<sup>1</sup>Prawar <sup>2</sup>Anjali Naithani <sup>3</sup>H.D. Arora <sup>4</sup>Ekata

# Optimizing Industrial Reliability: A Comparative Study of Hot and Cold Standby Configurations in Three-Unit Parallel Systems



*Abstract:* - This comparative research study explores the efficacy of two parallel systems, Model A employing a hot standby configuration and Model B utilizing a cold standby setup, each comprising three units operating based on demand. The investigation focuses on evaluating the reliability and cost-effectiveness of these systems, employing metrics such as Mean Time to System Failure, availability at full and reduced capacity, repairer busy periods, downtime, and profit analysis using Semi-Markov and regenerative point techniques. Results reveal that Model B (cold standby) outperforms Model A (hot standby) across multiple parameters, including Mean Time to System Failure, availability, repairer busy periods, downtime, and profitability. Specifically, Model B demonstrates higher values for Mean Time to System Failure, Availability at full and reduced capacity, Busy period for repairmen, and Profit in Rs. Consequently, it is inferred that Model B proves to be more reliable and efficient than Model A. These findings underscore the potential advantages of employing cold standby configurations in similar machinery setups, offering insights to enhance system performance and minimize downtime, thus suggesting avenues for optimizing cold standby parallel systems based on the study's outcomes.

Keywords: hot standby, reliability, profit analysis, cold standby, regenerative process, availability, maintainability.

## I. INTRODUCTION

In today's world, for a company or firm to grow, it needs a system that is both cost-effective and reliable and doesn't fail often. So, to maximise the profit and for the smooth functioning of the supply chain, the production must be of good quality, and supply should be regular; this brings the concept of reliability to light. System reliability can be rephrased as the probability of the system, i.e., how long it will last before it wears out entirely or time in between repairs. The firm will most likely consider a system with higher reliability and cost-effectiveness.

And to avoid unforeseen failures, companies use standby units to continue manufacturing their goods. In this study, a firm that manufactures plates is taken under consideration, and a study is done to find out which standby unit would be a better match for the firm, either cold standby or hot standby. Various researchers studied and assessed different models with different kinds of standby units under consideration. Researcher (1) examined the system parameters for a functional system in the paper industry by modelling using RPGT and applying specific situations using cold standby. Later (2) used a contemporary perspective on discrete state space and continuous time. Semi-Markov processes and their applications in reliability and maintenance. Researcher (3) studied the concepts of deterioration, inspection, preventive maintenance (PM), and priority were used to examine a two-unit cold standby system. Systems with different maintenance techniques was discussed (4). A formulated modelling of a 3-unit cold standby (induced draft fan) system operating at complete/reduced capacity is discussed by (5). A multi-state machines that have similar failure causes and their reliability using fuzzy probability and Bayesian networks was studied by (6). A study on the failure mechanism was done and analysed by (7). Depending on the triggering loads, failure mechanisms (FMs) were divided into three categories: environmental load-triggered (Etype), operational load-triggered (O-type), and combination load-triggered (C-type). System reliability-redundancy optimisation with cold-standby was discussed and studied by (8). The expanded nest cuckoo optimisation method is a novel approach to the system reliability-redundancy allocation with a cold-standby strategy discussed in this research (ENCOA). Regarding egg laying and survival cuckoos, ENCOA employs more realistic techniques than the cuckoo optimisation algorithm (COA), which is based on in-depth research of the European cuckoo's way of life that is accessible in the literature. Reverse osmosis and forward osmosis integrated desalination network accessibility and dependability was studied by (9). The probabilities were estimated using fuzzy set theory, and failure probabilities were calculated. Researchers (10) calculated and discussed reliability, availability and

<sup>4</sup>Professor, Department of Engineering, K.I.E.T. Ghaziabad (201206), INDIA. Email: ekata@kiet.edu Copyright © JES 2024 on-line : journal.esrgroups.org

<sup>&</sup>lt;sup>1</sup> Research Scholar, Department of Mathematics, Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida (201303) INDIA. Email: prawar1@s.amity.edu

<sup>&</sup>lt;sup>2\*</sup>Associate Professor, Department of Mathematics, Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida (201303) INDIA. Email: anathani@amity.edu

<sup>&</sup>lt;sup>3</sup>Professor, Head of Department of Mathematics, Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida (201303) INDIA. Email: hdarora@amity.edu

maintainability for the wine packaging industry. A theory of Using functional failures as a basis to make early predictions about the availability and reliability of hardware-software systems coupled was discussed by (11). Also, (12) proposed an assessment of the reliability and availability of a photovoltaic power plant. Later (13,14) proposed an evaluation of the reliability and availability of a photovoltaic power plant. Researchers also (15) discussed parallel system reliability modelling with maximum operation and repair durations. Study on dependability metrics for the performance evaluation of mobile communication systems to evaluate the performance of phone communications and their reliability measures done by (16). Two systems with a singular unit that is subjected to randomly generating shocks that occur randomly was analysed by (17). Also, (18) discussed an analysis of the costs for two-unit warm standby models. They believed an expert was only contacted when a regular repair person could not fix the problem within the allotted time. They had a regular repairperson and patience. The costs and benefits of operating two of the three induced draught fans at cold standby rather than at decreased capacity using a semi-Markov process was discussed by (19). Studies on reliability evaluation of large-scale industries such as steel plant production of biscuits, etc done by (20). (21) studied dependability metrics for the performance evaluation of serial chilled water system under m-out-of-n:G policy and their reliability measures. A study on hydroforming technology and its applications for manufacturing industries was discussed by (22).

This comparative study's primary goal is to examine two parallel systems, each with three components. Both Model A with hot standby and Model B with cold standby are the categories assigned to these systems. They function according to demand. The main goal of this research is to identify the most economical standby system for similar types of machines. The system under consideration is a plate manufacturing business with three identical units. The company manufactures two different kinds of plates: full plates and half plates. Full plates are produced by the first unit, half plates by the second, and either plate type can be produced by the third unit, which is on hot standby in model A and cold standby in model B. The other two can be utilised to finish the job if one of the units malfunctions. The dye will be changed if any unit malfunctions, ensuring that neither the output of either plate is hindered. The system is regarded as being utterly dysfunctional if every unit malfunctions.

This paper comprises the methodology used for both systems to analyse the best alternative, followed by a complete system description of both models, including the state transition diagram and assumptions. Further measures to find an effective model are calculated on the basis of MTSF, availability at total capacity and reduced capacity, time for which repairers are busy when the system is completely down or failed and profit analysis. Graphs are also made to support the argument of the best possible model, and finally, the article concludes which model is best suited for companies with similar system setups.

Op <sub>1</sub>	operative state for machine manufacturing plate 1.			
Op <sub>2</sub>	operative state for machine manufacturing plate 2.			
Fw	failed unit awaiting repair.			
F <sub>r</sub>	failed unit under repair.			
F <sub>R</sub>	unit is being repaired from its prior state.			
S	switching of plates is taking place.			
g(ł),G(ł)	p.d.f and c.d.f of repair time of the unit.			
$\overline{G}(\mathbf{k})$	survival function.			
$C_{\rm S}, H_{\rm S}$	cold standby and hot standby.			
© \$	laplace convolution, laplace stieltjes convolution.			
C <sub>0</sub>	revenue takings per up-time unit when the system is at maximum efficiency			
C <sub>1</sub>	Revenue takings per unit of up-time when the system is operating reduced capacity			
C <sub>2</sub>	revenue takings per up-time unit while the system is operating at decreased capacity			
C <sub>3</sub>	cost per unit of time when the system is not functioning			
$C_4$	payment per unit of time made to a repairman			
AF <sub>0</sub>	the probability that the system operates at full capacity under the condition that it is initially at state 0 at $\frac{1}{2} = 0$			
AR <sub>0</sub>	the probability that the system operates at reduced capacity under the condition that it is			
-	initially at state 0 at $\frac{1}{2} = 0$			
B <sub>0</sub>	Time when the repair man is busy			
DT <sub>0</sub>	expected system downtime			
Greek Symbols				
α	repair rate			
λ	constant rate of failure of the operative unit.			

II. ANNOTATIONS

constant rate of allowed time for switching the plates.

#### III. METHODOLOGY

The study compares the outcomes subsequently. Regenerative point method (RGPT) and Semi-Markov-process (SMP) theories have been used to study the system models to estimate reliability parameters and account for variation in their cost analysis. As examples of the aforementioned systems, two systems with various standby configurations were used, and numerical calculations were made to compare dependability metrics and profit estimation between the two systems.

Assumptions for Model A and Model B

- Initially, all three units are operative.
- All three units are identical.

β,

- The rate of failure of all three identical units is constant.
- Preference is given to switching instead of repair.
- The repairman is readily available for repair.
- An exponential distribution is followed by the failure times.
- The system works perfectly after each repair.
- A unit cannot fail while switching is taking place.
- If all units get failed, the system is considered to be completely failed.
- The unit cannot fail immediately after repair.



#### IV. ANALYSIS OF SYSTEM WITH HOT STANDBY (MODEL A)

Fig. 1: State transition diagram for Model A (Hot Standby)

The system uses Hot Standby with two fully operative units initially at state '0'. The system is fully working. States '1', '4', '5', '7' and '2' are reduced capacity states where hot stand-by takes over the job as per manufacturing demand and '6' is the failed state where no manufacturing takes place and '3' down states. The transition diagram in Fig. 1 further defines it. Fig. 1 shows the State Transition Diagram for model A. The regeneration states are 0, 1, 2, 3, 5, and 7, whereas the down state is 3. In states 0 and 2, the system operates at its peak performance. In states 1, 4, and 7, it functions with less efficiency. Six is the failing state.

A. Transition Probabilities and Mean Sojourn Time Transition probabilities are evaluated as follows:  $dQ_{01}(k) = \lambda e^{-3\lambda k} dk$  $dQ_{02}(i) = 2\lambda e^{-3\lambda i} d i$  $dQ_{12}(\mathbf{k}) = \beta_1 e^{-(\lambda + \beta_1)\mathbf{k}} d\mathbf{k}$  $dQ_{13}(\frac{1}{2}) = \lambda e^{-(\lambda + \beta_1)\frac{1}{2}} d\frac{1}{2}$  $dQ_{20}(h) = e^{-2\lambda h}g(h)dh$  $dQ_{26}^4(a) = (2\lambda e^{-2\lambda a} \odot \lambda e^{-\lambda a})\overline{G}(a)da$  $dQ_{21}^{4}(\frac{1}{2}) = (2\lambda e^{-2\lambda \frac{1}{2}} \odot e^{-\lambda \frac{1}{2}})g(\frac{1}{2}) d\frac{1}{2}$  $dQ_{27}^{4,6}(\frac{1}{2}) = (2\lambda e^{-2\lambda \frac{1}{2}} \odot \lambda e^{-\lambda \frac{1}{2}} \odot 1)g(\frac{1}{2}) d\frac{1}{2}$  $dQ_{35}(i) = \beta_1 e^{-\beta_1 i} di$  $dQ_{52}(h) = e^{-\lambda h}g(h)dh$  $dQ_{56}(\frac{1}{2}) = \lambda e^{-\lambda \frac{1}{2}} \overline{G}(\frac{1}{2}) d\frac{1}{2}$  $dQ_{57}^6(\frac{1}{2}) = (\lambda e^{-\lambda \frac{1}{2}} \odot 1)g(\frac{1}{2}) d\frac{1}{2}$ p<sub>ii</sub> transition probabilities are calculated as follows:  $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$  where  $\frac{dQ_{ij}(t)}{dt} = q_{ij}(t)$  $p_{01} = \frac{1}{3}$  $p_{02} = \frac{2}{3}$  $p_{12} = \frac{\lambda}{\beta_1 + \lambda}$  $p_{13} = \frac{\beta_1}{\beta_1 + \lambda}$  $p_{35} = p_{72} = 1$  $p_{20} = g^*(2\lambda)$  $p_{26}^4 \, = 1 \text{-} \, 2g^*(\lambda) + g^*(2\lambda)$  $p_{21}^4 = 2g^*(\lambda) - 2g^*(2\lambda)$  $p_{27}^{4,6} = 1 - 2g^*(\lambda) + g^*(2\lambda)$  $p_{52} = g^*(\lambda)$  $p_{56} = 1 - g^*(\lambda)$  $p_{57}^6 = 1 - g^*(\lambda)$ 

When time is measured from the epoch of admission into state I, the unconditional mean time taken by the system to transit for each regeneration state j can be expressed mathematically as follows:

$$\begin{split} m_{01} + m_{02} &= \mu_0 \\ m_{12} + m_{13} &= \mu_1 \\ m_{20} + m_{26}^4 + m_{21}^4 &= k_1 \; (say) \\ m_{20} + m_{27}^{4,6} + m_{21}^4 &= k_2 \; (say) \\ m_{35} &= \mu_3 \\ m_{52} + m_{56} &= \mu_5 \\ m_{52} + m_{57}^6 &= k \; (say) \\ m_{72} &= \mu_7 \end{split}$$

The amount of time spent in the regenerative state before changing to another state is known as the mean sojourn time, or µi.

$$\begin{split} \mu_0 &= \int_0^\infty e^{-3\lambda t} dt = \frac{1}{3\lambda} \\ \mu_1 &= \int_0^\infty e^{-(\lambda + \beta_1) t} dt = \frac{1}{\lambda + \beta_1} \\ \mu_2 &= \int_0^\infty e^{-2\lambda t} \, \overline{G}(t) dt = \frac{1 - g^*(2\lambda)}{2\lambda} \\ \mu_3 &= \int_0^\infty e^{-\beta_1 t} dt = \frac{1}{\beta_1} \\ \mu_5 &= \int_0^\infty e^{-\lambda t} \, \overline{G}(t) dt = \frac{1 - g^*(\lambda)}{\lambda} \end{split}$$

 $\mu_7 = \int_0^\infty {}^t\!g({}^t\!) d{}^t\! = - g^{*'}(0)$ 

## V. MEASURES OF SYSTEM EFFECTIVENESS FOR MODEL A

### A. Mean Time To System Failure

A maintenance metric called mean time to system failure (MTSF) counts how long an unrepairable item typically lasts before failing. MTSF can be regarded as an asset's average lifespan because it only applies to equipment and assets that cannot or should not be fixed.

$$\phi_0^{**}(s) = \frac{N_0(s)}{D_0(s)}$$
MTSF for Model A  
M.T.S.F. =  $\frac{D'_0(0) - N'_0(0)}{D_0(0)} = \frac{NA_1}{DA_1}$ 
Where

Where,

$$\begin{split} \mathsf{NA}_1 &= \mu_0 \quad ( \ p_{20} \mathsf{p}_{56} \mathsf{p}_{13} \mathsf{p}_{21}^4 + \mathsf{p}_{26}^4 \mathsf{p}_{13}) + \mu_1 \left( \mathsf{p}_{20} \mathsf{p}_{01} + \mathsf{p}_{01} \mathsf{p}_{21}^4 \mathsf{p}_{26}^4 \right) + k_1 \left( \mathsf{p}_{01} \mathsf{p}_{12} + \mathsf{p}_{01} \mathsf{p}_{52} \mathsf{p}_{13} + \mathsf{p}_{02} \right) + \\ \mu_3 \left( \mathsf{p}_{13} \mathsf{p}_{52} \mathsf{p}_{21}^4 + \mathsf{p}_{20} \mathsf{p}_{01} \mathsf{p}_{52} \mathsf{p}_{13} + \mathsf{p}_{01} \mathsf{p}_{56} \mathsf{p}_{13} + \mathsf{p}_{01} \mathsf{p}_{52} \mathsf{p}_{13} \mathsf{p}_{26}^4 + \mathsf{p}_{21}^4 \mathsf{p}_{02} \mathsf{p}_{56} \mathsf{p}_{13} \right) + \\ \mu_5 \left( \mathsf{p}_{01} \mathsf{p}_{13} + \mathsf{p}_{02} \mathsf{p}_{13} \mathsf{p}_{21}^4 \right) \\ \mathsf{DA}_1 = \mathsf{I} - \mathsf{p}_{21}^4 \mathsf{p}_{12} - \mathsf{p}_{21}^4 \mathsf{p}_{52} \mathsf{p}_{13} - \mathsf{p}_{20} \mathsf{p}_{01} \mathsf{p}_{12} - \mathsf{p}_{20} \mathsf{p}_{01} \mathsf{p}_{52} \mathsf{p}_{13} - \mathsf{p}_{20} \mathsf{p}_{02} \end{split}$$

## B. Availability Analysis of the System at Full Capacity

One of the most crucial metrics in dependability measuring is availability or system availability. It determines if a system is functioning regularly and how well it can recover from a crash, attack, or other sort of failure.

$$\begin{split} &AF_0^*\left(s\right) = \frac{N_2\left(s\right)}{D_2\left(s\right)} \\ &Availability at full capacity for Model A given by \\ &AF_0 = \lim_{s \to 0} sAF_0^*\left(s\right) = \frac{N_2\left(0\right)}{D_2'\left(0\right)} = \frac{NA_2}{DA_2} \ , \text{ where} \\ &DA_2 = k_2 \quad + \quad \mu_0 \, p_{20} \quad + \quad \mu_1(p_{20}p_{01} + p_{21}^4) + \mu_3(p_{20}p_{13}p_{01} + p_{13}p_{21}^4) + k(p_{20}p_{13}p_{01} + p_{13}p_{21}^4) + \\ &\mu_7\left(p_{20}p_{13}p_{01}p_{57}^6 + p_{13}p_{21}^4p_{57}^6 + p_{27}^{4,6}\right) \\ &NA_2 = \mu_0 \, p_{20} \end{split}$$

### C. Availability Analysis of the System at Reduced Capacity

When the system works partially ie not completing the desired work but can partially complete the work that is needed to be done is called availability at reduced capacity

$$\begin{split} AR_{0}^{*}(s) &= \frac{N_{3}(s)}{D_{2}(s)} \\ Availability at reduced capacity for Model A \\ AR_{0} &= \lim_{s \to 0} sAR_{0}^{*}(s) = \frac{N_{3}(0)}{D_{2}'(0)} = \frac{NA_{3}}{DA_{2}} \\ where, \\ NA_{3} &= \mu_{1}p_{01}p_{27}^{4,6} - \mu_{2}p_{01}p_{13}p_{57} + \mu_{5}p_{01}p_{13}p_{27}^{6} - \mu_{7}(p_{02}p_{27}^{4,6} + p_{01}p_{13}p_{57}^{6} + p_{01}p_{13}p_{52}p_{27}^{4,6} + p_{01}p_{12}p_{27}^{4,6} + p_{02}p_{13}p_{57}p_{21}^{4}) \\ DA_{2} &= k_{2} + \mu_{0}p_{20} + \mu_{1}(p_{20}p_{01} + p_{21}^{4}) + \mu_{3}(p_{20}p_{13}p_{01} + p_{13}p_{21}^{4}) + k(p_{20}p_{13}p_{01} + p_{13}p_{21}^{4}) + \mu_{7}(p_{20}p_{13}p_{01}p_{57}^{6} + p_{13}p_{21}^{4}p_{57}^{6} + p_{27}^{4,6}) \end{split}$$

### D. Busy Period Analysis of Repairman $(B_0)$

Let  $B_i(t)$  be the probability that a repairman is busy with the system in the interval (0,t), then in the long run the total fraction of time for which a repairman is busy

$$\begin{split} B_0^*(s) &= \frac{N_4(s)}{D_2(s)} \\ Busy \text{ period of repairman in Model A given by} \\ B_0 &= \lim_{s \to 0} sB_0^*(s) = \frac{N_4(0)}{D_2'(0)} = \frac{NA_4}{DA_2} \\ \text{where,} \end{split}$$

$$\begin{split} &\mathrm{NA}_{4} = (\mu_{7} + \mu_{3})(\,p_{01}p_{13}p_{27}^{46}) + \mu_{7}\,(\,-p_{02}p_{27}^{46} - p_{01}p_{12}p_{27}^{46} - p_{13}p_{01}p_{52}p_{27}^{46} - p_{01}p_{12}p_{57}^{6} - p_{02}p_{13}p_{57}^{6}p_{21}) + \\ &\mu_{1}(p_{27}^{46}p_{01}) + \mu_{2}(p_{57}^{6}p_{01}p_{13}) \\ &\mathrm{DA}_{2} = k_{2} + \mu_{0}\,p_{20} + \mu_{1}(p_{20}p_{01} + p_{21}^{4}) + \mu_{3}(p_{20}p_{13}p_{01} + p_{13}p_{21}^{4}) + k(p_{20}p_{13}p_{01} + p_{13}p_{21}^{4}) + \\ &\mu_{7}(p_{20}p_{13}p_{01}p_{57}^{6} + p_{13}p_{21}^{4}p_{57}^{6} + p_{27}^{4,6}) \end{split}$$

## E. Expected Down Time of the System $(DT_0)$

Denotes the total time the System is entirely unusable during the Scheduled Operation Time.  $DT_0^* (s) = \frac{N_5(s)}{D_2(s)}$ Downtime of Model A can be calculated by  $DT_0 = \lim_{s \to 0} sDT_0^* (s) = \frac{N_5(0)}{D'_2(0)} = \frac{NA_5}{DA_2}$ Where,  $NA_5 = \mu_3 p_{01} p_{13} p_{27}^{4,6}$   $DA_2 = k_2 + \mu_0 p_{20} + \mu_1 (p_{20} p_{01} + p_{21}^4) + \mu_3 (p_{20} p_{13} p_{01} + p_{13} p_{21}^4) + k(p_{20} p_{13} p_{01} + p_{13} p_{21}^4) + \mu_7 (p_{20} p_{13} p_{01} p_{57}^6 + p_{13} p_{21}^4 p_{57}^6 + p_{27}^{4,6})$ 

F. Profit Generated by the Model

Profit generated by Model A given by  $PA = C_0 (AF_0) + C_1 (AR_0) - C_2 (B_0) - C_3 (DT_0) - C_4$ Where,  $\beta = 0.5, \alpha = 0.3, \lambda = 0.09, C_0 = 2600, C_1 = 1600, C_2 = 500, C_3 = 4200, C_4 = 50$ 

#### VI. ANALYSIS OF SYSTEM WITH COLD STANDBY (MODEL B)

The Model B State Transition Diagram is displayed in Figure 2. The down state is 2, whereas the regeneration states are 0, 1, 2, 3, and 4. The system functions at its best in states 0 and 3. It operates less effectively in states 1 and 4. The failing state is five.



Figure 2: State transition diagram for Model B (cold standby).

VII. TRANSITION PROBABILITIES AND MEAN SOJOURN TIME (MODEL B) Transition probabilities are evaluated as follows:  $dQ_{01}(t) = 2\lambda e^{-2\lambda t} dt$   $dQ_{12}(t) = \lambda e^{-(\lambda + \beta_1)t} dt$   $dQ_{13}(t) = \beta_1 e^{-(\lambda + \beta_1)t} dt$   $dQ_{24}(t) = \beta_1 e^{-\beta_1 t} dt$  
$$\begin{split} dQ_{43}(t) &= e^{-\lambda t}g(t)dt \\ dQ_{44}^5(t) &= (\lambda e^{-\lambda t} \odot 1)g(t) dt \\ dQ_{45}(t) &= \lambda e^{-\lambda t} \overline{G}(t)dt \\ dQ_{30}(t) &= g(t)dt \\ p_{ij} \text{ transition probabilities are calculated as follows:} \\ p_{ij} &= \lim_{s \to 0} q_{ij}^*(s) \text{ where } \frac{dQ_{ij}(t)}{dt} = q_{ij}(t) \\ p_{12} \neq p_{13} = 1 \\ p_{30} &= p_{24} = p_{01} = 1 \\ p_{43} \neq p_{45} = 1 \end{split}$$

$$p_{43} + p_{44}^5 = 1$$

When time is measured from the epoch of admission into state I, the unconditional mean time taken by the system to transit for each regeneration state j can be expressed mathematically as follows:

$$m_{01} = \mu_0$$
  

$$m_{12} + m_{13} = \mu_1$$
  

$$m_{30} = \mu_3$$
  

$$m_{24} = \mu_2, m_{45} = k$$
  

$$m_{43} + m_{44}^5 = \mu_3$$
  

$$m_{43} + m_{45} = \mu_4$$

The amount of time spent in the regenerative state before changing to another state is known as the mean sojourn time, or µi.

$$\begin{split} \mu_{0} &= \int_{0}^{\infty} e^{-2\lambda t} dt = \frac{1}{2\lambda} \\ \mu_{1} &= \int_{0}^{\infty} e^{-(\lambda + \beta_{1})t} dt = \frac{1}{\lambda + \beta_{1}} \\ \mu_{2} &= \int_{0}^{\infty} e^{-\beta_{1}t} dt = \frac{1}{\beta_{1}} \\ \mu_{4} &= \int_{0}^{\infty} e^{-\lambda t} \bar{G}(t) dt = \frac{1 - g^{*}(\lambda)}{\lambda} \\ \mu_{5} &= \mu_{3} = \int_{0}^{\infty} tg(t) dt = -g^{*'}(0) \end{split}$$

### VIII. MEASURES OF SYSTEM EFFECTIVENESS FOR MODEL B

#### A. Mean Time To System Failure

A maintenance metric called mean time to system failure (MTSF) counts how long an unrepairable item typically lasts before failing. MTSF can be regarded as an asset's average lifespan because it only applies to equipment and assets that cannot or should not be fixed.

$$\begin{split} \hat{\phi}_{0}^{**}(s) &= \frac{N_{0}(s)}{D_{0}(s)} \\ \text{MTSF for Model B} \\ \text{M.T.S.F.} &= \frac{D_{0}'(0) - N_{0}'(0)}{D_{0}(0)} = \frac{NB_{1}}{DB_{1}} \\ \text{where} \\ \text{NB}_{1} &= \mu_{0} \quad (p_{13} + p_{12}p_{43}) + \mu_{3} \quad (p_{13} + p_{12}p_{43}) + \mu_{1} + \mu_{4}p_{12} + \mu_{2}p_{12}p_{43} + kp_{12} + \mu_{0}p_{12}p_{45} + \mu_{4}p_{45} + \mu_{2}p_{12}p_{45} \\ \text{DB}_{1} &= 1 - p_{13} - p_{12}p_{43} \end{split}$$

### B. Availability Analysis of the System at Full Capacity

One of the most crucial metrics in dependability measuring is availability or system availability. It determines if a system is functioning regularly and how well it can recover from a crash, attack, or other sort of failure.

$$\begin{split} AF_{0}^{*}(s) &= \frac{N_{2}(s)}{D_{2}(s)} \\ Availability at full capacity for Model B given by \\ AF_{0} &= \lim_{s \to 0} sAF_{0}^{*}(s) = \frac{N_{2}(0)}{D'_{2}(0)} = \frac{NB_{2}}{DB_{2}} , \text{ where} \\ NB_{2} &= \mu_{0} + \mu_{3}(p_{13}) + \mu_{3}(p_{12}p_{43}) + \mu_{0}(p_{44}^{5}) + \mu_{3}(p_{13}p_{44}^{5}) \end{split}$$

 $DB_{2=}(1 + p_{44}^5)(\mu_0 + \mu_1 + \mu_3) + \mu_2(p_{12} + p_{12}p_{44}^5)$ 

#### C. Availability Analysis of the System at Reduced Capacity

When the system works partially ie not completing the desired work but can partially complete the work that is needed to be done is called availability at reduced capacity

 $\begin{aligned} AR_0^*(s) &= \frac{N_3(s)}{D_2(s)} \\ Availability at reduced capacity for Model B \\ AR_0 &= \lim_{s \to 0} sAR_0^*(s) = \frac{N_3(0)}{D'_2(0)} = \frac{NB_3}{DB_2} \\ where, \\ NB_{3=}\mu_1 + \mu_4 p_{12} + \mu_1 p_{44}^5 \\ DB_{2=}(1 + p_{44}^5)(\mu_0 + \mu_1 + \mu_3) + \mu_2(p_{12} + p_{12}p_{44}^5) \end{aligned}$ 

## D. Busy Period Analysis of Repairman $(B_0)$

Let  $B_i(t)$  be the probability that a repairman is busy with the system in the interval (0,t), then in the long run the total fraction of time for which a repairman is busy

$$\begin{split} &B_0^* \left(s\right) = \frac{N_4 \left(s\right)}{p_2 \left(s\right)} \\ &\text{Busy period of repairman in Model B given by} \\ &B_0 = \lim_{s \to 0} sB_0^* \left(s\right) = \frac{N_4 \left(0\right)}{p_2' \left(0\right)} = \frac{NB_4}{DB_2} \\ &\text{where,} NB_4 = \mu_1 + \mu_2 (p_{12}) + \mu_3 (p_{13}) + \mu_3 (p_{43} p_{12}) + \mu_4 (p_{12}) + \mu_1 (p_{44}^5) + \mu_2 (p_{44}^5 p_{12}) + \mu_3 (p_{44}^5 p_{13}) \\ &DB_{2=} (1 + p_{44}^5) (\mu_0 + \mu_1 + \mu_3) + \mu_2 (p_{12} + p_{12} p_{44}^5) \end{split}$$

#### E. Expected Down Time of the System $(DT_0)$

Denotes the total time the System is entirely unusable during the Scheduled Operation Time.  $DT_0^* (s) = \frac{N_5(s)}{D_2(s)}$ Downtime of Model B calculated by  $DT_0 = \lim_{s \to 0} sDT_0^* (s) = \frac{N_5(0)}{D'_2(0)} = \frac{NB_5}{DB_2}$ where  $NB_5 = \mu_2 p_{12} p_{44}^5 + \mu_2 p_{12}$   $DB_{2=}(1 + p_{44}^5)(\mu_0 + \mu_1 + \mu_3) + \mu_2(p_{12} + p_{12} p_{44}^5)$ 

F. Profit Generated by the Model

Profit generated by Model B given by  $PA = C_0 (AF_0) + C_1 (AR_0) - C_2 (B_0) - C_3 (DT_0) - C_4$ Where,  $\beta = 0.5, \alpha = 0.3, \lambda = 0.09, C_0 = 2600, C_1 = 1600, C_2 = 500, C_3 = 4200, C_4 = 50$ 

### IX. COMPARISION BETWEEN MODEL A AND MODEL B

Based on the research findings listed in Table 1, it is evident that Model B (cold standby) surpasses Model A (hot standby) across various metrics including Mean Time to System Failure, Availability at full capacity, Availability at reduced capacity, Busy period for repairmen, Downtime of the system, and Profit in Rs. Model B exhibits notably higher values for Mean Time to System Failure (104372617 hours), Availability at full capacity (0.996434168), Availability at reduced capacity (0.035161232), Busy period for repairmen (0.093756692), and Profit in Rs (2014.082) in comparison to Model A. Thus, it can be concluded that Model B (cold standby) proves to be more reliable and efficient than Model A (hot standby) based on the evaluated measures.

Table 2 compares the profit generated by Model A (hot standby) and Model B (cold standby) for increasing failure rate values for three different repair rate values.

Table 3 is a comparison of values for availability expression with changing values of failure rate for three distinct values of repair rates where Model A is hot standby and Model B is cold standby.

uble 1. Comparison of various reliability and cost maleators for model if and model b					
S.no.	Measures	Model A	Model B		
1	Mean Time to System Failure (in hrs.)	4331866	104372617		
2	Availability at full capacity	0.9844752	0.996434168		
3	Availability at reduced capacity	0.00001787	0.035161232		
4	Busy period for repairmen	0.0000267	0.093756692		
5	Downtime of system	0.0000003174445	0.000062989		
6	Profit in Rs	168.881516	2014.082		

Table 1: Comparison of various reliability and cost indicators for Model A and Model B

Table 2. Profit generated by Model A and Model B for different repair rate values with respect to the failure rates.

Lambda	Alpha 0.2		Alpha 0.3		Alpha 0.4	
	Model A	Model B	Model A	Model B	Model A	Model B
0.04	151.8805	1701.583	199.4638	2154.806	234.7689	2625.409
0.05	105.3474	1030	140.3421	1265.905	167.3387	1515.222
0.06	77.9333	697.1994	104.7360	835.1481	126.0108	983.9122
0.07	60.3560	508.5133	81.5332	595.8143	98.7088	692.1101
0.08	48.3712	391.0402	65.5200	449.5151	79.6617	515.6458
0.09	39.8079	312.7072	53.9739	353.5701	65.8086	401.0676

Table 3. Availability of Model A and Model B for different values of repair rates concerning failure rate.

Lambda	Alpha 0.2		Alpha0.3		Alpha 0.4	
	Model A	Model B	Model A	Model B	Model A	Model B
0.04	0.188949487	0.892009	0.25080750	0.884936	0.30187414	0.880537
0.05	0.16472471	0.87344	0.21251785	0.864625	0.25791165	0.858934
0.06	0.14630640	0.856464	0.18471123	0.846192	0.22537949	0.839339
0.07	0.13181254	0.840708	0.16359465	0.829255	0.20032862	0.821384
0.08	0.12009671	0.825924	0.14700865	0.813532	0.18044101	0.804789
0.09	0.11042039	0.811933	0.13363296	0.798815	0.16426620	0.789335



Figure 3: Comparison of profit generated by System A and System B for different values of failure rate.

The graph plotted in Figure 3 compares the profit generated by the two models concerning increments in failure rates. Model A (hot standby) gives less profit when the failure rate is slightly increased; similarly, model B (cold standby) shows the same for identical values of failure rates.



Figure 4. Comparison of Mean Time To System Failure for different failure rates.

The graph plotted in Figure 4 is a comparison between the mean time to system failure of Model A (Hot Standby) and of Model B (Cold Standby) for increasing failure rates. This depicts that for increasing values of failure rate, both the Models shows decrement in values of mean time to system.

#### X. CONCLUSION

When the same system is analysed with two different kinds of standby units, model A with Hot standby and model B with cold standby, respectively, for the same system, there has been a huge difference when the system of a plate manufacturing firm is considered with two kinds of standby. In comparison, we found out that the mean time to system failure is higher when the system runs with a cold standby compared to a hot standby. Similarly, there has been a huge difference in profit generated by both systems. The one with cold standby generates more profit when compared to the one with hot standby. Graphs with proof have been attached, and a comparison table has been made for the same on different values of repair rates to support the argument of the same. The availability analysis for the same has been done, and values are then compared and inserted in the table.

In conclusion, the research conducted aimed to evaluate the availability and reliability of a system, with a specific focus on the mean time to system failure (MTSF). The findings indicate that among various system options, the three-unit cold standby parallel system, consisting of one cold standby unit operating based on demand, demonstrated the highest levels of dependability and availability. These results provide valuable insights for improving system performance and reducing downtime, suggesting that designing and optimizing cold standby parallel systems based on the study's findings can be a viable strategy.

This model will support the system with similar configurations and will be helpful to predict the mean time to system failure and maximise the profit for similar systems with minimum and optimal cost spent and to run the system with maximum efficiency and profit.

#### REFERENCES

- Kumar A. Cost-Benefit Study of a Cold Standby Structure With Waiting Time Aimed at Repair. SSRN Electronic Journal. 2022; DOI: 10.2139/ssrn.4044551
- Grabski F. Semi-Markov Processes: Applications in System Reliability and Maintenance. Elsevier; 2015. DOI: 10.1016/C2013-0-14260-2
- Kumar A, Gupta SK, Taneja G. Probabilistic analysis of a two-unit cold standby system with instructions at need. Microelectronics Reliability. 1995;35(5):829–32. DOI: 10.1016/0026-2714(94)00099-A
- [4] Gitanjali. Reliability Measurement of Complex Industrial Redundant Systems Using Semi-Markov Process and Regenerative Point Technique. In Recent Advances in Metrology . Lecture Notes in Electrical Engineering, vol 906. Springer, 2023. DOI: 10.1007/978-981-19-2468-2\_25
- [5] Kumar J, Goel M. Availability and profit analysis of a two-unit cold standby system for general distribution. Cogent Mathematics. 2016;3(1):1262937. DOI: 10.1080/23311835.2016.1262937
- [6] Li Z, Xu T, Gu J, Dong Q, Fu L. Reliability modelling and analysis of a multi-state element based on a dynamic Bayesian network. R Soc Open Sci. 2018;5(4):171438. DOI: 10.1098/rsos.171438

- [7] Chen Y, Wang Z, Li Y, Kang R, Mosleh A. Reliability analysis of a cold-standby system considering the development stages and accumulations of failure mechanisms. Reliab Eng Syst Saf. 2018;180:1–12. DOI: 10.1016/j.ress.2018.06.022
- [8] Mellal MA, Zio E. System reliability-redundancy optimization with cold-standby strategy by an enhanced nest cuckoo optimization algorithm. Reliab Eng Syst Saf. 2020;201:106973. DOI: 10.1016/j.ress.2020.106973
- [9] Safder U, Ifaei P, Nam K, Rashidi J, Yoo C. Availability and reliability analysis of integrated reverse osmosis forward osmosis desalination network. Desalination Water Treat. 2018;109:1–7. DOI: 10.5004/dwt.2018.22147
- [10] Tsarouhas P. Reliability, availability and maintainability (RAM) analysis for wine packaging production line. International Journal of Quality & Reliability Management. 2018;35(3):821–42. DOI: 10.1108/IJQRM-02-2017-0026
- [11] Li Y-F, Huang H-Z, Mi J, Peng W, Han X. Reliability analysis of multi-state systems with common cause failures based on Bayesian network and fuzzy probability. Ann Oper Res. 2022;311(1):195–209. DOI: 10.1007/s10479-019-03247-6
- [12] Stember LH, Huss WR, Bridgman MS. A Methodology for Photovoltaic System Reliability & Commic Analysis. IEEE Trans Reliab. 1982; R-31(3):296–303. DOI: 10.1109/TR.1982.5221344
- [13] Zhou N, López-Puente V, Wang Q, Polavarapu L, Pastoriza-Santos I, Xu Q-H. Plasmon-enhanced light harvesting: applications in enhanced photocatalysis, photodynamic therapy and photovoltaics. RSC Adv. 2015;5(37):29076–97. DOI: 10.1039/C5RA01819F
- [14] Abdulla H, Sleptchenko A, Nayfeh A. Photovoltaic systems operation and maintenance: A review and future directions. Renewable and Sustainable Energy Reviews. 2024;195:114342. DOI: 10.1016/j.rser.2024.114342
- [15] Malik SC, Rathee R. Reliability modelling of a parallel system with maximum operation and repair times. International Journal of Operational Research. 2016;25(1):131. DOI: 10.1504/IJOR.2016.073255
- [16] Ram M, Nagiya K. Performance evaluation of mobile communication system with reliability measures. International Journal of Quality & Reliability Management. 2016;33(3):430–40. DOI: 10.1108/IJQRM-01-2015-0013
- [17] Raghav YS, Mradula, Varshney R, Modibbo UM, Ahmadini AAH, Ali I. Estimation and Optimization for System Availability Under Preventive Maintenance. IEEE Access. 2022;10:94337–53. DOI: 10.1109/ACCESS.2022.3204394
- [18] Murari K, Goyal V, Rani S. Cost analysis in two-unit warm standby models with a regular repairman and patience time. Microelectronics Reliability. 1985;25(3):473–83. DOI: 10.1016/0026-2714(85)90199-4
- [19] Naithani A, Parashar B, Bhatia PK, Taneja G. Cost Benefit Analysis of a 2-Out-of-3 Induced Draft Fans System with Priority for Operation to Cold Standby over Working at Reduced Capacity. Vol. 15, AMO-Advanced Modeling and Optimization 2013.
- [20] Parashar B, Taneja G. Reliability and Profit Evaluation of a PLC Hot Standby System Based on a Master-Slave Concept and Two Types of Repair Facilities. IEEE Trans Reliab. 2007;56(3):534–9. DOI: 10.1109/TR.2007.903151
- [21] Yusuf I, Sanusi A, Mus' NA, Musa abu, Sulaiman S. Performance optimisation of serial chilled water system under m-out-of-n: G policy. International Journal of Process Management and Benchmarking. 2024;16(3):376–99. DOI: 10.1504/IJPMB.2024.136480
- [22] Grote K-H, Antonsson EK, editors. Springer Handbook of Mechanical Engineering. Berlin, Heidelberg: Springer Berlin Heidelberg; 2009. DOI: 10.1007/978-3-540-30738-9