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Construction of $Q - Ary(n, M, d) -$ Codes in PG (2,9)

Abstract: - This study's main objective is to offer the connection between the projective plane of order nine & the field of coding theory. An incidence matrix has been developed using the parameters n (code length), d (code min. distance), and e (code error – correcting). The greatest value of size code M across a limited field of the ninth order has also been determined. There are a few theorems and examples provided.

Keywords: coding theory, Algebraic geometry, incidence matrix.

I. INTRODUCTION

Breast The basic subjects of this work are group theory, field theory, coding theory, and projective geometry over a finite field. The brief history of this theme is shown below. Every word and research theorem comes from Hirshfeld [1], [2]. R Hill [3] examined an introductory course in coding theory. N.A.M. Al seraji [4] presented the arcs on the projective plane of order 17. B.A. Al-Zangana Emad [5] defined the arcs in the projective plane of order 19 and error codes. Ideal linear codes were studied by Yahya and Alzangana in [6], [7]. The projective plane over a finite field of order q, where q = 9, is the subject of this article.

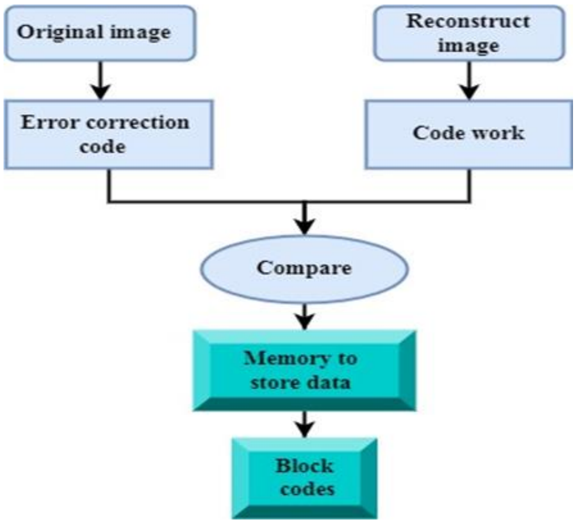

Figure 1 Block diagram of error correction code

Figure 1 shown in the channel becomes noisy, the channel encoder and decoder play critical roles in the entire channel error correction system. The channel error correction system comprises of an error correction coder before to the noisy channel and an error correction decoding after data reception. Error correction code is described is the addition of 'managed redundancy' to the source data in effort to lessen the effect of data transfer across a noisy channel and improve transmission channel dependability. The process of image error correction codes can be formulated as designing an error correction code and decoder, as shown in the S represents the original image, t represents the encoded image that can be transmitted over the noisy channel, and r represents the received image after noise was added during the noisy transmission channel.

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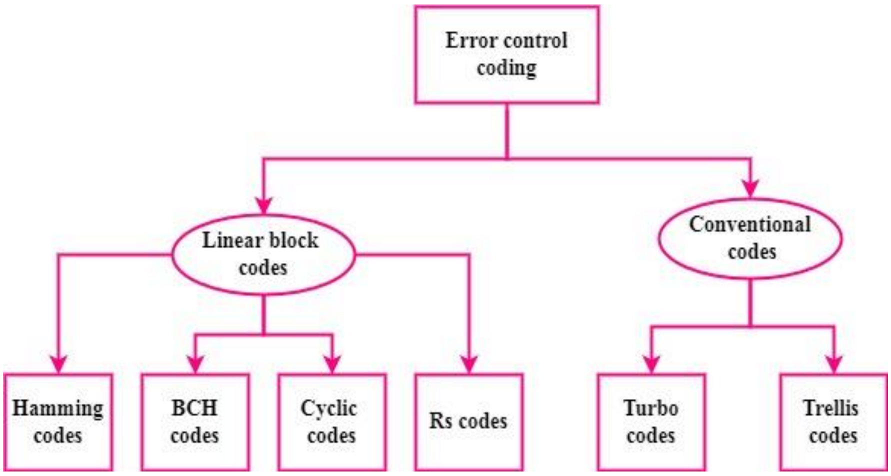

Figure 2 Block diagram of error control coding

All of the codes employ a different method or approach to detect the mistake bit and fix it by adding or deleting bits. Furthermore, many forms of cyclic codes are used to generate and identify errors in the channel caused by a variety of factors like as noise. Convolutional codes are error control codes that are used to identify and fix errors. Convolutional codes are error control codes that are used to identify and fix errors. Block codes are effective in detecting and preventing random mistakes. Convolutional codes are effective in detecting and preventing burst mistakes. The hardware component of Block codes is sophisticated, and the encoding procedure is quite challenging. The hardware component of Convolutional Codes is simpler, and the encoding procedure is straightforward. Convolutional code is a kind of error-correcting code in which the output bits are obtained by performing an optimal logical process on the current bitstream while thinking about the prior bit. The main problem of a communication system is to transmit messages without errors. The majority of the errors arise in the channel or noise. To address the issue, error detection and repair methods are implemented.

II. RELATED WORK

The research field will find the next conclusions fascinating.

Theorem(1.1) [1]. A q - ary $(n, M, d\ )$ - code C Satisfies:

$$M\left\{\binom{n}{0}+\binom{n}{1}\ (q-1)+\cdots+\binom{n}{e}(q-1)e\right\}\leq q^n\Big]$$

Whereas d= 2e+1 or 2e+2, [Theorem 1.1,1] provides a required requirement for the $[n,\ K,\ d]$q code to exist.

Definition (1.2) [2]:

Suppose that $F(X) = X^n + a_{n-1}X^{n-1} + \cdots + a_0$

be a degree n greater than or equal 1 monic polynomial over Fq. The n × n matrix provides the companion matrix C(f).

$$C\ (f) = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & \ldots & 0 & 1 \\ a_0 & a_1 & a_2 & \ldots & a_{n-2} & a_{n-1} \end{bmatrix}$$

The categorization of cubic shapes on an order 9 finite field

Suppose that the polynomial $g_1(x) = \sigma^2 - \sigma - 1$ and $F_9 = \frac{F3[x]}{(g1(x))}$

Which has 9 elements namely 0,1, -1, σ, - σ, σ^2, - σ^2, σ^3, - σ^3 here σ is the ideal $< g_1(x) >$ in addition to x. produced by a degree three polynomial with $F_3 = \{0\ ,1\ ,2\ \}$ for the coefficients.The companion matrix of

$g_2(x) = x^3 - \sigma^2 x^2 - 1$ generated the points and lines of PG (2,9) like comes:

$$P_i = [1 , 0 , 0] \, C(g)^i = [1 , 0 , 0] \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & \sigma^2 \end{pmatrix},$$

Where $i = 0 , 1 ,..., 90$

Definition: Let C be a $q-ary$ code of length n, with ρ as the covering radius. C is uniformly packed in the broad sense, i.e. in the sense of [1], if there are rational integers $\alpha 0, \ldots, \alpha\rho$ that correspond to any $v \in F^n$

$$\sum_{k=0}^{\rho} \boldsymbol{\alpha_k\, f_k(v) = 1},$$

where $fk(v)$ represents the number of codewords for a distance of k from v.

Let C be a $q-ary$ linear code, and $C\ (s)$ denotes the shorter code of C generated for selecting the codewords of C

C with a fixed coordinate identical to zero prior to removing the fixed coordinate. Let $C \perp$ be the dual code of C, and

and $C(p)$ the penetrated code of C, that is produced from C with eliminating the fixed coordinate. Then,

$\boldsymbol{C^{(S)\perp} = C^{\perp(p)}}$

As remains clear that like an optimum $Eq\ (n, d, N)$ code has the greatest feasible distance d [1], where

$$\boldsymbol{d = n\frac{N\,(q-1)}{q(N-1)}}$$

Deleting the empty codeword in an $Eq(n, d, N)$-code effectively produces a comparable constant-weight $Eq(n, w, d, N) -$ code integrating $w - d$.

The implementing is an overall approach for specifying linear codes from a mapping $F: Fpm \rightarrow Fpm$, where

$F(0) = 0$. The expressed code $CF \subset F\,pm - 1$ has been established as a result of

$$\boldsymbol{C_F = \{C_{a,u} = (Tr_1\ (ux)\ a\ \epsilon F_{pm}\}}$$

Where $F's$ image is included in Fp, and m otherwise. The CF has a maximum size and a length of $p\,m - 1$. For $p = 2$, the code $\boldsymbol{C_F}$ can be used to describe AB and APN functions. To prevent misunderstanding, nous can use capital letters to designate functions image is the included in the base field Fp $(l = m\,in)$, and lowercase letters to denote functions from Fpm to Fp $(l = t\,in)$ while dealing with either varieties of functions. The reports that codes with excellent error-correcting parameters can be achieved through implementing certain vectorially mappings from $F\ to\ F$. If F: $Fpm \rightarrow Fpm$ has zero linear components, the linear code $\boldsymbol{C_F}$ obtained from the general construction in a rate of has dimension 2m. The weights can be expressed with the Walsh transform of absolute trace functions of the track F: $Fpm \rightarrow Fpm$, as indicated using the preceding theorem.

Table 1.A points of PG (2,9) are:

P_0	(1 , 0 , 0)	P_{26}	$(1, \sigma, \sigma)$	P_{65}	$(1,-\sigma,0)$
P_1	(0 , 1 , 0)	P_{27}	$(1,-\sigma^3,-\sigma^3)$	P_{66}	$(0,1,-\sigma)$
P_2	(0 , 0 , 1)	⋮		P_{67}	(1,0,-1)
P_3	$(1,0,\sigma^2)$	P_{36}	(1,1,0)	P_{68}	$(1,-1,\sigma^2)$
P_4	$(1,-\sigma^2,\sigma^2)$	P_{37}	(0,1,1)	P_{69}	$(1,-\sigma^2,-\sigma^2)$
P_5	$(1,-\sigma^2,\sigma)$	P_{38}	$(1,0,-\sigma^3)$	P_{70}	$(1,\sigma^2,-\sigma^3)$
P_6	$(1,-\sigma^3,1)$	P_{39}	$(1,\sigma,\sigma^2)$	P_{71}	$(1,\sigma,-1)$
P_7	$(1,1,-\sigma)$	P_{40}	$(1,-\sigma^2,-\sigma)$	P_{72}	(1,-1,1)
P_8	$(1,\sigma^3,-1)$	P_{41}	$(1,\sigma^3,\sigma^3)$	P_{73}	$(1,1,\sigma)$
P_9	$(1,-1,-\sigma)$	P_{42}	$(1,-\sigma,-\sigma^3)$	P_{74}	$(1,-\sigma^3,-\sigma)$

P_{10}	$(1, \sigma^3,- \sigma)$	P_{43}	$(1, \sigma,0)$	P_{75}	$(1, \sigma^3,- \sigma^2)$
P_{11}	$(1, \sigma^3,0)$	P_{44}	$(0,1, \sigma)$	P_{76}	$(1, \sigma^2,1)$
P_{12}	$(0,1, \sigma^3)$	P_{45}	$(1,0,- \sigma)$	P_{77}	$(1,1,- \sigma^2)$
P_{13}	$(1,0,1)$	P_{46}	$(1, \sigma^3, \sigma^2)$	P_{78}	$(1, \sigma^2,- \sigma^2)$
P_{14}	$(1,1, \sigma^2)$	P_{47}	$(1,- \sigma^2, \sigma^3)$	P_{79}	$(1, \sigma^2, \sigma)$
P_{15}	$(1,- \sigma^2,0)$	P_{48}	$(1,- \sigma,-1)$	P_{80}	$(1,- \sigma^3, \sigma^3)$
P_{16}	$(0,1,- \sigma^2)$	P_{49}	$(1,-1, \sigma^3)$	P_{81}	$(1,- \sigma, \sigma)$
P_{17}	$(1,0,- \sigma^2)$	P_{50}	$(1,- \sigma, \sigma^3)$	P_{82}	$(1,- \sigma^3, \sigma)$
P_{18}	$(1, \sigma^2, \sigma^2)$	P_{51}	$(1,- \sigma,- \sigma^2)$	P_{83}	$(1,- \sigma^3,0)$
P_{19}	$(1,- \sigma^2,- \sigma^3)$	P_{52}	$(1, \sigma^2,- \sigma)$	P_{84}	$(0,1,- \sigma^3)$
P_{20}	$(1 , \sigma, - \sigma)$	P_{53}	$(1, \sigma^3,1)$	P_{85}	$(1,0, \sigma^3)$
P_{21}	$(1, \sigma^3, \sigma)$	P_{54}	$(1,1,-1)$	P_{86}	$(1,- \sigma, \sigma^2)$
P_{22}	$(1,- \sigma^3,- \sigma^2)$	P_{55}	$(1,-1, \sigma)$	P_{87}	$(1,- \sigma^2,-1)$
P_{23}	$(1, \sigma^2, \sigma)$	P_{56}	$(1,- \sigma^3,-1)$	P_{88}	$(1,-1,- \sigma^2)$
P_{24}	$(1,- \sigma,- \sigma)$	P_{57}	$(1,-1,-1)$	P_{89}	$(1, \sigma^2,0)$
P_{25}	$(1, \sigma^3,- \sigma^3)$	⋮		P_{90}	$(0,1, \sigma^2)$

Choose a PG (2,9) points with order for the 3rd component is equal to 0, this means belong to ℓ_0 = v (z) s.t v (z) = t_z = z ∀ t in F9/{0} therefore, and with Pi = i , i = 0 , 1 ,..., 90, we invest ℓ_0 = { 0, 1, 11, 15, 31, 36, 43, 65, 83, 89 } remove this,

$$\ell_i = \ell_0 \, c(g)i = \ell_0 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & \sigma^2 \end{pmatrix}$$

Table 2. the lines and equation of PG (2,9) .

$\boldsymbol{\ell i}$	$\boldsymbol{P_i}$										**Line equation**
ℓ_0	0	1	11	15	31	36	43	65	83	89	x_2
ℓ_1	1	2	12	16	32	37	44	66	84	90	x_0
ℓ_2	2	3	13	17	33	38	45	67	85	1	$-x_1$
ℓ_3	3	4	14	18	34	39	46	68	86	2	$-x_0- \sigma^2 x_2$
ℓ_4	4	5	15	19	35	40	47	69	87	3	$\sigma^2 x_0 + x_1$
ℓ_5	5	6	16	20	36	41	48	70	88	4	$\sigma^3 x_0 – \sigma^3 x_1 – \tau x_2$
ℓ_6	6	7	17	21	37	42	49	71	89	5	$X_0 + \sigma^2 x_1 – \sigma^2 x_2$
ℓ_7	7	8	18	22	38	43	50	72	90	6	$X_o + \sigma^3 x_1 – \tau x_2$
ℓ_8	8	9	19	23	39	44	51	73	0	7	$-x_1 – \sigma^3 x_2$
ℓ_9	9	10	20	24	40	45	52	74	1	8	$- \sigma x_0 – x_2$
ℓ_{10}	10	11	21	25	41	46	53	75	2	9	$- \sigma^3 x_0 + x_1$
ℓ_{11}	11	12	22	26	42	47	54	76	3	10	$\sigma^2 x_0 + \sigma^3 x_1 – x_2$
ℓ_{12}	12	13	23	27	43	48	55	77	4	11	$X_0 + \sigma^3 x_1 – x_2$
ℓ_{13}	13	14	24	28	44	49	56	78	5	12	$X_0 + \sigma x_1 – x_2$

ℓ_{14}	14	15	25	29	45	50	57	79	6	13	$\sigma x_0 - \sigma^3 x_1 x_2$
ℓ_{15}	15	16	26	30	46	51	58	80	7	14	$X_0 - x_1 - x_2$
ℓ_{16}	16	17	27	31	47	52	59	81	8	15	$\sigma^3 x_0 + \sigma^3 x_1 + \tau x_2$
ℓ_{17}	17	18	28	32	48	53	60	82	9	16	$\sigma^3 x_0 - \sigma x_1 + \sigma x_2$
ℓ_{18}	18	19	29	33	49	54	61	83	10	17	$- \tau x_0 + \sigma^2 x_1 + x_2$
ℓ_{19}	19	20	30	34	50	55	62	84	11	18	$\sigma x_0 + \sigma^2 x_1 - \sigma^3 x_2$
ℓ_{20}	20	21	31	35	51	56	63	85	12	19	$\sigma^3 x_0 + \sigma^3 x_1 - x_2$
ℓ_{21}	21	22	32	36	52	57	64	86	13	20	$\sigma^2 x_0 - \sigma^2 x_1 - \sigma^2 x_2$
ℓ_{22}	22	23	33	37	53	58	65	87	14	21	$-x_0 + \sigma^3 x_1 - \sigma^3 x_2$
ℓ_{23}	23	24	34	38	54	59	66	88	15	22	$- \sigma^3 x_0 - \sigma x_1 - x_2$
ℓ_{24}	24	25	35	39	55	60	67	89	16	23	$-x_0 - \sigma^2 x_1 - x_2$
ℓ_{25}	25	26	36	40	56	61	68	90	17	24	$-x_0 + x_1 + \sigma^2 x_2$
ℓ_{26}	26	27	37	41	57	62	69	0	18	25	$- \sigma^2 x_1 + \sigma^2 x_2$
ℓ_{27}	27	28	38	42	58	63	70	1	19	26	$- \sigma^3 x_0 - x_2$
ℓ_{28}	28	29	39	43	59	64	71	2	20	27	$- \sigma x_0 + x_1$
ℓ_{29}	29	30	40	44	60	65	72	3	21	28	$\sigma x_0 + x_1 + \sigma^3 x_2$
ℓ_{30}	30	31	41	45	61	66	73	4	22	29	$- \sigma^2 x_0 - \sigma^2 x_1 - \sigma x_2$
ℓ_{31}	31	32	42	46	62	67	74	5	23	30	$- \sigma^2 x_0 - \sigma^2 x_1 - \sigma^2 x_2$
ℓ_{32}	32	33	43	47	63	68	75	6	24	31	$- \sigma^3 x_0 + \sigma^2 x_1 + \sigma^2 x_2$
ℓ_{33}	33	34	44	48	64	69	76	7	25	32	$-x_0 - x_1 - \sigma^3 x_2$
ℓ_{34}	34	35	45	49	65	70	77	8	26	33	$\sigma x_0 + x_1 + x_2$
ℓ_{35}	35	36	46	50	66	71	78	9	27	34	$- \sigma^2 x_0 + \sigma^2 x_1 + \sigma x_2$
ℓ_{36}	36	37	47	51	67	72	79	10	28	35	$- \sigma x_0 + \sigma x_1 - \sigma x_2$
ℓ_{37}	37	38	48	52	68	73	80	11	29	36	$- \sigma^3 x_0 + x_1 - x_2$
ℓ_{38}	38	39	49	53	69	74	81	12	30	37	$\sigma^3 x_0 - \sigma^3 x_1 + x_2$
ℓ_{39}	39	40	50	54	70	75	82	13	31	38	$-x_0 - x_1 + x_2$
ℓ_{40}	40	41	51	55	71	76	83	14	32	39	$-x_0$
ℓ_{41}	41	42	52	56	72	77	84	15	33	40	$\sigma^3 x0 - \sigma^3 x_1 x_2$
ℓ_{42}	42	43	53	57	73	78	85	16	34	41	$- \sigma x_0$
ℓ_{43}	43	44	54	58	74	79	86	17	35	42	$-x_0 - \sigma^3 x_1 + \sigma^2 x_2$

ℓ_{44}	44	45	55	59	75	80	87	18	36	43	$\sigma^2x_0-\sigma^2x_1+\tau x_2$
ℓ_{45}	45	46	56	60	76	81	88	19	37	44	$-\sigma x_0+x_1-x_2$
ℓ_{46}	46	47	57	61	77	82	89	20	38	45	$-\sigma^3x_0+\sigma x_1-x_2$
ℓ_{47}	47	48	58	62	78	83	90	21	39	46	$-\sigma^3x_0+\sigma x_1-\sigma^2x_2$
ℓ_{48}	48	49	59	63	79	84	0	22	40	47	$-\sigma^3x_0+\sigma^2x_1-x_2$
ℓ_{49}	49	50	60	64	80	85	1	23	41	48	$-\sigma^3x_1+\sigma^2x_2$
ℓ_{50}	50	51	61	65	81	86	2	24	42	49	$-\sigma x_0-x_2$
ℓ_{51}	51	52	62	66	82	87	3	25	43	50	$-\sigma^3x_0+x_1$
ℓ_{52}	52	53	63	67	83	88	4	26	44	51	$-\sigma^2x_0-\sigma x_1+x_2$
ℓ_{53}	53	54	64	68	84	89	5	27	45	52	$\sigma^3x_0-\sigma x_1+\sigma^2x_2$
ℓ_{54}	54	55	65	69	85	90	6	28	46	53	$\sigma^3x_0+\sigma^2x_1-x_2$
ℓ_{55}	55	56	66	70	86	0	7	29	47	54	$-x_1-\sigma^3x_2$
ℓ_{56}	56	57	67	71	87	1	8	30	48	55	$-\sigma x_0-x_2$
ℓ_{57}	57	58	68	72	88	2	9	31	49	56	$-\sigma^3x_0+x_1$
ℓ_{58}	58	59	69	73	89	3	10	32	50	57	$\sigma^2x_0+\sigma^3x_1-x_2$
ℓ_{59}	59	60	70	74	90	4	11	33	51	58	$-\sigma x_0-\sigma^2x_1+x_2$
ℓ_{60}	60	61	71	75	0	5	12	34	52	59	$-\sigma x_1-\sigma^2x_2$
ℓ_{61}	61	62	72	76	1	6	13	35	53	60	X_0-x_2
ℓ_{62}	62	63	73	77	2	7	14	36	54	61	$-x_0+x_1$
ℓ_{63}	63	64	74	78	3	8	15	37	55	62	$\sigma x_0-\sigma^3x_1+\sigma^3x_2$
ℓ_{64}	64	65	75	79	4	9	16	38	56	63	$-\sigma^3x_0-x_1+x_2$
ℓ_{65}	65	66	76	80	5	10	17	39	57	64	$\sigma^2x_0+\tau x_1+x_2$
ℓ_{66}	66	67	77	81	6	11	18	40	58	65	$\sigma^2x_0+\sigma^3x1+\sigma^2x_2$
ℓ_{67}	67	68	78	82	7	12	19	41	59	66	$\sigma x_0+x_1+\sigma x_2$
ℓ_{68}	68	69	79	83	8	13	20	42	60	67	$X_0-\sigma x_1-x_2$
ℓ_{69}	69	70	80	84	9	14	21	43	61	68	$-x_0-\sigma^3x_1-x_2$
ℓ_{70}	70	71	81	85	10	15	22	44	62	69	$X_0-\sigma^2x_1+\sigma x_2$
ℓ_{71}	71	72	82	86	11	16	23	45	63	70	$X_0+\sigma x_1-\sigma^3x_2$
ℓ_{72}	72	73	83	87	12	17	24	46	64	71	$-\sigma^2x_0+\sigma^3x_1-x_2$
ℓ_{73}	73	74	84	88	13	18	25	47	65	72	$\sigma x_0+\sigma^2x_1-\sigma x_2$

ℓ_{74}	74	75	85	89	14	19	26	48	66	73	$-x_0-\sigma^2x_1-\sigma x_2$
ℓ_{75}	75	76	86	90	15	20	27	49	67	74	$-x_0+\sigma^2x_1-x_2$
ℓ_{76}	76	77	87	0	16	21	28	50	68	75	$\sigma^2x_1+x_2$
ℓ_{77}	77	78	88	1	17	22	29	51	69	76	$-\sigma^2x_0-x_2$
ℓ_{78}	78	79	89	2	18	23	30	52	70	77	$-\sigma^2x_0+x_1$
ℓ_{79}	79	80	90	3	19	24	31	53	71	78	$-\sigma^2x_0-\sigma^2x_1+x_2$
ℓ_{80}	80	81	0	4	20	25	32	54	72	79	$-\sigma^2x_1-\sigma^2x_2$
ℓ_{81}	81	82	1	5	21	26	33	55	73	80	σx_0-x_2
ℓ_{82}	82	83	2	6	22	27	34	56	74	81	$\sigma^3x_0+x_1$
ℓ_{83}	83	84	3	7	23	28	35	57	75	82	$\sigma^3x_0+x_1-\sigma x_2$
ℓ_{84}	84	85	4	8	24	29	36	58	76	83	$\sigma^3x_0-\sigma^3x_1-x_2$
ℓ_{85}	85	86	5	9	25	30	37	59	77	84	$\sigma-x_0-\sigma^3x_1+\sigma^3x_2$
ℓ_{86}	86	87	6	10	26	31	38	60	78	85	$-\sigma^3x_0-\sigma^3x_1-x_2$
ℓ_{87}	87	88	7	11	27	32	39	61	79	86	$\sigma^3x_0+x_1+x_2$
ℓ_{88}	88	89	8	12	28	33	40	62	80	87	$\sigma x_0+\sigma^3x_1-x_2$
ℓ_{89}	89	90	9	13	29	34	41	63	81	88	$X_0+\sigma^2x_1-x_2$
ℓ_{90}	90	0	10	14	30	35	42	64	82	89	$\sigma x_1+\sigma^3x_2$

Following theorem builds the parameters (n, M, d)

Theorem(1.3) The code C projective plane of grade nine has a parameter

[n = 91, M $\leq 9^{88}$,d = 10]

proof:

An incident matrix aij exists for the plane πq, where

$$aij = \begin{cases} 1 & if \quad P_j \in \ell\, \mathrm{i} \\ 0 & if \quad P_j \notin \ell\, \mathrm{i} \end{cases}$$

$\ell_0 = [1, 1, 0, 0, 0, \dots, 0, 0, 1, 0]$, $\ell_1 = [0, 1, 1, 0, 0, \dots, 0, 0, 0, 1]$, $\ell_2 = [1, 0, 1, 1, 0, \dots, 0, 0, 0, 0]$

$\ell_3 = [0, 1, 0, 1, 1, \dots, 0, 0, 1, 0]$, $\ell_4 = [0, 0, 1, 0, 1, \dots, 1, 0, 0, 0]$, … , $\ell_{87} = [0, 0, 0, 0, 0, \dots, 1, 1, 0, 0]$

$\ell_{88} = [0, 0, 0, 0, 0, \dots, 0, 1, 1, 0]$, $\ell_{89} = [0, 0, 0, 0, 0, \dots, 1, 0, 1, 1]$, $\ell_{90} = [1, 0, 0, 0, 0, \dots, 0, 1, 0, 1]$.

Let $\alpha = [0, 0, \dots, 0]$, $\mu = [1, 1, \dots, 1]$, $\varpi = [-1, -1, \dots -1]$, $\Upsilon = [\sigma, \sigma, \dots, \sigma]$, $s = [-\sigma, -\sigma, \dots, -\sigma]$,

$\hbar = [\sigma^2, \sigma^2, \dots, \sigma^2]$, $\kappa = [-\sigma^2, -\sigma^2, \dots, -\sigma^2]$, $\theta = [\sigma^3, \sigma^3, \dots, \sigma^3]$, $\mathscr{g} = [-\sigma^3, -\sigma^3, \dots, \sigma -^3]$

$\rho_i = \mu + \ell_i$

$\rho_0 = [-1, .1, 1, 1, 1, ..., 1, 1, -1, 1]$, $\rho_1 = [1, -1, -1, 1, 1, ..., 1, 1, 1, -1]$, $\rho_2 = [-1, 1, -1, -1, 1, \ldots, 1, 1, 1, 1]$

$\rho_3 = [1, -1, 1, -1, -1, \ldots, 1, 1, 1, 1]$, $\rho_4 = [1, 1, -1, 1, -1, \ldots, -1, 1, 1, 1]$, … , $\rho_{87} = [1, 1, 1, 1, 1, \ldots, -1, 1, 1, 1]$

$\rho_{88} = [1, 1, 1, 1, 1, ..., 1, -1, -1, 1]$, $\rho_{89} = [1, 1, 1, 1, 1, \ldots, -1, 1, -1, -1]$, $\rho_{90} = [-1, 1, 1, 1, 1, \ldots, 1, -1, 1, -1]$.

Let vi = $\varpi + \ell_i$ & $i = 0, 1, 2, \ldots, 90$

$v_0 = [-1, -1, -1, 0, 0, \ldots, -1, -1, 0, -1]$, $v_1 = [-1, -1, 0, 0, -1, \ldots, -1, -1, -1, 0]$, $v_2 = [0, -1, 0, 0, -1, \ldots, -1, -1, -1, -1]$,

$v_3 = [-1, 0, -1, 0, 0, \ldots, -1, -1, -1, -1]$, $v_4 = [-1, -1, 0, -1, 0, \ldots, 0, -1, -1, -1]$, … , $v_{87} = [-1, -1, -1, -1, -1, \ldots, 0, 0, -1, -1]$,

$v_{88} = [-1, -1, -1, -1, -1, \ldots, -1, 0, 0, -1]$, $v_{89} = [-1, -1, -1, -1, -1, \ldots, 0, -1, 0, 0]$, $v_{90} = [0, -1, -1, -1, -1, \ldots, 0, -1, -1, 0]$.

Let $z_i = \Upsilon + \ell_i$ & $i = 0, 1, 2, \ldots, 90$

$z_0 = [\sigma^2, \sigma^2, \sigma, \sigma, \sigma, \ldots, \sigma, \sigma, \sigma^2, \sigma]$, $z_1 = [\sigma, \sigma^2, \sigma^2, \sigma, \sigma, \ldots, \sigma, \sigma, \sigma, \sigma^2]$, $z_2 = [\sigma^2, \sigma, \sigma^2, \sigma^2, \sigma, \ldots, \sigma, \sigma, \sigma, \sigma]$

$z_3 = [\sigma, \sigma^2, \sigma, \sigma^2, \sigma^2, \ldots, \sigma, \sigma, \sigma, \sigma]$, $z_4 = [\sigma, \sigma, \sigma^2, \sigma, \sigma^2, \ldots, \sigma^2, \sigma, \sigma, \sigma]$, … , $z_{87} = [\sigma, \sigma, \sigma, \sigma, \sigma, \ldots, \sigma^2, \sigma^2, \sigma, \sigma]$

$z_{88} = [\sigma, \sigma, \sigma, \sigma, \sigma, \ldots, \sigma, \sigma^2, \sigma^2, \sigma]$, $z_{89} = [\sigma, \sigma, \sigma, \sigma, \sigma, \ldots, \sigma^2, \sigma, \sigma^2, \sigma^2]$, $z_{90} = [\sigma^2, \sigma, \sigma, \sigma, \sigma, \ldots, \sigma, \sigma^2, \sigma, \sigma^2]$

$f_i = s + \ell_i$ & $i = 0, 1, 2, \ldots, 90$

$f_0 = [\sigma^3, \sigma^3, -\sigma, -\sigma, -\sigma, \ldots, -\sigma, -\sigma, \sigma^3, -\sigma]$, $f_1 = [-\sigma, \sigma^3, \sigma^3, -\sigma, -\sigma, \ldots, -\sigma, -\sigma, -\sigma, \sigma^3]$, $f_2 = [\sigma^3, -\sigma, \sigma^3, \sigma^3, -\sigma, \ldots, -\sigma, -\sigma, -\sigma, -\sigma]$

$f_3 = [-\sigma, \sigma^3, -\sigma, \sigma^3, \sigma^3, \ldots, -\sigma, -\sigma, -\sigma, -\sigma]$, $f_4 = [-\sigma, -\sigma, \sigma^3, -\sigma, \sigma^3, \ldots, \sigma^3, -\sigma, -\sigma, -\sigma]$, … , $f_{87} = [-\sigma, -\sigma, -\sigma, -\sigma, -\sigma, \ldots, \sigma^3, \sigma^3, -\sigma, -\sigma]$

$f_{88} = [-\sigma, -\sigma, -\sigma, -\sigma, -\sigma, \ldots, -\sigma, \sigma^3, \sigma^3, -\sigma]$, $f_{89} = [-\sigma, -\sigma, -\sigma, -\sigma, -\sigma, \ldots, \sigma^3, \sigma, \sigma^3, \sigma^3]$, $f_{90} = [\sigma^3, -\sigma, -\sigma, -\sigma, -\sigma, \ldots, -\sigma, \sigma^3, -\sigma, \sigma^3]$.

$c_i = \hbar + \ell_i$, $i = 0, 1, 2, \ldots, 90$

$c_0 = [-\sigma^3, -\sigma^3, \sigma^2, \sigma^2, \sigma^2, \ldots, \sigma^2, \sigma^2, -\sigma^3, \sigma^2]$, $c_1 = [\sigma^2, -\sigma^3, -\sigma^3, \sigma^2, \sigma^2, \ldots, \sigma^2, \sigma^2, \sigma^2, -\sigma^3]$, $c_2 = [-\sigma^3, \sigma^2, -\sigma^3, -\sigma^3, \sigma^2, \ldots, \sigma^2, \sigma^2, \sigma^2, \sigma^2]$

$c_3 = [\sigma^2, -\sigma^3, \sigma^2, -\sigma^3, -\sigma^3, \ldots, \sigma^2, \sigma^2, \sigma^2, \sigma^2]$, $c_4 = [\sigma^2, \sigma^2, \sigma^3, \sigma^2, \sigma^3, \ldots, -\sigma^3, \sigma^2, \sigma^2, \sigma^2]$, … , $c_{87} = [\sigma^2, \sigma^2, \sigma^2, \sigma^2, \sigma^2, \ldots, -\sigma^3, -\sigma^3, \sigma^2, \sigma^2]$

$c_{88} = [\sigma^2, \sigma^2, \sigma^2, \sigma^2, \sigma^2, \ldots, \sigma^2, -\sigma^3, -\sigma^3, \sigma^2]$, $c_{89} = [\sigma^2, \sigma^2, \sigma^2, \sigma^2, \sigma^2, \ldots, -\sigma^3, \sigma, -\sigma^3, -\sigma^3]$, $c_{90} = [-\sigma^3, \sigma^2, \sigma^2, \sigma^2, \sigma^2, \ldots, \sigma^2, -\sigma^3, \sigma^2, -\sigma^3]$.

In the same way we find, $x_i = \kappa + \ell_i$, $b_i = T + \ell_i$, $n_i = g + \ell_i$ where $i = 0, 1, 2, \ldots, 90$

Now, we will give $D = d(c)$ min. distance as follows:

d (c_i, m_i) = d (c_i, v_i) = d (c_i, z_i) = d (c_i, f_i) = d (c_i, x_i) = d (c_i, b_i) = d (c_i, n_i) = d (n_i, m_i) = 91 ,

d (μ, α) = d (μ, ϖ) = d (μ, Υ) = d (μ, s) = d $(\mu, \hbar)$ = d (μ, κ) = d (μ, θ) = d (μ, g) = 91,

d (Υ, α) = d (Υ, ϖ) = d (Υ, Υ) = d (Υ, s) = d $(\Upsilon, \hbar)$ = d (Υ, κ) = d (Υ, θ) = d (Υ, g) = 91,

d $(\hbar, \alpha)$ = d $(\hbar, \varpi)$ = d $(\hbar, \Upsilon)$ = d $(\hbar, s)$ = d $(\hbar, \hbar)$ = d $(\hbar, \kappa)$ = d (Υ, θ) = d (Υ, g) = 91,

d (θ, α) = d (θ, ϖ) = d (θ, Υ) = d (θ, s) = d $(\theta, \hbar)$ = d (θ, κ) = d (θ, θ) = d (θ, g) = 91,

d (ϖ, α) = d (ϖ, ϖ) = d (ϖ, Υ) = d (ϖ, s) = d $(\varpi, \hbar)$ = d (ϖ, κ) = d (ϖ, θ) = d (ϖ, g) = 91,

d (s, α) = d (s, ϖ) = d (s, Υ) = d (s, s) = d $(s, \hbar)$ = d (s, κ) = d (s, θ) = d (s, g) = 91, …, In a similar manner, the remaining.

If we subrogate with n =91, d = 10, e = 4 In inequality 1.1 we get M = 9^{88} hence C is a (91, 9^{88}, 10)-code

9^{88} { $\binom{91}{0}+\binom{91}{1}(9-1)+\binom{91}{2}(9-1)^2+\binom{91}{3}(9-1)^3+\binom{91}{4}(9-1)^4 \succ 9^{91}$

As a result of the corollary, C is not perfect.

Algorithm :

We can get geometry on, p_i = i , $i = 0, 1, 2, \dots, 90$

Then $\ell_i + \ell_j$ = ai where $i, j = 0, 1, 2, \dots, 90$

$\alpha_r = 1 \Leftrightarrow p_r$ lies on precisely one of , ℓ i, ℓ j

$\alpha_r = 0 \Leftrightarrow p_r$ lies on the third line through ℓ i$\cap\ell$ j

here

$\ell_i + \ell_j$, $\ell_i + \mu +$ w, $\ell_i + \Upsilon$, ℓ_i +s , $\ell_i + \hbar$, ℓ_i +k, ℓ_i +t , $\ell_i + g \ell_i + \rho$ i, $\ell_i + v_i$, $\ell_i + z_i$, $\ell_i + f_i$, $\ell_i + c_i$, $\ell_i + x_i$, $\ell_i + b_i$, ℓ_i

$+n_i$ in C

$\rho_i + \rho_j$, $\rho_i + \mu$, ρ_i +w, $\rho_i + \Upsilon$, ρ_i +s, $\rho_i + \hbar$, $\rho_i + \kappa$, ρ_i +t, ρ_i +g, $\rho_i + v_i$, $\rho_i + z_i$, $\rho_i + f_i$, $\rho_i + c_i$, $\rho_i + x_i$, $\rho_i + b_i$, $\rho_i + n_i$

in C

v_i +vj, $v_i + \mu$, v_i +w, $v_i + \Upsilon$, v_i +s, $v_i + \hbar$, $v_i + \kappa$, v_i +t, $v_i + g$ $v_i + z_i$, $v_i + f_i$, $v_i + c_i$, $v_i + x_i$, $v_i + b_i$, $v_i + n_i$ in C

z_i +zj, $z_i + \mu$, z_i +w, $z_i + \Upsilon$, z_i +s, $z_i + \hbar$, $z_i + \kappa$, z_i +t, $z_i + g$ in C

$z_i + f_i$, $z_i + c_i$, $z_i + x_i$, $z_i + b_i$, $z_i + n_i$

f_i +fj, $f_i + \mu$, f_i +w, $f_i + \Upsilon$, f_i +s, f_i +h, $f_i + \kappa$, f_i +t, f_i +g $f_i + c_i$, $f_i + x_i$, $f_i + b_i$, $f_i + n_i$ in C

c_i +cj, $c_i + \mu$, c_i +w, $c_i + \Upsilon$, c_i +s, $c_i + \hbar$, $c_i + \kappa$, c_i +t, $c_i + g$ $c_i + x_i$, $c_i + b_i$ $c_i + n_i$ in C

x_i +xj, $x_i + \mu$, x_i +w, $x_i + \Upsilon$, x_i +s, $x_i + \hbar$, $x_i + \kappa$, x_i +t, $x_i + g$ $x_i + b_i$, $x_i + n_i$ in C

$\mathcal{B}_i + \mathcal{B}$j, $\mathcal{B}_i + \mu$, $\mathcal{B}_i$ +w, $\mathcal{B}_i + \Upsilon$, $\mathcal{B}_i$ +s, $\mathcal{B}_i + \hbar$, $\mathcal{B}_i + \kappa$, $\mathcal{B}_i$ +t, $\mathcal{B}_i$ +g $\mathcal{B}_i + T_i$ in C

$T_i + T$j, $T_i + \mu$, T_i +w, $T_i + \Upsilon$, T_i +s, $T_i + \hbar$, $T_i + \kappa$, T_i +t, T_i +g in C

Theorem (1.5) the code C = [n =91 , M ≤ 9^{88}, d = 10]

Is the vector space's subspace. $((F_9)^{91}$,+,.)

Over a finite (F_9, $+_3$, $._3$)

Proof :

The vector $\alpha = 0, 0, \dots, 0 \in C$ Thus $C \neq \emptyset$

∀X = ($x_1, x_2, \dots, x_{21}$) , $\Upsilon = (\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_{21}) \in C$.

From theorem (1.4) , we get x+y ∈ C.

∀ k ∈ F9 , and X ∈ C , we have

K . X = k ($x_1, x_2, x_3, \dots, x_{91}$) = ($k._3 x_1$, $k._3 x_2$, …, $k._3 x_{91}$) ∈ C

Thus, by theorem (1.3) , C is a subspace of $((F_9)^{91}$, +,.) over (F_{91}, +3,$._3$).

Result and discussion

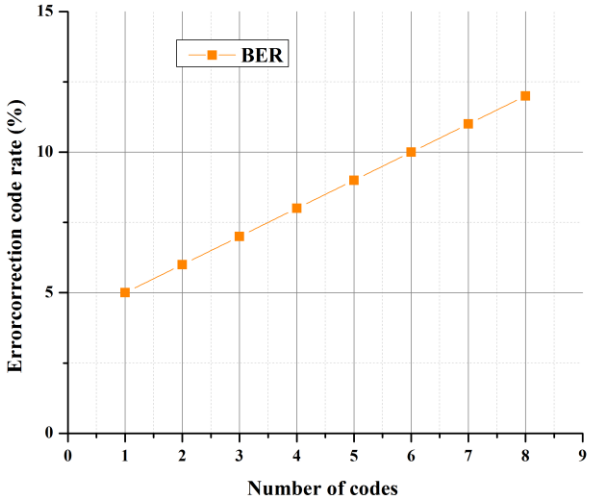

Figure 3 Error correction code rate

Figure 3 shown in an error correcting code is an encoding system that sends messages as binary integers and allows the message to be retrieved even if some bits are incorrectly flipped. They are utilized in almost every scenario of message transmission, particularly in data storage, where error-correcting code protects against data corruption. Error coding is a way of ensuring dependable digital data transmission and storage when the communication channel utilized has an unacceptable bit error rate (BER) and a poor signal-to-noise ratio. Error-correction codes identify and rectify single-bit mistakes automatically, resulting in lower standby and refresh power while increasing memory dependability. Error-correcting codes are mathematical approaches that help digital communication systems discover and rectify faults during data transfer. Interfaces are critical for enabling dependable and effective information transmission via noisy channels like wireless networks, optical fibre, and satellite connections.

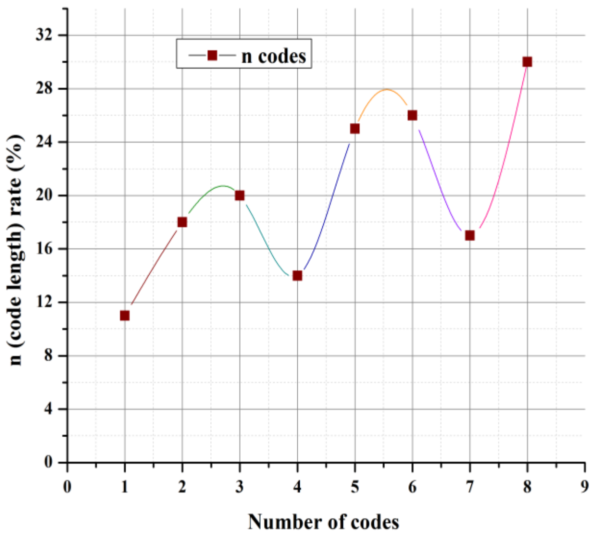

Figure 4 Analysis of n (code length) rate

Figure 4 shown is the measure code length in bits, the code length of an integer n may be estimated. The average length of a codeword in a Huffman encoded alphabet is mostly determined by the alphabet's probability distribution and the number of symbols. As previously stated, for integer codeword lengths, the source entropy serves as the lower constraint on the average codeword length. A code in which a fixed number of source symbols are converted to a fixed number of output symbols. Anything that may change length is referred to as variable length. In databases, a variable-length field is one that does not have a set length. Instead, the field length fluctuates according on the data put in it. Linear codes are used in forward error correction and in techniques for delivering symbols across a communications channel such that, if mistakes occur during transmission, the receiver of a message block may correct and identify.

III. CONCLUSION

We have also explored the relationship between coding theory and the finite Projective plane, where the columns of the generator matrix of any liner code are regarded as points in the Projective plane, in addition to constructing projective linear codes with parameters (n, k, d) based on the Galois Field Fq order.

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