\{u*\}$ and $D_{SDM}\{v*\}$ are $D_{SDM}k*$-adjacent vertices, we say that $D_{SDM}u* \in D_{SDM}N_k*(v*).|D_{SDM}N_k*(v*)|$ yields the $D_{SDM}k*$-degree, $D_{SDM}(deg_k(v*))$, of $D_{SDM}(v*)$ in $D_{SDM}(G*)$. Hence $D_{SDM}N_k(v) \cup D_{SDM}(v)$, or $\{D_{SDM}u|D_{SDM}d(u,v) \leq D_{SDM}k\}$, is the definition of the closed $D_{SDM}k-$neighborhood $D_{SDM}N_k(v)$, of v*. When $D_{SDM}u*$ and $D_{SDM}v*$ are k-adjacent vertices, we say that if $D_{SDM}u* \in D_{SDM}N_k(v).|N_k(v)|$ yields the k-degree, $deg_k(v)$, of $D_{SDM}v$ in $D_{SDM}G$.

Consequently, let $D_{SDM}k* \geq 1$ be an integer and let $D_{SDM}(G) = (D_{SDM}(V), D_{SDM}(E))$ be considered a diagram. The open $D_{SDM}k*$- neighbourhood $D_{SDM}N_k(v*)$ become an vertex in $D_{SDM}(G)$ described as that collection of $D_{SDM}(G)$ vertices $D_{SDM}(G)$ disjoint of $D_{SDM}(v)$ and at degree at maximum $k*$ from $v*$ in $D_{SDM}(G)$. Take $D_{SDM}(G) = (D_{SDM}(V), D_{SDM}(E))$ become Consequently, let $D_{SDM}k* \geq 1$ be an integer. A vertex v in G contains an open k-neighborhood $N_k(v)$ It represents a group of every one of points in $D_{SDM}(G)$ disjoint of $D_{SDM}(v)$ and at degree at maximum $k*$ from $v*$ in $D_{SDM}(G)$.

If every single vertex of $D_{SDM}S \subseteq VD_{SDM}$ encompasses a minimum of two neighbors in $D_{SDM}S$, then a small portion $VD_{SDM} \setminus D_{SDM}S$ have become 2-dominant collection. When $D_{SDM}S$ have become 2-dominant collect and that subcollection induced by $D_{SDM}S$ has no separate vertex then $D_{SDM}S$ have double dominant set.

The cardinality of one minimal double dominant collection of $D_{SDM}G$ and then minimal 2-dominant set that $D_{SDM}G$ are represented by then double dominant collection $\gamma_{2\times2}D_{SDM}$ after the 2-dominant collection $\gamma_2 D_{SDM}(G)$. It is evident that for someone graph $D_{SDM}G$ not in separate points, $\gamma_{2\times2}D_{SDM}G \geq \gamma_2 D_{SDM}(G)$ holds true. Any dominant set possessing

the quality that each vertex $u \in VD_{SDM} \backslash D_{SDM}S$ is next to a vertex $VD_{SDM} \backslash D_{SDM}S$ such that $(D_{SDM}S \subseteq VD_{SDM} \cup UD_{SDM}$ being dominant set as called a secure dominant set that graph $D_{SDM}G$.

When every single vertex in $VD_{SDM} \cap D_{SDM}(G)$ It's near through $k*$ compared to a point of $D_{SDM}(G)$, then a $D_{SDM} \leq D_{SDM}V(G)$ is referred to as a distance $k*$-dominant collection of $D_{SDM}(G)$. That classic dominant size of $D_{SDM}(G)$ is $\gamma_k D_{SDM}(G)$ in that special case of $k*=1$. Even if $D_{SDM}(G)$ is bipartite, determining is $\gamma_k D_{SDM}(G)$ for any graph $D_{SDM}(G)$ is an NP-hard problem [5]. Numerous authors have thoroughly examined the idea of a $k*$-dominating set in order to take into account the distance parameters in a variety of contexts and structures that result in graphs; see, for example, [18–22].

For non-cyclic abelian groups $Al(p^n) \times Al(q^m)$ and $Al(p^n) \times \mathbb{Z}_m$, where p and q are distinct primes, Arora et al. [23] computed the spectrum of the secure distance matrix domination of the enhanced power graph of non-abelian groups of order pq, dihedral groups, dicyclic groups, and elementary abelian groups $El(p^n)$.

Many types of dominance criteria have been studied by placing different constraints on dominant sets [5]. Parameters characterise the most innovative dominance is the quantity of vertices over which a vertex is dominant. Additionally, a study of the dominance polynomial of a particular graph is presented in [6]. The degree of v represented as $deg(v)$, represents the cardinality of G. Numerous studies have been conducted in several domains, including linear algebra, laplacian, and distance matrices [7–12]. This paper computes new domination results in graphs using a technique called secure distance matrix domination. A few dominating set theorems for secure distance matrices are described.

The lowest cardinality associated with secure overpowering $D_{SDM}G$ equal to secure dominance number $\gamma_S D_{SDM}(G)$. Cockayne et al. [13] introduced secure domination, which is examined, for instance, in [14–17].

A table that displays the distance between two objects distance . Square that shows the separations between each element of a set in pairs. There are four types of topologies: ring, star, mesh, and hybrid. These networks have advantages and disadvantages. As a result, all of these networks are becoming topological graphs and using the distance domination theory on them.Although the chemical formulas for substances like ethanol and methane were known, it was unclear how the constituent elements mixed to create these compounds.Alexander Crum Brown developed the concepts of chemistry and presented his visual formulas for depicting molecules in 1850. Additionally, he displayed the typical drawing, the matching tree graph, and the representation of ethanol.

2. Preliminaries

Definition 2.1

A subset $D_{SDM}(S) \subseteq VD_{SDM}(G)$ that is referred to as Secure Distance matrix dominat set of G if each vertices $D_{SDM}(v) \in VD_{SDM}(G) \setminus D_{SDM}(G)$ there exists $D_{SDM}(u) \in D_{SDM}(S)$ such that $D_{SDM}(uv) \in D_{SDM}(E)$ and $D_{SDM}(S) = (D_{SDM}(S) -$

$D_{SDM}(u) \cup D_{SDM}(v)$ is a dominating set and the minimum cardinality of secure distance matrix dominating set is the secure Distance matrix dominating number which is denoted by $\gamma_s D_{SDM}(G)$

Definition 2.2

Consider $D_{SDM}(G) = (V(D_{SDM}), E(D_{SDM}))$ be a normal graph. A secure distance matrix dominating set $D_{SDM}(G)$ is defined a secure distance matrix dominating set if for every set $D_{SDM} V_1 \subseteq D_{SDM} V \setminus D_{SDM}(D)$ there is a set that is not empty $D_{SDM} D_1 \subseteq D_{SDM} D$ in a way that generated a subgraph $D_{SDM} < V_1 \cup D_{SDM} D_1 >$ Resulting from $D_{SDM} V_1 \cup D_{SDM} D_1$ has a connection. The cardinality minimum of secure Distance matrix dominating set is called the secure Distance matrix domination number of $D_{SDM}(G)$ and is denoted by $\gamma_s D_{SDM}(G)$.

Definition 2.3

The upper secure distance matrix dominating number, represented by $\gamma_s D_{SDM}(G)$, is the cardinality maximum of a minimal secure distance matrix dominating set of $D_{SDM}(G)$. It is obvious that a dominating set $D_{SDM}(D)$ is only a secure Distance matrix dominating set if and when the set $D_{SDM}(D)$ itself is a secure distance matrix dominating set.

Definition 2.4

A line graph is formed by $D_{SDM}(G) = (V(D_{SDM}), E(D_{SDM}))$, when the set of edges is represented by $E(D_{SDM})$ as well as the one that powers the set of points is implied from $(V(D_{SDM})$. Each of the edge, usually referred to as simply vivj, consists of an unorganized established of two unique vertices, $\{(v_i(D_{SDM}), (v_j(D_{SDM})\}$ for $1 \leq D_{SDM}(i) \neq D_{SDM}(j) \leq n$. If there is a path from u to v for every $u, v \in V D_{SDM}(G)$, then graph $D_{SDM}(G)$ is connected.

Definition 2.5

Given an asymmetrical graph $D_{SDM}(G)$ on its vertex set $\{v_1, v_2, \ldots, v_\rho\}$, the $p \times p$ matrix is the Secure Distance matrix an $D_{SDM}(G)$.

$$D_{SDM}(G) = \begin{cases} d(x,y) & \text{if } v_x \leftrightarrow v_y, \\ \\ 0 & , \quad \text{otherwise.} \end{cases}$$
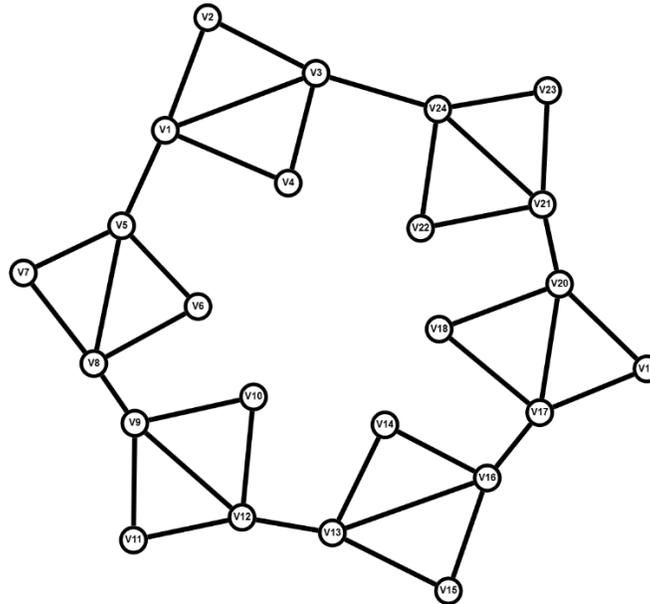
EXAMPLE 2.5.1

Figure 1: Diamond necklace Graph

Secure Distance Matrix Domination of a Diamond necklace Graph

0 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 5, 6, 6

1, 0, 1, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 5, 5, 5, 4, 5, 5, 5, 7, 6, 6, 7

1, 1, 0, 1, 2, 1, 3, 3, 3, 2, 2, 2, 4, 5, 5, 5, 3, 4, 4, 4, 6, 6, 5, 6

1, 2, 1, 0, 2, 2, 3, 3, 3, 3, 3, 3, 4, 5, 5, 5, 4, 5, 5, 5, 7, 6, 6, 7

1, 2, 2, 2, 0, 3, 1, 1, 1, 4, 4, 4, 2, 3, 3, 3, 5, 6, 6, 6, 5, 4, 5, 5

2, 2, 1, 2, 3, 0, 4, 4, 4, 1, 1, 1, 5, 6, 6, 6, 2, 3, 3, 3, 5, 5, 4, 5

2, 3, 3, 3, 1, 4, 0, 2, 1, 5, 5, 5, 2, 3, 3, 3, 6, 7, 7, 6, 5, 4, 5, 5

2, 3, 3, 3, 1, 4, 2, 0, 1, 5, 5, 5, 2, 3, 3, 3, 6, 7, 7, 6, 5, 4, 5, 5

2, 3, 3, 3, 1, 4, 1, 1, 0, 5, 5, 5, 1, 2, 2, 2, 6, 6, 6, 5, 4, 3, 4, 4

3, 3, 2, 3, 4, 1, 5, 5, 5, 0, 2, 1, 6, 7, 7, 6, 2, 3, 3, 3, 5, 5, 4, 5

3, 3, 2, 3, 4, 1, 5, 5, 5, 2, 0, 1, 6, 7, 7, 6, 2, 3, 3, 3, 5, 5, 4, 5

3, 3, 2, 3, 4, 1, 5, 5, 5, 1, 1, 0, 6, 6, 6, 5, 1, 2, 2, 2, 4, 4, 3, 4

3, 4, 4, 4, 2, 5, 2, 2, 1, 6, 6, 6, 0, 1, 1, 1, 5, 5, 5, 4, 3, 2, 3, 3

4, 5, 5, 5, 3, 6, 3, 3, 2, 7, 7, 6, 1, 0, 2, 1, 5, 5, 5, 4, 3, 2, 3, 3

4, 5, 5, 5, 3, 6, 3, 3, 2, 7, 7, 6, 1, 2, 0, 1, 5, 5, 5, 4, 3, 2, 3, 3

4, 5, 5, 5, 3, 6, 3, 3, 2, 6, 6, 5, 1, 1, 1, 0, 4, 4, 4, 3, 2, 1, 2, 2

4, 4, 3, 4, 5, 2, 6, 6, 6, 2, 2, 1, 5, 5, 5, 4, 0, 1, 1, 1, 3, 3, 2, 3

5, 5, 4, 5, 6, 3, 7, 7, 6, 3, 3, 2, 5, 5, 5, 4, 1, 0, 2, 1, 3, 3, 2, 3

5, 5, 4, 5, 6, 3, 7, 7, 6, 3, 3, 2, 5, 5, 5, 4, 1, 2, 0, 1, 3, 3, 2, 3

5, 5, 4, 5, 6, 3, 6, 6, 5, 3, 3, 2, 4, 4, 4, 3, 1, 1, 1, 0, 2, 2, 1, 2

6, 7, 6, 7, 5, 5, 5, 5, 4, 5, 5, 4, 3, 3, 3, 2, 3, 3, 3, 2, 0, 1, 1, 2

5, 6, 6, 6, 4, 5, 4, 4, 3, 5, 5, 4, 2, 2, 2, 1, 3, 3, 3, 2, 1, 0, 1, 1

6, 6, 5, 6, 5, 4, 5, 5, 4, 4, 4, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1, 1, 0, 1

6, 7, 6, 7, 5, 5, 5, 5, 4, 5, 5, 4, 3, 3, 3, 2, 3, 3, 3, 2, 2, 1, 1, 0

Floyd-Warshall Algorithm of Secure Distance Matrix Domination of a Diamond necklace Graph

EXAMPLE 2.5.2

Figure 2: Diamond chain Graph

Secure Distance Matrix Domination of a Diamond chain Graph

$$
D_{SDM}(G) = \begin{vmatrix}
0 & 1 & 1 & 2 & 2 & 3 & 3 & 3 & 3 \\
1 & 0 & 2 & 1 & 3 & 2 & 3 & 3 & 3 \\
1 & 2 & 0 & 3 & 1 & 2 & 2 & 2 & 2 \\
2 & 1 & 3 & 0 & 2 & 1 & 2 & 2 & 2 \\
2 & 3 & 1 & 2 & 0 & 1 & 1 & 1 & 1 \\
3 & 2 & 2 & 1 & 1 & 0 & 1 & 1 & 1 \\
3 & 3 & 2 & 2 & 1 & 1 & 0 & 1 & 1 \\
3 & 3 & 2 & 2 & 1 & 1 & 1 & 0 & 1 \\
3 & 3 & 2 & 2 & 1 & 1 & 1 & 1 & 0
\end{vmatrix}
$$

Floyd-Warshall Algorithm of Secure Distance Matrix Domination of a Diamond chain Graph
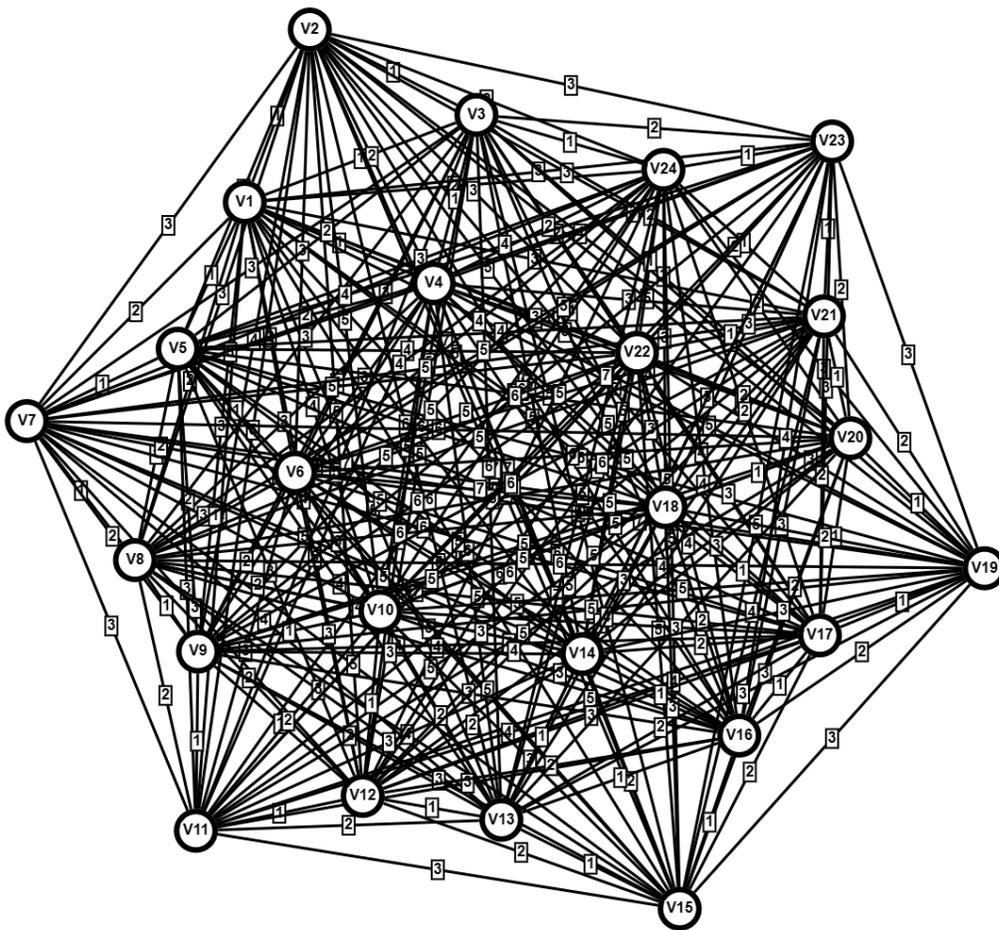
3. Secure Distance Matrix Domination

Example 3.1 : A Set $D_{SDM}(S) = \{1,4\}$. The secure dominant set in graph G. For, $V(D_{SDM}(G)) = \{1,2,3,4\}$. It is the dominant set. $V(D_{SDM}(G) - D_{SDM}(S) = \{2,3\}$. Therefore, Figure 1(b) displays that the D_{SDM} become a secure distance matrix dominating set of D_{SDM} (G).
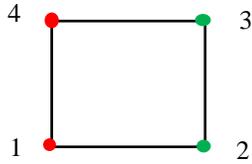
$$SDM\ (G) = \begin{array}{c} v_2 \\ v_3 \end{array}\begin{pmatrix} v_2 & v_3 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$
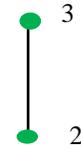
Figure 1(a). Secure dominating set   **Figure 1(b).** Secure distance matrix dominating set

Theorem 3.3

For the complete graph K_6, $\frac{d}{da}\left(\frac{D_{SDM_{m,n}}(K_6,a)}{6}\right) = D_{SDM_{m,n}}(K_{6-1},a) + 1$.

Proof:

A Set $D_{SDM}(S) = \{6\}$ is the firmly established dominant set. For, $VD_{SDM} = \{1,2,3,4,5,6\}$ become a dominating set. $VD_{SDM} - D_{SDM}(S) = \{1,2,3,4,5\}$.
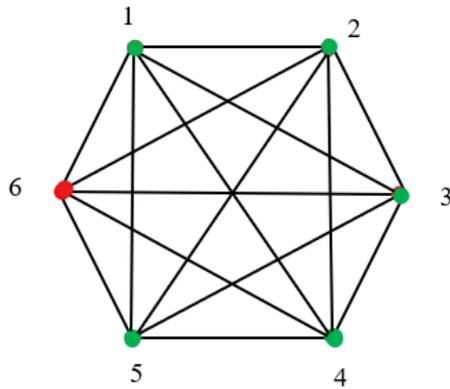

Figure 2(a). Secure dominating set of complete graph K_6

$$SDM\ (G) = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{pmatrix} v_1 & v_2 & v_3 & v_4 & v_5 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$
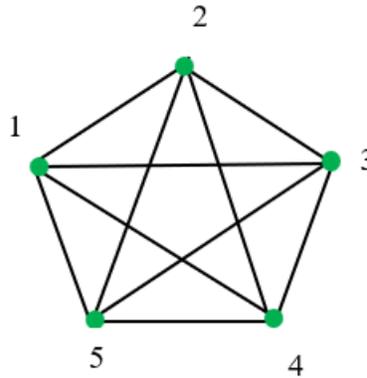

Figure 2(b). Secure distance matrix dominating set of complete graph K_5

We have $D_{SDM_{m,n}}(K_6, a) = (1 + a)^6 - 1$.

Therefore, $\frac{d}{da}\left(D_{SDM_{m,n}}(K_6, a)\right) = 6(1 + a)^{6-1}$

$$\frac{d}{da}\left(\frac{d(v_m, v_n)\ (K_6, a)}{6}\right) = (1 + a)^5$$

$$\frac{d}{da}\left(\frac{d(v_m, v_n)\ (K_6, a)}{6}\right) - 1 = (1 + a)^5 - 1$$

$$= d(v_m, v_n)\ (K_5, a)$$

Hence,

$$\frac{d}{da}\left(\frac{D_{SDM_{m,n}}(K_6, a)}{6}\right) = SDM_{m,n}(K_5, a) + 1.$$

Theorem 3.4

Let D_{SDM} be the secure distance matrix such that $D_{SDM}\ S \subseteq V(G)$ and let K_n become a complete graph having $D_{SDM}n$ nodes. Following that, given an algebraic multiplicity of $n - 1$, the eigenvalues of D_{SDM} are $n - 1$ and -1.

Proof.

Initially, we demonstrate that -1 is an D_{SDM} eigenvalue by taking into account $D_{SDM_{m,n}} = d(v_m, v_n)$, clearly

$$(D_{SDM} - (-1)I_n) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}$$

The other eigenvalue of D_{SDM} is found in the second part of the proof, which assumes that The total amount of each and every their eigenvalue in D_{SDM} has become equivalent to the process of tracing.

Given the standing of $(D_{SDM} - (-1)I_n)$ is 1, this implies that det $(D_{SDM} - (-1)I_n) = 0$ and that negative One of the eigenvalues of D_{SDM} with algebraic variance of $n - 1$ indexed by $D_{SDM}S \subseteq VD_{SDM}(G)$.

$$e * + \sum_{i=1}^{n*-1} (-1) = 0$$
$$e * -(n*-1) = 0$$
$$e * = (n*-1).$$

As result, D_{SDM}'s eigenvalues are $n*-1$ and -1, with $n*-1$ algebraic multiplicity.

Theorem 3.5

Assume that $D_{SDM}(G)$ is going on secure distance matrix for a line graph $D_{SDM}G$ with $n \geq 2$ vertices that is implies that $D_{SDM}S \subseteq D_{SDM}V(G)$. With $n*-1$ negative eigenvalues and one positive eigenvalue, $D_{SDM}(G)$ is then described.

Proof.

Using the induction approach on the number of vertices $(n*)$, we shall demonstrate this. Starting there is only one straightforward graph with two vertices when $(n*) = 2$.

$$SDM(G) = \begin{matrix} v_1 \\ v_2 \end{matrix}\begin{pmatrix} v_1 & v_2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$
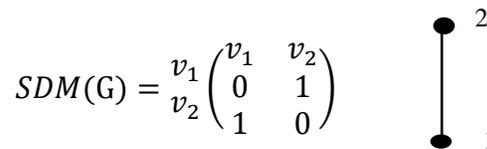
Figure 3. Secure distance matrix dominating set

Let Simple Graph be and Secure distance matrix be $D_{SDM}(G)$.

We calculate the eigenvalues of $SDM(G)$, and we refer to $D_{SDM}(G)$ as D_{SDM} to keep things simple.

Next, we solve,

$$
\begin{aligned}
\det(D_{SDM} - \psi I) &= 0, \\
\begin{vmatrix} -\psi & 1 \\ 1 & -\psi \end{vmatrix} &= 0, \\
\psi^2 - 1 &= 0, \\
(\psi + 1)(\psi - 1) &= 0.
\end{aligned}
$$

As a result, we have one eigenvalue that is positive, $\psi = 1$, and one that is negative, $\psi = -1$.

We now suppose that the theory applies to graphs having $n - 1$ vertices and examine $n > 2$.

By removing a vertex, which we refer to as v_a, or a vertex of degree one, from D_{SDM}, we create a subgraph (which is once more a graph) with $n - 1$ vertices.

$D_{SDM_{v_a}}$ is the secure distance matrix for the generated subgraph.

Keep in consideration that the separations between the remaining vertices don't change if v_a is removed since it is pendant. This indicates that the submatrix of D_{SDM} called $D_{SDM}M_{v_a}$ is created by taking out the columns and rows and column that match the point of intersection v_a.

We assume that the eigenvalues of $D_{SDM_{v_a}}$ are $\rho_1, \rho_2, \cdots, \rho_{n-1}$ such that ρ_1 is positive and the remaining eigenvalues are negative.

Let us now assume that the D_{SDM} eigenvalues are $\psi_1, \psi_2, \cdots, \psi_n$. Cauchy's Interlacing Theorem may be applied to the Hermitian matrix's eigenvalues.

The introduction informs us that D_{SDM} is Hermitian. After that, we obtain $\psi_1 \geq \rho_1 \geq \psi_2 \geq \rho_2 \geq \cdots \geq \psi_{n-1} \geq \rho_{n-1} \geq \psi_n$ by using the interlacing theorem. We note that ψ_2 might have a positive or negative value. D_{SDM} has two positive eigenvalues if it is positive; if it is negative, D_{SDM} has only one positive eigenvalue. The sign is supported by the The reality that the matrices determinant's consider is proportional to the product of the eigenvalues of ψ_2.

Hence,

$$
\frac{\det D_{SDM}}{\det D_{SDM_{v_a}}} = \frac{\psi_1 \cdot \psi_2 \cdots \psi_n}{\rho_1 \cdot \rho_2 \cdots \rho_{n-1}}.
$$

The sign of $\frac{\det D_{SDM}}{\det D_{SDM_{v_a}}}$ relies on the sign of ψ_2, since ψ_1, ρ_1 are positive, ρ_2 is negative, and $\rho_2 \geq \psi_3 \cdots \geq \psi_{n-1} \geq \rho_{n-1} \geq \psi_n$.

By using the determinant of a graph formula, we obtain

$$
\begin{aligned}
\frac{\det D_{SDM}}{\det D_{SDM_{v_a}}} &= \frac{(-1)^{n*-1}(n*-1)2^{n*-2}}{(-1)^{n*-1-1}(n*-1-1)2^{n*-1-2}} \\
&= \frac{(n*-1)}{(-1)(n*-2)2^{-1}} \\
&= \frac{-2(n*-1)}{(n*-2)} < 0.
\end{aligned}
$$

This suggests that SDM has a single positive eigenvalue since ψ_2 is negative.

Theorem 3.6

If a graph G consists of p components $G_1, G_2, \ldots, G_p$, then $D_{SDM}(G, x) = D_{SDM}(G_1, x)D_{SDM}(G_2, x) \ldots D_{SDM}(G_p, x)$, for any natural number p.

Proof:

When $p = 2, G = G_1 \cup G_2$.

Therefore $D_{SDM}(G, x) = D_{SDM}(G_1, x)D_{SDM}(G_2, x)$.

Hence, $D_{SDM}(G, x) = D_{SDM}(G_1, x)D_{SDM}(G_2, x) \ldots D_{SDM}(G_p, x)$, for any natural number m.

Corollary 3.7

Assume that the null graph, $\bar{K}_n$, has $n*$ vertices.

Then $D_{SDM}(\bar{K}_{n*}, x) = x^{n*}$.

Proof:
Since $D_{SDM}(\bar{K}_1, x) = x$, by Theorem 3.6, $D_{SDM}(\bar{K}_{n*}, x) = x^{n*}$.

Perfect Distance Matrix Dominating Set

We examine how a set becomes a vertex cut outstanding commanding set, and distance matrix dominant set to a given graph in this section. In the event that there are more of dominating sets that are available, we then identify the best set among all the

dominating sets due to a certain requirement or legitimate goal. The goal is to increase the effectiveness of electrical circuit connectivity by reducing distance.

An overwhelming group If each vertex in V has a single dominant vertex in D_m, then D become a perfect dominating set or else a dominant set A perfect dominant set (D) is defined as follows: each vi in D, $N_{(v_1)} \cap N_{(v_2)} \cap ... \cap N_{(v_k)} = \emptyset$ As well as an alternative presentation method a powerful group . If for every in D_m there is exactly one vertex $N_{(v_i)} \cap D_m$, or if for every vi in D there is exactly one vertex $| N_{(v_i)} \cap D_m| = 1$, then D_m is considered perfect dominating set. where $N_{(v)}$ is $v's$ neighboring vertex.

Dominance of Distance Matrix in Biomolecule Structures

Dehydration synthesis is the process of joining two monomers to create a covalent bond. Synthesis means "to join together," and dehydration means "removal of water." Consequently, two monomers in this process form a covalent bond through the removal of a molecular water. On one side of every organic monomer is a hydrogen (H) atom, and on the other is a hydroxyl group $(-OH)$. These two functional groups $(H \& OH)$ will be facing each other when two monomers are lined up side by side.

A water molecule is formed when the H and the OH separate from their corresponding monomers and form a bond. This completes the process's dehydration phase. Now that every monomer has a carbon atom that needs to form a covalent bond, they attach themselves to one another.

Now that every monomer has a carbon atom that needs to form a covalent bond with something, the monomers bind to one another to form polymers. That amounts to 115.the process's synthesis phase. The process of dehydration synthesis requires energy.Each time a cell needs to assemble a protein or a carbohydrate, it must invest energy in creating those chemical bonds. Building all organic polymers requires the universal process of dehydration synthesis.

Dominance of Distance Matrix in Networks
Networks of Computers Having Common Connectivity
A telecommunications network that enables data exchange between computers is called a computer network, sometimes referred to as a data network. Networked computing devices in computer networks

Using a data link, they can exchange data with one another. Either wireless or cable media are used to connect the nodes to one another. The Internet is the most widely used computer network.

Network nodes are the computer systems on a network that start, route, and end data (vertices).Networking hardware and hosts like PCs, phones, servers, and other devices can be considered nodes, or vertices. These two gadgets can be. When two devices can exchange information, regardless of whether they are directly connected to one another, they are said to be joined forces through a network.

Computer networks differ in their size, topology, transmission medium, communications protocols used to control traffic, and organizational objectives. Networks of computers enable a wide range.

Using the Internet, electronic audio and video transmission, sharing printer and fax machine use, application and storage servers, email and instant messaging are just a few examples of applications and services. Generally, application-specific protocols layer (or carry over) more general communications protocols as payloads.

Properties

Computer networking can be categorized as a subfield of information technology, computer science, electrical engineering, or telecommunications. Engineering, as it is dependent on the application of the theoretical and practical allied fields of study. Interpersonal communications are made easier and more efficient by computer networks, which enable users to communicate by phone, video conference, chat rooms, email, instant messaging, and other means.

One essential element of many networks is the ability to access data on communal storage devices. Authorized users can access information stored on other computers connected to the network by sharing files, data, and other kinds of information over the network. Sharing network and computer resources is made possible by a network. Networked devices offer resources that users can access and utilize. Users can print documents, for instance, from a network printer that is shared.Distributed computing makes use of a network to accomplish tasks by utilizing computer resources. Computer crackers may use a computer network. Desktop computers crackers to infect connected devices with computer viruses or worms, or to use denial-of-service attacks to stop connected devices from connecting to the network

Network Packet:

Unlike traditional one point to another (the edge) data connections for communications is transmitted as a packet over computer systems network connections (edges) that do not support packets. stream of bits. However, the majority of data in computer networks is transported via packets. Formatted data is transferred over a packet-switched network as a list of bits or bytes, typically with sizes ranging from a few tens to several kilobytes.

Data is formatted into packets in packet networks and sent to its destination via the network. The packets are assembled back into the original message once they arrive. When using packets of information, the communication medium's bandwidth can be more evenly dispersed across individuals as opposed to circuit switching the network. The link (edge) can fill with packets from other users if it isn't overused and one user isn't sending any, allowing the price to be split as evenly as possible.Packets contain two types of data: user data (payload) and control data. The control information provides the network with error detection codes, source and destination network addresses, and other information required to send user data informational ordering. Usually, payload data is situated between the control information-containing trailers and packet headers. The necessary path for a packet

through a network is often not known at all times. After that, the packet is queued and waits for a link (edge) to open.

A community Nodes (Vertices):

Modems, hubs, switches, routers, bridges, firewalls, and network interface controllers (NICs) are some of the additional essential system components that make up networks in addition to any physical transmission media that may be present.

Conclusion

Assume we have a basic line graph $G = (V, E)$. Dominance in graphs is one aspect of graph theory that has been extensively researched. A subset S of vertex contains a graphs can be said to become a dominant set if every one of vertex in the subset either resides in S or is adjacent to a separate vertex within S. The present work obtains new domination results in graphs employing an approach called secure distance matrix domination. A few dominating set theorems for secure distance matrices are described.

References

[1] P. J. Campbell, Gauss and the eight queens problem: A study in miniature of the propagation of historical error. Historia mathematica, *4*(4), 397-404 (1977).

[2] O. Ore, Theory of Graphs, American Mathematical Society, Provedence, R.I, (1962).

[3] F. Harary, Graph Theory, Addison- Wesley, Reading Mass, (1969).

[4] M. S. Rahman, Basic Graph Theory, Springer, India, (2017).

[5] T. W. Haynes and S.T. Hedetniemi, P. J. Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., New York, (1998).

[6] R.B. Bapat, Determinant of the distance matrix of a tree with matrix weights, Linear Algebra Appl. 416, 2–7 (2006).

[7] R.B. Bapat, S.J. Kirkland, M. Neumann, On distance matrices and Laplacians, Linear Algebra Appl. 401, 193–209 (2005).

[8] R.B. Bapat, A.K. Lal, S. Pati, A q-analogue of the distance matrix of a tree, Linear Algebra Appl. 416, 799–814 (2006).

[9] M. Edelberg, M.R. Garey, R.L. Graham, On the distance matrix of a tree, Discrete Math. 14, 23–39 (1976).

[10] M. Fiedler, Algebraic connectivity of graphs, Czechoslovak Math. J. *23*(98), 298–305 (1973).

[11] P.W. Fowler, G. Caporossi, P. Hansen, Distance matrices, wiener indices, and related invariants of fullerenes, J. Phys. Chem. A 105, 6232–6242 (2001).

[12] R.L. Graham, L. Lovász, Distance matrix polynomials of trees, Adv. Math. 29, 60–88 (1978).

[13] E.J. Cockayne, P.J.P. Grobler, W.R. Grundlingh, J. Munganga, J.H. van Vuuren, Protection of a graph, Util. Math. 67 (2005) 19–32. 790

[14] W.F. Klostermeyer, C.M. Mynhardt, Secure domination and secure total domination in graphs, Discuss. Math., Graph Theory 28 (2008) 267–284.

[15] C.M. Mynhardt, H.C. Swart, L. Ungerer, Excellent trees and secure domination, Util. Math. 67 (2005) 255–267.

[16] E. J. Cockayne, Irredundance, secure domination and maximum degree in trees, Discrete Math., 307 (2007), 12–17.

[17] Merouane, H. B., & Chellali, M. (2015). On secure domination in graphs. *Information Processing Letters*, *115*(10), 786-790.

[18] A. Hansberg, D. Meierling and L. Volkmann, Distance Domination and Distance Irredundance in Graphs, Elec. J. Combin. (2007), R35.

[19] J.H. Hattingh and M. A. Henning, The ratio of the distance irredundance and domination numbers of a graph, J. Graph Theory 18 (1994), 1-9.

[20] S. G. Li, On connected k-domination numbers of graphs, Discrete Math. 274 (2004), 303-310.

[21] D. Rautenbach and L. Volkmann, On $\alpha_r\gamma_s(k)$-perfect graphs, Discrete Math. 270 (2003), 241-250.

[22] Tian, F., & Xu, J. M. (2009). A note on distance domination numbers of graphs. *Australas. J Comb.*, *43*, 181-190.

[23] Arora, A., Dey, H. K., & Goel, S. (2023). Distance matrix of enhanced power graphs of finite groups. *arXiv preprint arXiv:2304.04288*.

[24] R.L. Graham, H.O. Pollack, On the addressing problem for loop switching, Bell System Tech. J. 50 (1971) 2495–2519.

[25] M. Edelberg, M.R. Garey, R.L. Graham, On the distance matrix of a tree, Discrete Math. 14 (1976) 23–39

[26] R.L. Graham, L. Lovász, Distance matrix polynomials of trees, Adv. Math. 29 (1978) 60–88.

[27] C. M. Jones, Security and secure-dominating sets in graphs (Doctoral dissertation, Auburn University) (2014).

[28] Randy Davila, Caleb Fast, Michael A. Henning and Franklin Kenter, Lower Bounds on the Distance Domination Number of a Graph, arXiv:1507.08745v1 [math.co], 31 (2015).

[29] Risan Nur Santi, IkaHesti Agustin, Dafik, Ridho Alfarisi, On the locating domination number of corona product, IOP Conf. Series: Journal of Physics: Conf. Series, 1008 (2018), 12–53.

[30] D.P. Salve, E.L. Enriquez, Inverse Perfect Domination in the Composition and Cartesian Product of Graphs, Global Journal of Pure and Applied Mathematics, 2(1) (2016), 1–10.

[31] Santiago Canales, Gregorio Hernaˊndezb, Mafalda Martins and Ineˆs Matosc, Distance domination, guarding and covering of maximal outerplanar graphs, Discrete Applied Mathematics, 181 (2015), 41–49.

[32] D.A.R. Wardani, Dak, I.H. Agustin, E.Y. Kurniawati, On Locating Independent Domination Number of Amalgamation Graphs, IOP Conf. Series: Journal of Physics: Conf. Series, 943 (2017), 12–27.

[33] Lars Jaffke, joungkwon, Torstein J.F.Strmme, Jan Arne Telle, Mim-width III. Graph powers and generalized distance domination problems , Theoretical Computer Science ,796 (2019) 216–236.

[34] Canan Ciftci, Disjunctive Total Domination of Some Shadow Distance Graphs , Fundamental Journal of Mathematics and Applications , 3(2) (2020) 185–193.

[35] E.L. Enriquez, Super Fair Dominating Set in Graphs, Journal of Global Research in Mathematical Archives, 6(2) (2019), 8–14.

[36] E.L. Enriquez , G.T. Gemina, Super Fair Domination in the Corona and Lexicographic Product of Graph, International Journal of Mathematics Trends and Technology, 66(4) (2020), 203–210.

[37] E.L. Enriquez and S.R. Canoy, Jr. Secure Convex Domination in a Graph, International Journal of Mathematical Analysis, 9(7) (2015), 317–325.

[38] E.L. Enriquez, and S.R. Canoy, Jr., Secure Convex Domination in a Graph, International Journal of Mathematical Analysis, 9(7) (2015), 317–325.

[39] G.M. Estrada, C.M. Loquias, E.L. Enriquez and C.S. Baraca, Perfect Doubly Connected Domination in the Join and Corona of Graphs, International Journal of Latest Engineering Research and Applications, 4(7) (2019), 17–21.