¹ Boyi Venkata Ramana

² Dr. T. Narasimhulu

³ Prof. P. Mallikarjuna Rao

Using Affine Arithmetic to Design a Controller for Large Uncertain System

Abstract: - This study presents a approach for creating the supervisor of expansive uncertain systems. The Controller for a particular high order system is designed using a reduced order model. The numerator and denominator polynomial in the suggested reduction approach is derived using modified polynomial differential method. A lower order model with least ISE optimization is obtained. Assuming that the initial high-order system is stable, the proposed approach guarantees the stability of the streamlined model. Through With reference to the original high-order systems, a PID controller is created for the suggested low-order model. In order to explain the method's efficiency, a few numerical examples were taken into consideration. It has been demonstrated that applying Control from the lower order model to the higher order system is improved and the controlled system's performance. Common numerical illustrations seen in the literature have been used to test the method, and the results show that it works satisfactorily.

Keywords: Controller; Affine Arithmetic; Order Reduction; Uncertain Systems

1. INTRODUCTION

It is intrinsically exceedingly difficult to build, analyze, and govern a higher order system. This makes it possible to model lower order systems that are generated from higher order systems in order to represent higher order systems. In engineering and science technologies, uncertainty is involved to variable degrees, which gives rise to analysis and design problems. Issues with the data may have an impact on the kind of uncertainty that exists in the system. As an illustration, consider the following scenarios: data that has vanished or is unavailable; data that is present but unpredictable or uncertain as a result of computation errors; data that is inaccurately or unpredictably represented; etc. There are several approaches to characterize uncertainty, including probabilistic, rounded, or fuzzy descriptions. Still, a lot of systems have constant elements. [1-4].

The primary benefit of interval arithmetic is its ability to regulate internal computation flaws like rounding errors and to design and analyze potential magnitude uncertainties. Its main drawback is its tendency to encourage more conservative or conventional processes. Often to the point of inefficiency, the interval provides a far more complete range than the actual range of the related quantities. This issue is particularly problematic in lengthy computation sequences since, in this case, the intervals are calculated at one step and used as inputs for a subsequent stage. [5-7].

The primary motivation behind using affine arithmetic is to progress the dependence issue, which arises particularly when evaluating a function's range. Every numerical issue that involves crucial guaranteed enclosures to continuous functions, including solving non-linear equations, analyzing dynamical systems, solving differential equations and integrating functions, etc., most likely uses algebraic arithmetic. Ray tracing, curve modeling, parametric surfaces, range analysis, dependency problems, process management, and electric circuit analysis are a few examples of applications using Affine arithmetic. Affine arithmetic is intended to be a specific advancement of interval arithmetic (IA). It is comparable to Taylor arithmetic in the first-order, the closed on-slope model,

¹* Research Scholar, AUCE, venkat.boyi@gmail.com

²Associate Professor, BITS, narsim.223@gmail.com

³Professor, AUCE, electricalprofessor@gmail.com

ellipsoid calculus, and stable is established interval arithmetic in that it is a self-regulating technique to derive first-order approved approximations to [9-10].

In many cases, affine arithmetic is able to produce interval approximations that are much better than those obtained with standard interval arithmetic because of the suggested method for solving the interval arithmetic problem by using some computational models and maintaining correlations between input and computed quantities of first-order system. These correlations are therefore exploited in original operations. Additionally, interval approaches can be made more efficient by using affine arithmetic, which effectively gives a geometric representation for the total range of associated quantities. Similar to interval arithmetic, affine arithmetic automatically affects each calculated size to provide the round-off and transition errors. [11-12]. Next, by changing the interpolation points, an ideal model (with least ISE) is produced An PID controller is created for the original high order systems with the proposed method.

PID controllers can be used in a variety of control system applications and function well enough without requiring any modifications or only coarse tweaking. However, they lack direct process knowledge, therefore their overall performance is compromise and reactive. The best controller available for an observer is this one, which provides superior performance when the process actor is explicitly modeled without the need for an observer. When Reduced PID loop gains are necessary to keep the control system from oscillating, overshooting, or hunting about the control set point value; PID controllers by themselves may not work well in certain situations. They also react slowly, suffer in the presence of non-linearities, and may be unable to respond quickly enough for control.

II. REDUCTION PROCEDURE

This section provides an interpretation of the suggested lower order reduction technique. The Modified Polynomial Differentiation method [13-15] using a procedure, the lower order coefficients are obtained, without taking into account the original system's time instants. The necessary Polynomial Differentiation method is computed using the following equations in the suggested manner.

Take a look at the interval system's higher-order transfer function as follows:

$$G(s) = \frac{[d_1^-, d_1^+]s^{n-1} + [d_2^-, d_2^+]s^{n-2} + \cdots + [d_n^-, d_n^+]}{[c_0^-, c_0^+]s^n + [c_1^-, c_1^+]s^{n-1} + \cdots + [c_n^-, c_n^+]} \quad \ldots \ldots \quad (1)$$

The rth order reduced model $R_r(s)$ is characterized by

$$R_r(s) = \frac{D_r(s)}{C_r(s)} \quad \ldots \ldots \quad (2)$$

Where, $C_r(s) = s^2 C_{r-2}(s) + [\gamma_r^-, \gamma_r^+]C_{r-1}(s)$

$$D_r(s) = [\delta_r^-, \delta_r^+]s^{r-1} + s^2 D_{r-2}(s) + [\gamma_r^-, \gamma_r^+]D_{r-1}(s) \quad \ldots \ldots \quad (3)$$

III. DESIGN PROCEDURE

The following are the steps in the design process:

1) Determine the constants for a system by finding the closed-loop transfer function T(S) from the controlled system (Gc(s))..

2) Select the constants from the closed-loop's characteristic equation based on the ITAE performance

index optimum coefficient for the lowest ITAE. for step input is given in table 4.1.

3)   The transfer function for universal closed loops in the instance of the zero steady state step error system is

$T(w) = \dfrac{C(w)}{R(w)} = \dfrac{a_n}{w^n + a_1 w^{n-1} + \ldots + a_{n-1} w + a_n}$
$w + w_n$
$w^2 + 1.4 w_n w + w_n^2$
$w^3 + 1.75 w_n w^2 + 2.15 w_n^2 w + w_n^3$
$w^4 + 2.1 w_n w^3 + 3.4 w_n^2 w^2 + 2.7 w_n^3 w + w_n^4$

Table.1 Forms of the closed loop transfer function that are optimal based on the

ITAE criterion (zero steady –state step error system)

$T(w) = \dfrac{C(w)}{R(w)} = \dfrac{a_{n-1} w + a_n}{w^n + a_1 w^{n-1} + \ldots + a_{n-1} w + a_n}$
$w^2 + 3.2 w_n w + w_n^2$
$w^3 + 1.75 w_n w^2 + 3.25 w_n^2 w + w_n^3$
$w^4 + 2.41 w_n w^3 + 4.93 w_n^2 w^2 + 5.14 w_n^3 w + w_n^4$

Table.2: versions of the transfer function in a closed loop that are ideal according to the ITAE standard (zero steady –state Ramp Error system)

4)   From the above step we can write T(S) in form of known coefficient except for Wn. From these we can get the constants of $G_c(S)$ in the terms of W_n.

5)   For the specified setting time, choose a suitable value of ξ and then find W_n.

6)   Now full $G_c(S)$ is known as so in T(S).

7)   Presence of zeros of $G_c(S)$ in T(S) normally does not allow the % overshoot requirement to be met.

8)   By determining a pre-filter $G_p(S)$, we can eliminate the zeros of T(S).

IV. CASE STUDY

Consider the 4th order system as follows:

$$G(s) = \frac{d_n(s)}{e_n(s)}$$

$$d_n(s) = [6.2164,1]s^3 + [4.6146,6]s^2 + [1.7134,11]s + [0.25,6]$$

$$e_n(s) = [73.018,1]s^4 + {}'[50.03,17]s^3 + [17.104,82]s^2 + [1.919,130]s + [0.25,100]$$

Using the suggested method described in section 2 in the subsequent stages, For the higher order system stated earlier, a second order reduced model is generated.

$$R_2(s) = \frac{a_k(s)}{b_k(s)}$$

After applying the Routh approach and normalizing the above, the reduced order denominator is:

$$b_k(s) = [0.9,1]s^2 + [0.07881,1.4409]s + [0..0186,1.3449]$$

Reduced order numerator (by applying modified routh approximation method):

The reduced order model is obtained as,

$$R_2(s) = \frac{[0.05244, 0.1205]s + [0.0194, 0.0806]}{[0.9, 1]s^2 + [0.07881, 1.4409]s + [0..0186, 1.3449]}$$
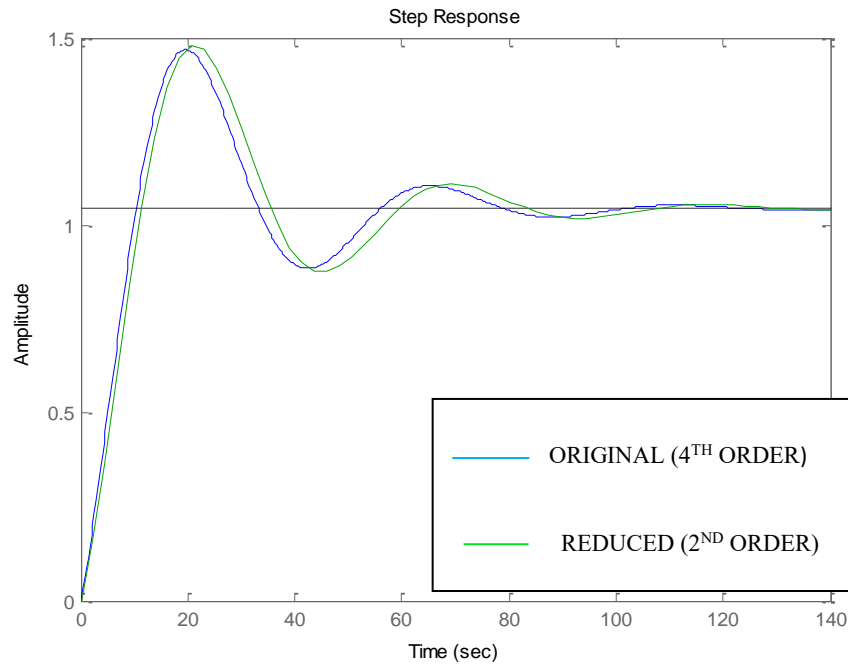

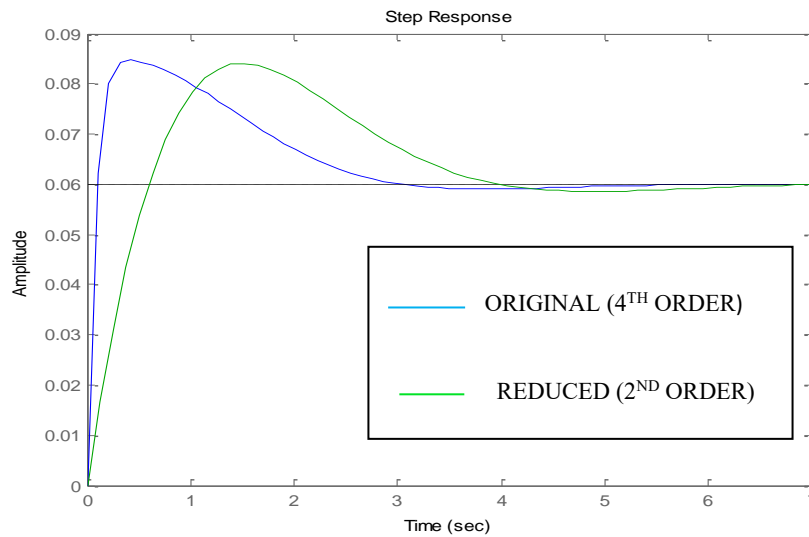Fig.1   Original and lower order systems' step responses

Fig. 2 Step reactions of original and lower order systems

The following is the design of the PID controller for the reduced order model:

Let the transfer function of the PID controller be as

$$G_c(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$

Utilizing the reduced model and the ITAE performance index approach, the values of Kp, Ki and Kd are obtained

$$R_2(s) = \frac{0.05244s + 0.0194}{s^2 + 0.07881s + 0.0186}$$

To now acquire the reduced order closed-loop transfer function

The tuned PID values are

$$K_p = -9.1331, \qquad K_i = 7.654, \qquad K_d = 23.899$$

Contrasting the compensated system's characteristic equation with the ideal ITAE characteristic equation as,

$$w^3 + 1.75w_n w^2 + 3.25w_n^2 w + w_n^3$$

The following is the system's closed-loop transfer function with unity feedback after the PID controller is added to the forward path:

$$T_c(s) = \frac{G_C(S)G(S)}{1 + G_C(S)G(S)}$$

where Gc(s) is the PID controller transfer function and G(s) is the high order system.

Utilizing the reduced model and the ITAE performance index approach, Kp, Ki, and Kd values are acquired.

$$R_2(s) = \frac{0.1205s + 0.0806}{s^2 + 1.4409s + 1.3449}$$

To now acquire the reduced order closed-loop transfer function

The tuned PID values are

$$K_p = 0.889, \qquad K_i = 133.33, \qquad K_d = 19.183$$

contrasting the compensated system's characteristic equation with the ideal ITAE characteristic equation as,

$$s^3 + 1.75w_n s^2 + 3.25w_n^2 s + w_n^3$$

The following is the system's closed-loop transfer function with unity feedback after the PID controller is added to the forward path:

$$T_c(s) = \frac{G_C(S)G(S)}{1 + G_C(S)G(S)}$$

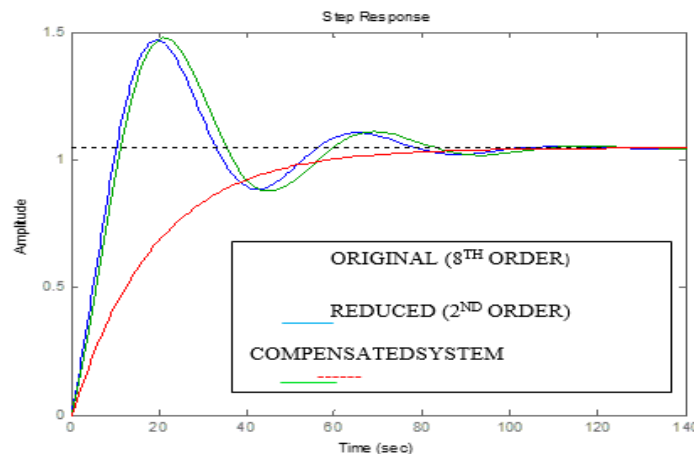Where Gc(s) is the PID controller transfer function and G(s) is the high order system.

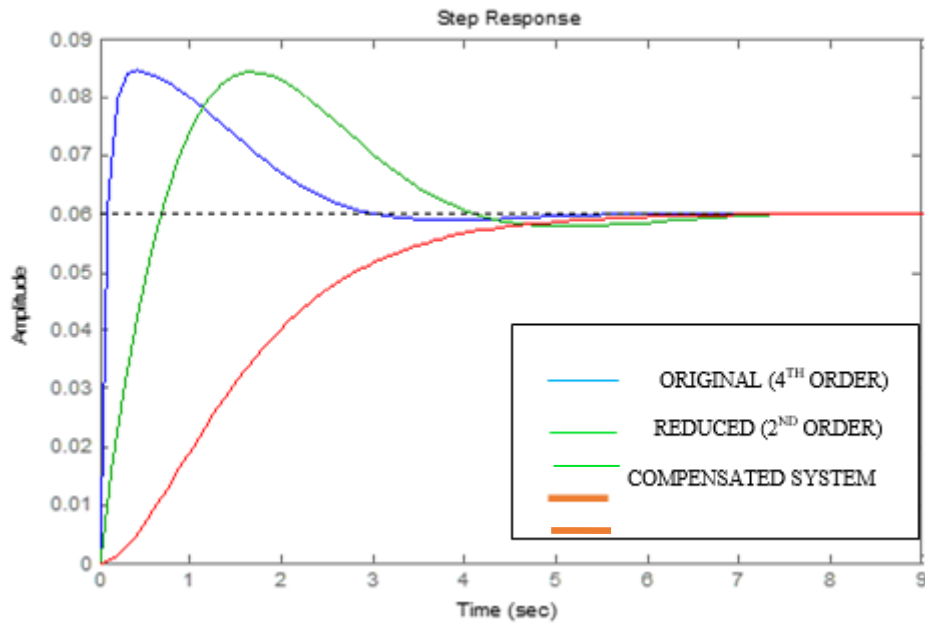Fig. 3 Comparing compensated and uncompensated systems' step response

Fig.4   Comparing compensated and uncompensated systems' step response

CONCLUSION

For these kinds of systems, controller design and analysis become laborious and expensive. The suggested approach has improved as a tool for high-order uncertain system analysis and simulation. For these reasons, it is preferable to operate from a lower-order model to a higher-order system while preserving the primary functionalities of the initial system. The original system's performance is quite expensive, as the created lower-order system from it illustrates. Thus, model order reduction is utilized in higher-order system simulation. Compared to interval arithmetic, Affine Arithmetic produces better steady intervals and preserves the associations between those quantities.

REFERENCES

[1]   Bandyopadhyay, B. Upadhye, A. and Ismail, O., "γ-δ Routh approximation for interval systems", IEEE Transactions on Automatic Control, Vol. 42, No. 8, pp. 1127 – 1130, 1997.

[2]   G.V.K.R. Sastry, G. Raja Rao and P. Mallikarjuna Rao Large scale interval system modelling using Routh approximants Electronics Letters 13th April 2000 Vol. 36 No.

[3]   G V K R Sastry & P Mallikarjuna Rao A New Method for Modelling of Large Scale Interval Systems JETE Journal of Research Vol 49, No.6, November-December, 2003, pp 423-430.

[4]   A.Saxena and R.Chaudhary,"Model Reduction Using improved Bilinear Routh Approximation Technique"   Advance in Electronic and Electric Engineering.2013,pp 497-502.

[5]   D. K.Saini and Dr.R.Prasad,"Order Reduction of Linear Interval Systems Using Particle Swarm Optimization", IT International Journal of Electrical and Instrumentation Engineering Vol.1,No.1, Jan 2011,pp 16-19.

[6]   K.K,Deveerasetty, Graduate student Member,IEEE and S. K.Nagar,"Order Reducing Order of Interval System Using Direct Routh Approximation Method".

[7]   D. K. Kumar ,S. K.Nagar and J.P.Tiwari ,"Model Order Reduction of Interval Systems Using Routh Approximation and Cauer Second Form", International Journal of Advances in Science and Technoogy,Vol.3,No.2,2011.

[8]   Sastry G V K R ,"State Space Model Using Simplified Routh Approximation Method ",Electronics Letters,Vol.23,pp 1300-1302,1987.

[9]   Sastry G V K R and K.MurthyV,"Biased Model Reduction by Simplified Routh Approximation Method", Electronics Letters,Vol.23,Issue:20,pp 1045-1047,Sept.,87.

[10] L.H.de Figueiredo and J.Stolfib, "Affine arithmetic: concepts and applications", Numerical Algorithms 37:147-158, 2004.

[11] J.Stolfi, and L.H.de Figueiredo, IMPA,"An Introduction to Affine Arithmetic", TEMA Tend. Mat.Apl. comput, 4, No.3, 297-312, 2003.

[12] J.L.D.I Combaan Jorge Stolfi,:"Affine Arithmetic and its Applications to Computer Graphics".

[13] Boyi Venkata Ramana, Dr. T. Narasimhulu, Prof. P. Mallikarjuna Rao, **"An Improved Method for Order Reduction of High Order Uncertain SISO Dynamic Systems by Affine Arithmetic"**, International Journal of Membrane Science and Technology, 2023, Vol. 10, No. 2, pp 01-07. (Scopus)

[13] Boyi Venkata Ramana, Dr. T. Narasimhulu and P. Mallikarjuna Rao. "An Affine Arithmetic Applied for Model Order Reduction on Discrete Uncertain Systems", International Journal for Modern Trends in Science and Technology, 2023, 9(11), pages. 22-26, https://doi.org/10.46501/IJMTST0911005 (UGC)

[14] Boyi Venkata Ramana, Dr. T. Narasimhulu and P. Mallikarjuna Rao, "An Affine Arithmetics for Large Scale Multi Input Dynamic System Model Order Reduction", Industrial Engineering Journal 0970-2555 Vol. 52, Issue 10, No. 3, October 2023, Pp. 79-85.(WoS)

ACKNOWLEDGEMENT

POTENTIAL CONFLICT OF INTEREST

The authors affirm that there isn't any conflict of interest with this article's publishing.