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Electromagnetic Transverse Modes in Periodic Structures: Mathematical Analysis and Engineering Applications



Abstract: - This study explores the electromagnetic transverse modes in periodic structures, combining mathematical analysis with practical engineering applications. We showcase how the deliberate arrangement of materials, using Maxwell's equations and Bloch's theorem, may control and guide electromagnetic waves for novel applications in devices. We emphasize the progress made in photonic crystals and metamaterials, which have resulted in significant developments in fields such as optical computing and stealth technologies. The debate encompasses the potential of computational methods to further improve the functions of materials and devices. This paper highlights the important interaction between theoretical physics and applied engineering, which lays the foundation for future technical advancements in wave manipulation.

Keywords: Electromagnetic Transverse Modes; Periodic Structures; Mathematical Analysis; Engineering Applications

I. INTRODUCTION

The investigation of electromagnetic transverse modes in periodic structures is a crucial convergence of physics and engineering, resulting in profound understanding and various practical uses. These structures alter electromagnetic waves by making periodic changes in material properties. This allows for the creation of advanced photonic and metamaterial devices that are more capable than traditional media. Recent theoretical advancements have greatly enhanced our comprehension of wave propagation in periodic media. The advancement has been expedited by intricate mathematical formulations and computational simulations. The utilization of Maxwell's equations, in conjunction with Bloch's theorem, has played a crucial role in this specific area [1,2]. In addition, the development of topological photonics has brought about new and creative viewpoints, specifically highlighting the important function of edge states in topological photonic crystals and their influence on wave manipulation [3,4]. These ideas are crucial for creating systems that can withstand flaws and fluctuations in the environment [5,6]. Significant progress has been achieved in converting these theoretical models into practical applications on the technological front. Photonic crystals have been utilized to create all-optical computing systems, resulting in significant advancements in speed and energy efficiency compared to conventional electronic systems [7,8]. In addition, there have been notable breakthroughs in the field of metamaterials, which have shown the ability to manipulate light patterns and make objects invisible. This concept has progressed from theoretical inquiry to experimental confirmation [9,10]. The uses of these groundbreaking materials are wide-ranging and significant. Photonic bandgap materials have been shown to boost antenna and filter performance in telecommunications [11,12]. Additionally, they have been shown to increase solar cell efficiency in renewable energy applications by improving light trapping [13,14]. Advancements in periodic structures have greatly improved the field of sensors, enabling the development of designs that can detect even the smallest changes in ambient conditions with exceptional precision [15,16]. In the future, the combination of machine learning and computational physics has the potential to completely transform the process of finding and improving novel materials [17,18]. Moreover, the continuous merging of quantum technologies with photonic systems is ready to unlock novel possibilities in information processing and

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secure communications [19,20]. This paper aims to thoroughly investigate the mathematical principles and practical uses of electromagnetic transverse modes in periodic structures. It provides a detailed analysis of recent theoretical advancements, practical implementations, and potential future developments in this rapidly evolving field.

The paper is structured as follows: Section 2 discusses the theoretical background of Electromagnetic Transverse Modes in Periodic Structures. Following this, Section 3 provides a detailed mathematical and numerical analysis. In Section 4, we present a case study focusing on rectangular waveguides. Finally, in Section 5, we explore the engineering applications enabled by these modes in periodic structures.

II. THEORETICAL BACKGROUND

Maxwell's Equations in Periodic Media

Maxwell's equations provide the fundamental principles of classical electromagnetism, electromagnetics, and optics. They provide a detailed description of the interactions between electric and magnetic fields, as well as their interactions with charges and currents. When considering periodic media, these equations need to take into consideration the regular variations in material properties. The periodic nature of the dielectric function, $\epsilon(\mathbf{r})$, has an impact on the solutions to Maxwell's equations, leading to distinct wave propagation properties.

Maxwell's equations in their differential form, in the absence of any sources in the medium, consist of the following equations:

- Gauss's law for electricity:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (1)$$

- Gauss's law for magnetism:

$$\nabla \cdot \mathbf{B} = 0 \quad (2)$$

- Faraday's law of induction:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3)$$

- Ampere's law with Maxwell's addition:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (4)$$

Both the permittivity $\epsilon(\mathbf{r})$ and perhaps the permeability $\mu(\mathbf{r})$ vary periodically in periodic media. By utilizing Bloch's theory [21], we streamline these equations to accurately represent the characteristics of the periodic structure.

An Overview of Bloch's Theorem

The utilization of Bloch's theorem is essential for the analysis of wave propagation in structures that exhibit periodicity. The concept suggests that wavefunctions in a periodic potential can be represented as a plane wave that is influenced by a periodic function:

$$\psi(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}} u_{\mathbf{k}}(\mathbf{r}) \quad (5)$$

where $u_{\mathbf{k}}(\mathbf{r})$ has the same periodicity as the lattice, and \mathbf{k} represents the wave vector. The formulation gives rise to the notion of electromagnetic Bloch waves, which are essential for describing fields within photonic crystals.

Principles of Transverse Modes

Transverse modes exhibit oscillations that occur at right angles to the direction in which the wave is propagating. The modes referred to as transverse electric (TE) and transverse magnetic (TM) are crucial in the design and operation of waveguides and optical fibers. The modes are determined by the equation:

$$\nabla^2 E_z + \left(\frac{\omega^2}{c^2} \varepsilon(r) - \beta^2 \right) E_z = 0 \quad (6)$$

In this context, ω represents the angular frequency, c denotes the speed of light, β stands for the propagation constant, and E_z represents the electric field component aligned with the direction of propagation.

Introduction to Photonic Band Theory

Photonic band theory, similar to electrical band theory in solids, is used to describe the behavior of light as it travels through a photonic crystal. This text elucidates the relationship between the crystal's structure and the permission or prohibition of specific light frequencies. This connection is established by solving Maxwell's equations with periodic boundary conditions. The band structure, which shows the bands that are allowed and disallowed, is commonly computed using the plane wave expansion technique:

$$\mathbf{A}\mathbf{H} = \omega^2 \mathbf{B}\mathbf{H} \quad (7)$$

The equation (7) represents a relationship between matrices \mathbf{A} and \mathbf{B} , which are formed from the spatial distributions of permittivity and permeability within the unit cell. \mathbf{H} represents the magnetic field vector, while ω represents the frequency.

III MATHEMATICAL AND NUMERICAL ANALYSIS

The primary approach for solving Maxwell's equations in periodic systems involves utilizing Fourier series and special functions to address the periodicity and boundary constraints. The plane wave expansion method is a frequently used methodology in which the electric field \mathbf{E} and the magnetic field \mathbf{H} are described as sums of plane waves.

The expressions for $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are given by the equations:

$$\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{G}} \mathbf{E}_{\mathbf{G}} e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}} \quad (8)$$

$$\mathbf{H}(\mathbf{r}) = \sum_{\mathbf{G}} \mathbf{H}_{\mathbf{G}} e^{i(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}} \quad (9)$$

In this context, \mathbf{G} denotes the reciprocal lattice vectors, while \mathbf{k} represents the Bloch wave vector. The process of expansion converts Maxwell's differential equations into a matrix eigenvalue problem. Solving for the eigenvalues ω , which represent the angular frequency, provides valuable information on the material's band structure.

The method of various scales is an additional analytical methodology employed to address electromagnetic problems that involve varying length scales in their properties. This approach entails expressing the electromagnetic fields by using a tiny parameter that represents the ratio between the periodicity of the structure and the wavelength of the incident wave.

The numerical simulation techniques, including Method of Moments (MoM), Finite Difference Time Domain (FDTD), Finite Element Method (FEM), and the Advanced Transverse Wave Approach (A-TWA), are highly important methods for solving Maxwell's equations in various electromagnetics applications. Each technique has unique benefits and specific applications, making them essential in their respective fields.

Finite Difference Time Domain (FDTD)

FDTD [22] is a flexible computational method employed to provide approximate solutions to Maxwell's equations. This approach employs time-domain numerical modeling, where a grid is established in both space and time, and finite differences are used to approximate the derivatives in Maxwell's equations. The update equations for the electric field (\mathbf{E}) and magnetic field (\mathbf{H}) are expressed as follows:

$$\mathbf{E}^{n+1} = \mathbf{E}^n + \frac{\Delta t}{\varepsilon} (\nabla \times \mathbf{H}^n - \sigma \mathbf{E}^n) \quad (10)$$

$$\mathbf{H}^{n+1} = \mathbf{H}^n - \frac{\Delta t}{\mu} \nabla \times \mathbf{E}^n \quad (11)$$

Here, σ denotes the electrical conductivity.

Finite Element Method (FEM)

FEM [23] is a robust numerical technique well-suited for intricate geometrical and material domains. It involves partitioning the computational area into small, finite elements, such as triangles in two dimensions or tetrahedra in three dimensions. Maxwell's equations are subsequently restated as a collection of algebraic equations using these components. The relationship for the electric fields in FEM can be expressed using the stiffness matrix (\mathbf{K}) and mass matrix (\mathbf{M}):

$$\mathbf{KE} = \omega^2 \mathbf{ME} \quad (11)$$

The equation (11) represents the eigenvalue problem used in FEM, where \mathbf{E} is the vector of electric field unknowns, \mathbf{K} is the stiffness matrix, and \mathbf{M} is the mass matrix.

Method of Moments (MoM)

The Method of Moments (MoM) [24-25] is highly efficient for solving problems that include structures with basic geometric forms, such as antennas and scatterers. This process converts Maxwell's equations from their original differential forms into integral forms by utilizing Green's functions. These integral forms are further discretized into algebraic equations:

$$\int_V \mathbf{G}(r, r') \cdot \mathbf{J}(r') dV' = \mathbf{V}(r) \quad (12)$$

Where \mathbf{G} is the Green's function, \mathbf{J} is the current distribution, and \mathbf{V} is the induced voltage. MoM converts the problem into a matrix equation that can be solved numerically for \mathbf{J} .

Advanced Transverse Wave Approach (A-TWA)

The Advanced Transverse Wave Approach (A-TWA) is a specialized technique designed to optimize the simulation of transverse electromagnetic waves, particularly in Radio Frequency (RF) and Microwave (MW) applications. This method adapts Maxwell's equations to more efficiently address challenges in environments dominated by transverse wave propagation, such as in waveguides and antenna structures. By focusing exclusively on the transverse components of electromagnetic fields, A-TWA reduces both computational overhead and the dimensionality of the problem. This streamlined approach not only speeds up simulations but also enhances the capacity to manage larger or more complex systems without increasing computational resource demands.

The fundamental modification in A-TWA can be exemplified by the adapted wave equation:

$$\nabla^2 \mathbf{E}_\perp - k^2 \mathbf{E}_\perp = -\omega^2 \mu \epsilon \mathbf{E}_\perp \quad (13)$$

Here, \mathbf{E}_\perp represents the transverse components of the electric field, ∇^2 is the Laplacian reflecting the spatial derivatives, k is the wave number related to the medium, and ω , μ , and ϵ denote the angular frequency, magnetic permeability, and electric permittivity, respectively. This equation emphasizes the focus on transverse electric fields, simplifying calculations by isolating these components and reducing the effects of longitudinal field components. More details of this method can be found in [26-32].

The following bar chart (Figure1) compares the accuracy and computational cost of key electromagnetic simulation methods—MoM, FDTD, FEM, and A-TWA. It highlights the trade-offs between effectiveness and resource demand, helping to identify the most suitable method based on specific project needs.

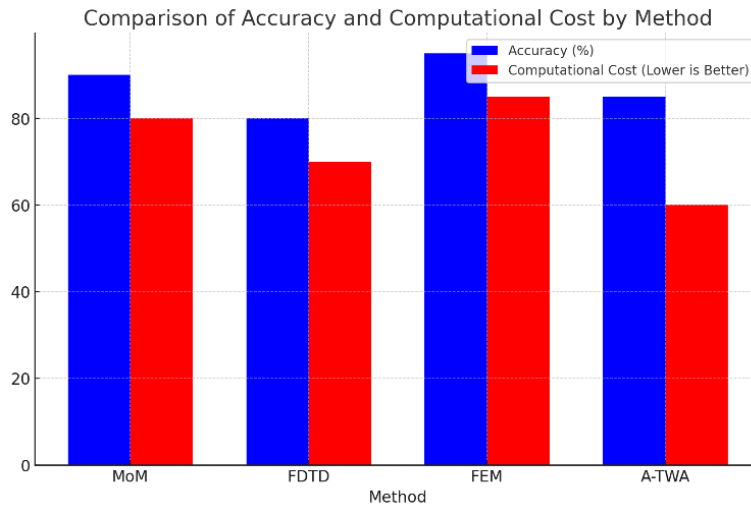


Figure 1 Accuracy vs. Cost in EM Simulation Methods

In addition, Table 1 offers a brief, yet insightful comparison of key numerical methods used for solving Maxwell’s equations. It succinctly outlines the primary applications, time complexity, and essential characteristics of each method. This overview facilitates a quick understanding of which method might be most suitable for specific electromagnetic simulation needs, considering factors like computational efficiency and problem complexity.

Table 1. Overview of Numerical Methods for Maxwell’s Equations

Method	Application	Time Complexity	Details
Method of Moments (MoM)	EM scattering and radiation problems	$\mathcal{O}(N^2)$ to $\mathcal{O}(N^3)$	Quadratic or cubic scaling with number of unknowns. Computationally intensive for large systems.
Finite Difference Time Domain (FDTD)	Broad range of electromagnetic problems	$\mathcal{O}(N_t \times N_x)$	Linear with the number of grid points and time steps. Cost escalates for finer grids.
Finite Element Method (FEM)	Complex geometries in electromagnetics	$\mathcal{O}(N^3)$ to $\mathcal{O}(N \log N)$	Depends on the solver; cubic for direct solvers but can be reduced with efficient iterative methods.
Advanced Transverse Wave Approach (A-TWA)	RF/MW structures focusing on transverse waves	Potentially better than $\mathcal{O}(N^2)$	Optimized for specific scenarios, can be more efficient than traditional methods.

IV. CASE STUDY: RECTANGULAR WAVEGUIDE

Let us examine a waveguide with rectangular cross-section and periodic walls, as shown in Figure C1. The waveguide has inner dimensions a and b . The waveguide can either be hollow or filled with dielectric material. In this analysis, we focus on the lossless scenario and assume that the walls of the waveguide have perfect conductivity. Additionally, we take the inner region of the waveguide to be either empty or filled with a uniform and non-conductive dielectric material.

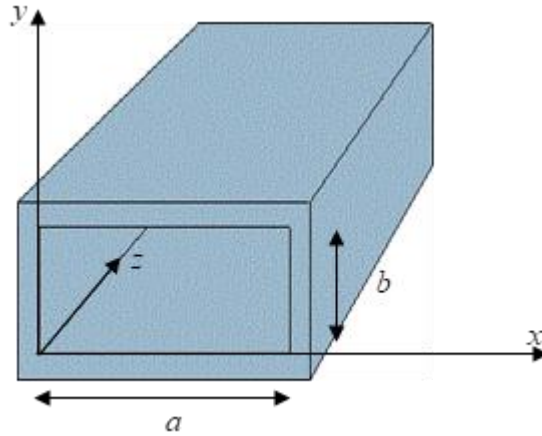


Figure 2 Waveguide with rectangular-section

TE Mode ($\underline{E}_z = 0; \underline{H}_z \neq 0$), TM Mode ($\underline{H}_z = 0; \underline{E}_z \neq 0$)

The TE mode (respectively TM mode) has a selfic behavior (resp. capacitive) defined as:

$$\underline{E}_z = 0 \text{ (resp. } \underline{H}_z = 0) \quad (14)$$

The EM field TE (resp. TM) modes can be derived from a differential equation in \underline{H}_z (resp. \underline{E}_z) where the solutions are satisfied to boundary conditions on waveguide walls.

For TE mode, we obtain from (A.1) (see appendix A)

$$\nabla_t^2 \underline{H}_z - \frac{\partial \nabla_t \underline{H}_t}{\partial z} = (j\omega\epsilon + \sigma) \nabla_t (e_z \times \underline{E}_t) \quad (15)$$

With (A.5), the latter equation becomes:

$$\nabla_t^2 \underline{H}_z + \frac{\partial^2 \underline{H}_z}{\partial z^2} = -(j\omega\epsilon + \sigma) e_z \nabla_t \times \underline{E}_t \quad (16)$$

It yields:

$$\nabla_t^2 \underline{H}_z + \frac{\partial^2 \underline{H}_z}{\partial z^2} = j\omega\mu(j\omega\epsilon + \sigma) \underline{H}_z \quad (17)$$

Let k be wavenumber (Bloch wave vector) defined as:

$$\underline{k}^2 = -j\omega\mu(j\omega\epsilon + \sigma) \quad (18)$$

Therefore, the differential equation in \underline{H}_z can be expressed as follows:

$$\underbrace{\nabla_t^2 \underline{H}_z}_{transv.} + \underbrace{\frac{\partial^2 \underline{H}_z}{\partial z^2}}_{longit.} + \underline{k}^2 \underline{H}_z = 0 \text{ A/m}^3 \quad (19)$$

In the same way for TM mode, the differential equation in \underline{E}_z can be written as:

$$\underbrace{\nabla_t^2 \underline{E}_z}_{transv.} + \underbrace{\frac{\partial^2 \underline{E}_z}{\partial z^2}}_{longit.} + \underline{k}^2 \underline{E}_z = 0 \text{ V/m}^3 \quad (20)$$

To obtain solution of equations (19) and (20), the following system must be satisfied:

$$\begin{cases} (1) \nabla_t^2 \underline{X}_z + \underline{\rho}^2 \underline{X}_z = 0 & \frac{A}{m^3} \\ (2) \frac{\partial^2 \underline{X}_z}{\partial z^2} - \underline{\gamma}^2 \underline{X}_z = 0 & \frac{A}{m^3} \\ (3) \underline{\rho}^2 - \underline{\gamma}^2 = \underline{k}^2 & \frac{1}{m^3} \end{cases} \quad (21)$$

Where X denotes E or H.

The equation (1) from (21) represents the vectorial Helmholtz equation, (2) the longitudinal wave equation and (3) the dispersion equation defining *transverse* wavenumber $\underline{\rho}$ having, in presence of boundary conditions on waveguide walls no solutions only for some values of $\underline{\rho}$ which represent the problem eigenvalues corresponding to propagation modes in the waveguide.

Based on (10) and (11), we obtain:

- For TE mode ($\underline{E}_z = 0$), (A.8) becomes:

$$\nabla_t \times \underline{H}_t = 0 \quad (22)$$

The transverse magnetic field can be then derived from a potential:

$$\underline{H}_t = -\nabla_t \underline{\psi} \quad (23)$$

This potential $\underline{\psi}$ must be satisfied to Helmholtz equation:

$$\nabla_t^2 \underline{\psi} + \underline{\rho}^2 \underline{\psi} = 0 \quad (24)$$

- For TM mode ($\underline{H}_z = 0$), (A.8) (See Appendix) becomes:

$$\nabla_t \times \underline{E}_t = 0 \quad (25)$$

Hence, a potential $\underline{\varphi}$ exists such as:

$$\underline{E}_t = -\nabla_t \underline{\varphi} \quad (26)$$

This potential $\underline{\varphi}$ must also fulfill the Helmholtz equation:

$$\nabla_t^2 \underline{\varphi} + \underline{\rho}^2 \underline{\varphi} = 0 \quad (27)$$

Resolution of Helmholtz equation:

$$\nabla_t^2 \underline{\theta} + \underline{\rho}^2 \underline{\theta} = 0 \quad (\theta = \varphi \text{ or } \psi) \quad (28)$$

Using the separation of variables method, we can write:

$$\underline{\theta}(x, y) = \underline{X}(x)\underline{Y}(y) \quad (29)$$

Thus, Helmholtz equation becomes:

$$\underbrace{\frac{1}{X} \frac{d^2 X}{dx^2}}_{\text{as function of } x} + \underbrace{\frac{1}{Y} \frac{d^2 Y}{dy^2}}_{\text{as function of } y} + \rho^2 = 0 \quad (30)$$

A non-trivial solution can be existed if we have:

$$\begin{cases} \frac{1}{X} \frac{d^2 X}{dx^2} = cte \\ \frac{1}{Y} \frac{d^2 Y}{dy^2} = cte \end{cases} \quad (31)$$

This can be rewritten as the following manner:

$$\begin{cases} (1) \frac{d^2 X}{dx^2} + u^2 X = 0 \\ (2) \frac{d^2 Y}{dy^2} + v^2 Y = 0 \\ (3) u^2 + v^2 = \rho^2 \end{cases} \quad (32)$$

where u, v are two constants.

The differential equation (1) of system (32) has as solution:

$$X = Ae^{-j|u|x} ; (A: \text{constant}) \quad (33)$$

The differential equation (2) of system (32) has as solution:

$$Y = Be^{-j|v|y} ; (B: \text{constant}) \quad (33)$$

The general solution can be therefore expressed as:

$$\underline{\theta} = A_{mn} e^{-j|u|x} e^{-j|v|y} \quad (35)$$

In order to determine u and v , we establish the boundary conditions for periodic walls with dimensions a and b .

In fact, on the walls the solution is periodic:

- For $x=a$: $|u|a = 2m\pi$; m positive integer number

$$\Rightarrow |u| = \frac{2m\pi}{a} = \beta_{xm} \quad (36)$$

- For $y=b$: $|v|b = 2n\pi$; n positive integer number

$$\Rightarrow |v| = \frac{2n\pi}{b} = \beta_{yn} \quad (37)$$

The equation (3) of system (32) can be written as:

$$\rho_{mn}^2 = u^2 + v^2 = \beta_{xm}^2 + \beta_{yn}^2 \quad (38)$$

$$\Rightarrow \rho_{mn} = \sqrt{\beta_{xm}^2 + \beta_{yn}^2} \quad (39)$$

Inserting (36) and (37) in (35), the general solution becomes:

$$\underline{\theta}_{mn} = A_{mn} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (40)$$

where A_{mn} is a constant so-called *normalization's constant*.

Expression of the normalization's constant A_{mn} :

Let \underline{A} be a transverse vector defined by:

$$\underline{A} = \underline{\theta}^* \nabla_t \underline{\theta} \quad (41)$$

Applying ∇_t to \underline{A} , we obtain:

$$\nabla_t \underline{A} = \nabla_t \underline{\theta}^* \cdot \nabla_t \underline{\theta} + \underline{\theta}^* \cdot \nabla_t^2 \underline{\theta} \quad (42)$$

From Helmholtz equation given by (22), (40) takes the following form:

$$\nabla_t \underline{A} = |\nabla_t \underline{\theta}|^2 - \rho^2 |\underline{\theta}|^2 \quad (43)$$

The divergence's theorem is defined as:

$$\int_s \nabla \underline{A} \, dA = \oint_c \underline{A} \cdot n \, dl \quad (44)$$

dA : being element of surface.

With (41), (42) and (43), we have therefore:

$$\int_s |\nabla_t \underline{\theta}|^2 \, dA - \rho^2 \int_s |\underline{\theta}|^2 \, dA = \underbrace{\oint_c \underline{\theta}^* \cdot \nabla_t \underline{\theta} \cdot n \, dl}_{=0} \quad (45)$$

$$\Rightarrow \rho^2 = \frac{\int_s |\nabla_t \underline{\theta}|^2 \, dA}{\int_s |\underline{\theta}|^2 \, dA} \quad (46)$$

To normalize the E_T and H_T functions, the transmitted power on the line must be equivalent to the transported power in the guide, this yields:

$$\int |E_T|^2 \, dA = \int |H_T|^2 \, dA = \int |\nabla_t \underline{\theta}|^2 \, dA = 1 \quad (47)$$

Further, based on (40) we obtain:

$$\int_s |\underline{\theta}_{mn}|^2 \, dA = \int_0^b \int_0^a |A_{mn}|^2 \, dA = ab |A_{mn}|^2 \quad (48)$$

From (46), (47) and (48), the expression of normalization's constant is given by:

$$|A_{mn}| = \frac{1}{\sqrt{ab} \sqrt{\beta_{xm}^2 + \beta_{yn}^2}} \quad (49)$$

Expressions of transverse EM fields TE and TM modes for periodic walls:

In satisfying (49), we can choose as normalization's constant:

$$A_{mn} = \frac{-j}{\sqrt{ab} \sqrt{\beta_{xm}^2 + \beta_{yn}^2}} \quad (50)$$

TM mode:

From (26), we can deduce the transverse electric and magnetic field for TM mode.

Using (40), (26) becomes:

$$\underline{E}_t = -A_{mn} [e_x (-j\beta_{xm} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y}) + e_y (-j\beta_{yn} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y})] \quad (51)$$

This allows us to identify the components of the transverse electric fields E_x and E_y as follows:

$$E_x^{TM} = \frac{\beta_{xm}}{\sqrt{ab} \sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (52)$$

$$E_y^{TM} = \frac{\beta_{yn}}{\sqrt{ab} \sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (53)$$

To notify the components of the transverse magnetic fields, it suffices to write:

$$H_T = e_z \times E_T \quad (54)$$

Therefore, we obtain:

$$H_x^{TM} = \frac{-\beta_{yn}}{\sqrt{ab} \sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (55)$$

$$H_Y^{TM} = \frac{\beta_{xm}}{\sqrt{ab}\sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (56)$$

TE Mode:

In the same way and from (23), the components of the transverse electric and magnetic fields for TE mode can be written hence as:

$$H_X^{TE} = \frac{\beta_{xm}}{\sqrt{ab}\sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (57)$$

$$H_Y^{TE} = \frac{\beta_{yn}}{\sqrt{ab}\sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (58)$$

$$E_X^{TE} = \frac{\beta_{yn}}{\sqrt{ab}\sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (59)$$

$$E_Y^{TE} = \frac{-\beta_{xm}}{\sqrt{ab}\sqrt{\beta_{xm}^2 + \beta_{yn}^2}} e^{-j\beta_{xm}x} e^{-j\beta_{yn}y} \quad (60)$$

This study provides clear evidence of the substantial influence of periodic structural alterations on the propagation characteristics of TE and TM modes in a rectangular waveguide. Through the utilization of the separation of variables technique, we have thoroughly verified the theoretical models and demonstrated how these periodic adjustments may successfully manipulate the behaviors of electromagnetic waves.

The analysis has demonstrated that these modifications result in noticeable changes in modal dispersion and cutoff frequencies, hence creating novel opportunities for waveguide design. This encompasses the capacity to manipulate and control specific bandgap characteristics and improve the ability to pick modes, which are essential for the advancement of sophisticated technologies such as filtering, isolation, and waveguiding.

Furthermore, the success of this mathematical methodology strengthens the practicality of employing organized waveguides in advanced applications, spanning from telecommunications to microwave engineering and beyond. Subsequent studies are expected to investigate other enhancements and possible uses of this technology, creating new opportunities for precise manipulation of electromagnetic waves in many industrial and scientific fields.

V. ELECTROMAGNETIC TRANSVERSE MODES IN PERIODIC STRUCTURES FOR ENGINEERING APPLICATIONS

Start by emphasizing the importance of theoretical understanding of electromagnetic transverse modes in driving the development of new technologies. Emphasize how this comprehension enables the manipulation of electromagnetic waves in innovative manners that are advantageous to diverse industries, such as telecommunications, healthcare, and computers.

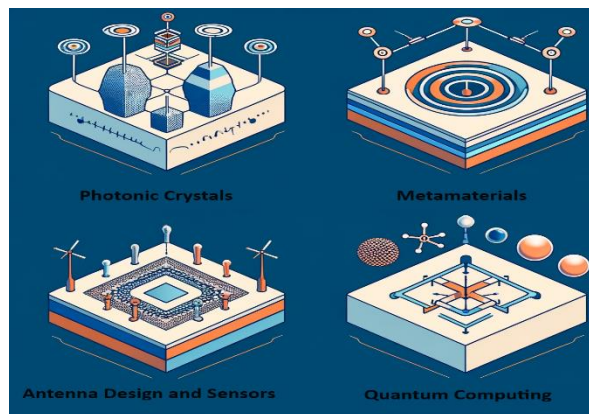


Figure 3. Key applications of electromagnetic transverse modes in periodic structures

The figure above (Figure 3) clearly depicts the fundamental uses of electromagnetic transverse modes in periodic structures. It accurately classifies the technology into four crucial domains: Photonic Crystals for Optical Filtering, Metamaterials for Superlensing, Antennas for Enhanced Directivity, and Quantum Computing with Localized Modes. The categorization is well-defined, offering a quick comprehension of the significant

functions that these innovative materials and designs play in several technical domains.

A. *Photonic Crystals: Transforming Optical Filtering and Waveguiding*

Photonic crystals are at the forefront of optical technology due to their capacity to create photonic bandgaps, which prevent the transmission of light at certain frequencies. This quality is crucial for technologies that necessitate precise control over the trajectory of light. The fundamental concept underlying this phenomenon can be described by the equation:

$$\nabla \times (\nabla \times \mathbf{E}) - \frac{\omega^2}{c^2} \epsilon(\mathbf{r}) \mathbf{E} = \mathbf{0} \quad (61)$$

This equation characterizes the interaction of electromagnetic waves in materials with varying dielectric constants. Photonic crystals exhibit bandgaps, which are places where specific frequencies of light are obstructed, allowing for exact control over the modulation of light pathways. Photonic crystals have a transformative impact on the development of optical filters and waveguides in real-world applications.

Practical applications:

- Photonic crystals are used in *optical filters* to efficiently reject unwanted frequencies and allow desired ones. This is an important feature in optical communications as it helps decrease channel crosstalk and improve signal integrity.

- *Waveguides*, however, take advantage of the capability of photonic crystals to efficiently guide light along intricate paths, minimizing energy loss. This makes them essential in the advancement of small, integrated optical circuits.

These applications demonstrate how the theoretical knowledge gained from photonic crystal research is applied to create essential components in modern optical technology. This enables advancements in the manipulation of light and the creation of signaling pathways beyond previous limitations.

B. *Metamaterials: Enabling Negative Refraction and Superlensing*

Metamaterials utilize electromagnetic transverse modes to reinterpret conventional optical characteristics and create phenomena such as negative refraction. This potential is not only theoretical, but it has the power to revolutionize by allowing the development of superlenses and cloaking technologies. These materials operate based on the idea that the designed structure enables accurate manipulation of electromagnetic wave movement, causing light to bend in unexpected directions.

The mathematical expression that represents this behavior is:

$$\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* \quad (62)$$

The Poynting vector, denoted as \mathbf{S} , represents the direction of energy flow. \mathbf{E} and \mathbf{H} represent the electric and magnetic fields, respectively. The equation presented here demonstrates the ability of metamaterials with negative refractive indices to alter the flow of light in a direction opposed to its propagation, hence providing exceptional control over the routes of light.

Practical applications:

- Superlenses exploit this distinctive characteristic to concentrate light beyond the diffraction limit, enabling imaging capabilities that extend beyond the microscale to the nanoscale. This is particularly valuable in disciplines like medical, security, and materials science.

- Cloaking Devices utilize the phenomenon of negative refraction to manipulate electromagnetic waves, causing them to curve around objects and render them invisible under certain circumstances. This technology not only stimulates the mind but also possesses substantial potential for practical applications in several areas, spanning from military to entertainment.

The study of metamaterials and their interaction with electromagnetic transverse modes is advancing the boundaries of scientific possibility, transforming ideas that were once considered science fiction into practical and influential technology.

C. *Enhanced Antenna Design and Sensors*

Utilizing electromagnetic transverse modes within periodic structures has greatly enhanced the development of antennas and sensors, enabling unparalleled accuracy in manipulating wave propagation. Precise control is

essential for the development of antennas with improved directivity and sensors with increased sensitivity. Periodic architectures modify the electromagnetic environment in which these devices function, allowing them to operate more efficiently in a smaller size.

The following fundamental equation expresses the relationship between wavelength, frequency, and speed of light:

$$\lambda = c/f \quad (63)$$

Here λ , representing the wavelength, c , representing the speed of light, and f , representing the frequency.

When dealing with periodic structures, changing the periodicity can effectively modify the effective wavelength (λ) in the medium. This modification facilitates the creation of more compact antennas by allowing them to function at lower frequencies without the need to increase their physical dimensions. Similarly, the sensitivity of sensors can be increased by tailoring their design to detect and react to the specific wavelengths that are most influenced by variations in the environment.

Practical applications:

- Antennas: The capability to manipulate and adjust electromagnetic transverse modes results in antennas that are not only smaller in size but also demonstrate enhanced performance attributes, including extended range and improved signal reception quality. This is especially advantageous in mobile devices and satellite communications, where limited space and optimal efficiency are crucial.
- Sensors: The increased sensitivity of sensors is another important use case. Sensors can detect more minute fluctuations in situations by adjusting the transverse modes to be responsive to specific environmental changes. This capability is particularly important in applications like security surveillance and environmental monitoring. These sensors have the capability to accurately quantify variations in temperature, pressure, or other crucial variables, thereby offering dependable data that is essential for upholding safety and operational soundness.

The progress highlights the significant influence of electromagnetic transverse modes on the continuous development of device downsizing and functional improvement in diverse high-tech applications. The interaction between theoretical ideas and practical engineering solutions continues to **propel substantial advancements in this ever-changing industry.**

D. Quantum Computing and Photonic Information Processing

The mastery of photonic modes is essential for the development of quantum computing technologies, which considerably improve the capabilities of information processing beyond what can be accomplished by classical systems. Through the strategic manipulation of electromagnetic transverse modes within photonic structures that have been precisely built, it is possible to acquire a highly accurate control over the interactions between photons. The management of photons, which are the fundamental carriers of quantum information, can therefore be accomplished with an accuracy that has never been seen before, which enables advanced quantum processes and advancements. Having this level of control is essential in order to fully use the capabilities of quantum physics in real-world applications.

Practical applications:

- Quantum Gates: The utilization of confined patterns within photonic bandgap structures is crucial for the precise and accurate functioning of quantum gates. These structures skillfully utilize electromagnetic transverse modes to limit light into specific patterns, thus allowing precise control over photon pathways and interactions. Accurate manipulation is essential for meeting the intricate demands of quantum computation. The equation governing mode localization in photonic crystals is given in (63); this equation illustrates the ability of photonic crystals to create settings that efficiently confine light, enabling the encoding and manipulation of qubits in a quantum computing system.
- Quantum networking relies on the skillful manipulation and interconnection of photons using specifically designed photonic modes. Networks constructed using this technology employ entangled photons, which possess the ability to sustain quantum coherence over extensive distances. Quantum cryptography is crucial for the implementation of secure communication systems that protect against eavesdropping.

The equation for photon entanglement in quantum networking is:

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}}(\phi(\mathbf{r}_1)\chi(\mathbf{r}_2) + \phi(\mathbf{r}_2)\chi(\mathbf{r}_1)) \quad (64)$$

where, the symbol ψ represents the entangled state of two photons located at \mathbf{r}_1 and \mathbf{r}_2 , with ϕ and χ corresponding to their respective wavefunctions. The exact manipulation of photonic modes within the structure allows for entanglement, demonstrating the advanced level of control necessary for quantum networking applications.

The deliberate control of electromagnetic transverse modes in photonic structures enables revolutionary possibilities in quantum computing and information processing, marking the beginning of a new era of technological progress and improved computational skills.

VI CONCLUSION

Our work highlights the significance of Electromagnetic Transverse Modes in Periodic Structures, providing a comprehensive overview of theoretical understanding and real-world uses. By doing thorough mathematical and numerical research, we have established a strong basis for identifying the fundamental features and behaviors that are crucial in understanding these modes. The examination we conducted on engineering applications, specifically focusing on rectangular waveguides as a case study, demonstrates their important significance in practical applications.

In addition, our investigation across several disciplines such as communication, healthcare, environmental monitoring, and materials science demonstrates the adaptability of transverse modes. These applications showcase their capacity to tackle real-world problems and propel technical advancement.

In the future, our research can provide a starting point for improving mathematical models, investigating innovative engineering solutions, and tackling practical problems. By fostering interdisciplinary collaboration, we aim to enhance our comprehension of electromagnetic field manipulation further and drive innovation at the intersection of mathematics and engineering.

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APPENDIX

Separation of Variables

The concept of "separation of variables" refers to the process of solving a differential equation by expressing the variables as separate functions.

The perpendicularity of the transverse planes to the direction of propagation is evident. The transverse and longitudinal variations of electromagnetic fields are independent, enabling the differentiation of the differential operator (nabla) into its transverse and longitudinal components.

In Cartesian coordinate:

$$\nabla = \nabla_t + e_z \frac{\partial}{\partial z} \quad (\text{A.1})$$

$$\text{where } \nabla_t = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y} \quad (\text{A.2})$$

For any phasor vector X can be separated as:

$$X = X_t + e_z X_z \quad (\text{A.3})$$

The Separation of Maxwell's equations into transversal and longitudinal components yields:

$$\text{Div}E = 0 \Rightarrow \nabla_t \underline{E}_t + e_z \frac{\partial E_z}{\partial z} = 0 \quad \text{v/m}^2 \quad (\text{A.4})$$

$$\text{Div}H = 0 \Rightarrow \nabla_t \underline{H}_t + e_z \frac{\partial H_z}{\partial z} = 0 \quad \text{A/m}^2 \quad (\text{A.5})$$

$$\overrightarrow{\text{Rot}} E = -j\omega\mu H \Rightarrow \nabla_t \times \underline{E}_t + e_z \times \left(\frac{\partial \underline{E}_t}{\partial z} - \nabla_t E_z \right) = -j\omega\mu \underline{H}_t - j\omega\mu \underline{H}_z e_z \quad \text{v/m}^2 \quad (\text{A.6})$$

$$\overrightarrow{\text{Rot}} H = (j\omega\varepsilon + \sigma)E \Rightarrow \nabla_t \times \underline{H}_t + e_z \times \left(\frac{\partial \underline{H}_t}{\partial z} - \nabla_t H_z \right) = (j\omega\varepsilon + \sigma)\underline{E}_t + (j\omega\varepsilon + \sigma)\underline{E}_z e_z \text{A/m}^2 \quad (\text{A.7})$$

Regrouping, hence, transverse part and longitudinal part, we obtain by identification:

$$\nabla_t \times \underline{E}_t = -j\omega\mu \underline{H}_z e_z \quad (\text{A.8})$$

$$\nabla_t \times \underline{H}_t = (j\omega\varepsilon + \sigma)\underline{E}_z e_z \quad (\text{A.9})$$

$$\nabla_t \underline{E}_z - \frac{\partial \underline{E}_t}{\partial z} = -j\omega\mu e_z \times \underline{H}_t \quad (\text{A.10})$$

$$\nabla_t \underline{H}_z - \frac{\partial \underline{H}_t}{\partial z} = (j\omega\varepsilon + \sigma)e_z \times \underline{E}_t$$

(A.11)