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Mathematical Thinking Among Pre-Service Teachers: A Critical Component of Teacher Preparation

Abstract: - This research examined pre-service mathematics teachers' mathematical thinking abilities, focusing on problem-solving, mathematical representation, reasoning, mathematical communication, and mathematical connection. Additionally, the study aimed to determine whether there were significant differences in mathematical thinking between pre-service students majoring and minoring in mathematics. The research employed a quantitative approach and involved 277 pre-service mathematics teachers enrolled in the Mathematics Teaching course at the Faculty of Science and Mathematics, Universiti Pendidikan Sultan Idris in Malaysia. The research design utilized a survey method, and data were collected through a mathematical thinking questionnaire. The collected data were analyzed using statistic descriptive including mean and standard deviation, while inferential statistics use t-test analyses. The study results revealed that the overall mean is 4.506, followed by standard deviation is 0.541, indicating that the pre-service teachers show a higher positive mathematical thinking. The result also shows a significant relationship between problem-solving, mathematical representation, reasoning, mathematical communication and mathematical connection. The findings also indicated significant differences in mathematical thinking between pre-service mathematics teachers based on gender. Lastly, the result shows that the study recommended the inclusion of mathematical thinking development in teacher education programs to enhance the mathematical thinking abilities of pre-service mathematics teachers. In conclusion, mathematical thinking is an essential aspect of teacher preparation, as it contributes to ensuring the quality of the next generation.

Keywords: Mathematical thinking, higher-order thinking, reasoning, mathematical thinking, problem-solving skills, pre-service mathematics teachers

INTRODUCTION

Mathematics is a field of knowledge that trains the mind to think manically and systematically in solving problems and making decisions (Tarmizi & Abdullah, 2009). Besides, mathematics develops through the use of scaling and logic, from calculation, counting, measuring, and studying the systematic position, shape, and motion of physical objects (Kheong, 2011). Therefore, mathematics is a necessary element for development, playing an essential role in all human endeavours. As Goos and Kaya (2019) stated, mathematical thinking can be obtained from examining the study framework and the curriculum framework that tries to describe its prominent features. In 2017, Mathematics Curriculum in Malaysia has been revised and rearranged from Integrated Secondary School Curriculum (KBSM) to Secondary School Standard Curriculum (KSSM) accordingly with the need to provide more knowledge and various mathematics skills to students (Nor Azlina, 2014). KSSM was introduced by the Ministry of Education Malaysia (MOE), also intended to produce mathematically thinking students that are able to do mathematics and understand mathematical ideas, as well as responsibly apply mathematical knowledge and skills in daily life that are based on attitudes and values Standard Document for Curriculum and Assessment (DSKP), 2016). For this reason, it is necessary to carry out mathematics in a way to help students in increasing their ability to solve real-life problems (Murat Tezer & Meryem Cumhur, 2017).

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In order to achieve the objective of DSKP, which is to produce students that who are enhanced in mathematical thinking, the mathematical thinking abilities of teachers also need to be taken seriously. This is due to it being possible that educator's attitudes toward mathematical thinking shows the complicated connection between their knowledge and their effectiveness as educators (Aljaberi & Gheith, 2018). If a teacher always performs a negative belief toward mathematical thinking, the quality of the teaching and learning session will be affected and decline. In order to satisfy the aforementioned gap, this research attempted by creating an instrument for assessing mathematics pre-service teachers' mathematical thinking skill.

LITERATURE REVIEW

Several studies have concentrated and kept focusing on mathematics teacher's mathematical thinking abilities. Recently, mathematical thinking has been widely used as a significant indicator to assess the ability of somebody toward mathematical skills in mathematics education. In fact, mathematical thinking assists individuals in obtaining and interpreting the necessary knowledge as well as problem-solving skills such as customization, hypothesis formulation, estimation, generalization and accuracy evaluation which are needed in highly thinking process (Avni Yildiz, Serdal Baltaci & Büşra Kartal, 2020; Celik & Ozdemir, 2020). Individuals could practice mathematical thinking in daily or non-daily problem solving by discovering problem-solving tactics, analyzing the available facts in the problem, rationalizing the problem solution, and persuading others who disagree (Breen & O'Shea, 2010; Schoenfeld, 1992). Since mathematical thinking is involved with higher order thinking skills, individual can improve their ability to understand and solve the problems they encounter effectively within a short period. This statement is supported by the study from Kargar, Tarmizi and Bayat (2010) which stated that individuals with higher mathematical thinking can solve math-related problems with low anxiety about math and show more positive attitudes towards mathematics.

There are five mathematics skills related to mathematical thinking abilities which are problem-solving, mathematical representation, reasoning, mathematical communication and mathematical connection that National Council of Teaching Mathematics has standardized for teaching mathematics in schools. Based on the research of Ismel et al. (2020), problem-solving ability can be summed up as an individual's capacity to be involved in thinking processes when identifying and resolving problems for which the solution is not readily obtainable. Problem solving is also considered a procedure of overcoming real-life problems through making assumptions, gaining knowledge, and figuring out mathematical ideas (Fiona F et al., 2020). It was well recognized that problem solving involves various mentally stimulating activities, such as analysis, contemplating, evaluation, interpretation, estimation, and reflection. In several nations, especially Malaysia, Hong Kong, Singapore, Australia and England were essentially problem solving skill as a fundamental concept or a significant focus of the mathematics curriculum in schools (Saoud, Kamisah & Naser, 2021; Abdullah et al., 2019; Acar-Erdol & Yıldız, 2018). According to several studies, many educators agree that problem solving allows students to strengthen their actual-life abilities as well as their mathematical thinking abilities (Palraj et al., 2017; Xenofontos & Kyriakou, 2017; Yavuz & Cansz, 2019). Therefore, it can be concluded that problem solving assists learners in developing experience in every aspect of general mathematical technique (Celiki & Ozdemir, 2020). The study by Palraj et al. (2017) and Uyangör (2019) has been believed that problem solving to be the highest level of mental ability. Therefore, problem-solving should be implemented as one of the 21st-century teaching strategies or skills because they are crucial in mathematics and daily life.

Mathematical representation can be referred to as the process of symbolizing, encoding, conveying or translating mathematical concepts from a specific form to another visual or actual items such as drawings, numerical lines, graphs, arrangements of actual objects or tools for learning, tangible models, written texts, expressions of mathematics, formulae and equations, or images displayed on a computer or calculator screen (Goldin, 2014; Polat, 2020; Saoud, Kamisah & Naser, 2021). Moreover, it also refers to the act of organizing a mathematical idea or concept into a particular form or representation itself (Mainali, 2021). There are two types of representation which are internal representation and external representation. Internal representation is a mental representation that can be interpreted as an image created within the mind of an individual instead of an actual item (Mainali, 2021). This results in knowledge being stored in various forms, collectively referred to as mental imagery, such as frameworks, products, networked neurons (Zhang, 1997). On the other hand, external representation involves the reflection or visual system of an individual through which learners' ideas can be

presented verbally, visually, and symbolically. It is a way to represent the answers students discover regarding problems they meet (Minarni & Napitupulu, 2017; Solikhah & Budiharso, 2019; Utomo & Syarifah, 2021). The external representation can also refer to tangible product developed by either a teacher or a student that is able to be utilized effectively for explaining concepts in mathematics, such as a triangle, algebraic formula or a numerical line that was created by pupils on a sheet (Mainali, 2021). Representation skills play a role in assisting students in illustrating mathematical concepts (Minarni & Napitupulu, 2017). The process of representation includes the reconfiguration of problems or concepts into new configurations applying tangible objects or illustrations as symbols and expressions to convey the concepts (NCTM, 2014). Therefore, students may express their ideas or provide solutions to a problem using their representational skills (Nurhayati, 2013). This means that the main intention of mathematical representation is to allow the visualization of mathematical concepts through the use of a set of symbols (Utomo & Syarifah, 2021). For instance, translators utilize their representation skills to analyze and recreate the meaning of verbal problems more clearly. Not only visualization of the mathematical concept, mathematical representation skill is also important in transforming images or tangible objects into spoken words and phrases that deliver the intended message (NCTM, 2014; Utomo & Syarifah, 2021). Based on the mathematical representation of students, it became an instrument by which teachers are able to evaluate the understanding and achievement of their students in mathematics (Utomo & Syarifah, 2021). Teachers may modify and enhance their teaching strategy based on the evaluation of students' representation skill.

Mathematical reasoning and proving skill are a key aspect of the mathematical skills that are utilized in every phase and activities, as well as having a connection to literacy in mathematics. According to Jeinne and Benidiktus (2019), mathematics reasoning is the procedure of generating a sense of and comprehending mathematical concepts and ideas that are fundamental to operations. It is also can be defined as a framework of thought that includes five interrelated phases of mathematical thinking which are making sense, conjecturing, persuasive, reviewing, and making generalizations in order to generate statements and establish conclusions (Bjuland, 2007; Boesen, Lither& Palm, 2010; Jeinne& Benidiktus, 2019). In other hands, proofing can be defined as an individual behavior that necessitates being convinced of the truth of a proof structure (Selden& Selden, 2016; Fuat, Nusantara, Irawan & Irawati, 2019). A proof is mostly seen as a particular type of claim, with no further clarification regarding what a claim is (NCTM, 2000). The action of proving can interpret as demonstration of a proof as well as the protracted process of discovering, assessing, and revising the proof and the argument that it is intended to support, which may involve predicting skill and making generalizations (Bergwall, 2021; Stylianides et al., 2017). In general, reasoning and proof play a key role in mathematics learning because they boost learners' ability to evaluate whether an argument is accurate or not as well as promote learners' efficiency when answering or expressing mathematics problems (Celiki & Ozdemir, 2020; Maoto et al., 2018). Thus, substantial chances to improve fast discoveries about an extensive variety of phenomena are made accessible by mathematical reasoning and proof (Saoud, Kamisah& Naser, 2021). This is because reasoning and proof can be beneficial in the study of all fields of mathematics, including algebra, probability, and statistics (Siemon et al., 2017). These skills had success producing analytical thinkers and reasoners which are more likely to pay attention to structure, designs, or contrasts in both mathematical and actual-world circumstances. Not only that, students' instinctive and imaginative skills can be further developed by developing their mathematical reasoning. The following process might assist them strengthen and analyze how well they are able to utilize mathematical reasoning and proofs (Reyes-Cedeno et al., 2019).

The National Council of Teachers of Mathematics (NCTM, 2000) recognized mathematical communication as one of the fundamental methods of learning in mathematics. Mathematical communication skills can be defined as the ability to represent concepts related to mathematics in the written word (via the use of equations, tables, and pictures/graphs) and spoken form (via discussion and explanation) (Lestari & Yudhanegara, 2015). Mathematical communication skill is also known as the ability to grasp and agree with the views of others mathematical concepts, critical analysis and evaluative thinking in order to sharpen knowledge (Zulkarnain et al., 2021). Moreover, Diah and Haninda (2018) opinionated that communication is a process that occurs when two or more individuals generate or share knowledge with one another, which leads to a profound comprehension. Therefore, communication is an essential component in playing an important role in assisting individual to explore concepts that are associated with mathematics (Indah, Minarni & Surya, 2019; Zulfa, Kartono &

Cahyono, 2020). Additionally, the practice of communicating is also beneficial in conceptualization, interpretation, and generalization of ideas. In general, mathematics communication aids in the emergence of ideas into objects for reflection, discussion, refinement and modification (Saoud, Kamisah & Naser, 2021). In order to motivate individual to think and reason regarding what they are engaged in, a learning atmosphere which enables them to share their ideas about mathematical concepts must be built (Zulkarnain et al., 2021). Thus, it can be simply said that mathematical communication skills are important since mathematics serves as a language, not simply a tool for thinking, it may be applied as well to identify patterns, solve problems and make conclusions. It's also an important tool for accurate and careful communication of ideas (Ismail, Siagian & Zul, 2018). Equally important, several distinct forms of communication have been studied that can be applied in mathematics learning in order to investigate and deepen the comprehension of learners towards mathematical concepts, as well as to create connections between other mathematical concepts and other fields of study (Diah & Haninda, 2018). Regarding this, it had contributed to helping students comprehend more deeply toward mathematical concepts, problem solving, and made a correction toward misconceptions about mathematical ideas (Kamid et al, 2020).

There are several definitions that have been defined by previous researchers toward mathematical connection skill. According to Garcia-Garca & Dolores-Flores (2018), a mathematical connection has emerged as a mental process that occurs when one's mind links a variety of concepts, ideas, definitions, theories, procedures, depictions, and meanings with one another, with other fields of study, or with actual life. Secondly, the ability of students to make connections between different problems in mathematics, whether within mathematics or between mathematics and challenges other than mathematics, is commonly referred to as mathematical connection skill (Siregar & Surya, 2017). A mathematical connection skill can be understood as the ability to interconnect knowledge of concepts and procedures, apply mathematics in other contexts and daily activities, as well as create connections among various mathematical topics (Coxford, 1995; Madia & Holimombo, 2022; Aloisius, 2022). Mathematical connections are also defined as a part of the web of interrelated information along with different knowledge encompassing significant principles in order to comprehend and develop linkages between mathematical concepts, ideas, and processes (Kenedi et al., 2019). In this regard, the competency of mathematical connection can be considered one of the skills that students demand to develop in order to be allowed to grow their capacities of mathematical thinking (Hani Juita Sari, 2020). This means that mathematical connection skill is important and must be fully strengthened for the purpose of enabling students to figure out concepts more thoroughly and implement them when faced with a variety of real-life scenarios. In other words, the comprehension of learners is going to become more profound and remain greater duration as long as they are able to connect to mathematical concepts, which resulting potentially helps them to understand the relationships of mutually beneficial implications between mathematics concepts in contexts that relate mathematics with other fields of study (Jahring, 2020; Madia & Holimombo, 2022).In addition, being fulfilled with the ability of mathematical connection may strengthen intellectual abilities of students such as memory, knowledge of ways to utilize knowledge regarding the environment, and so on. Despite the fact that mathematics is overflowing with countless principles, individual could discover it easier to remember each concept (Menanti, Sinaga & Siregar, 2018).

RESEARCH OBJECTIVE

1. To determine the level of mathematical thinking abilities focusing on problem-solving, mathematical representation, reasoning, mathematical communication and mathematical connection among pre-service mathematics teachers.

2. To identify a significant relationship between problem-solving, mathematical representation, reasoning, mathematical communication and mathematical connection.

3. To determine the significant differences in the level of mathematical thinking among pre-service mathematics teachers based on gender.

METHODOLOGY

The research approach that applied in this research is quantitative design. Quantitative research approach can be defined as the method that involves the collection, utilization and analysis of numerical data by using specific statistical methods (Oberiri Destiny Apuke, 2017). With the purpose of describing, predicting, and regulating a phenomenon, quantitative design has been applied to respond to questions regarding the relationships in variables that may be measured (Haradhan, 2020). There are many research designs that are classified as quantitative research approaches such as true experimental design, quasi- experimental design, survey, correlational and causal-comparative design. The design that is used in this current research is survey design. It is the process used to collect data from a sample of respondents through their responses to a set of questions to collect data and identify the characteristics of a wide sample of interesting individuals immediately (Julie P, 2015). The reason for choosing this research design is due to the larger size of the target population. The population of mathematics pre-service teachers whether taking mathematics as major or minor is greater and it is only suitable to minimize the numbers to relatively small sample sizes in order to draw conclusions for the entire population by using survey design. This can be reduced in cost and more efficiency in collecting data with saving the time by implementing the online survey.

A population is an entire group about which some information is required to be found out by someone while a sample is any part of the fully defined population (Banerjee & Chaudhury, 2010; Rahi, 2017). The population in this study is all of the UPSI that undergoes the course of Implementation of Mathematics Teaching before pursuing their teaching practice either as major or minor. Since this research is only conducted in Universiti Pendidikan Sultan Idris (UPSI) located in Tanjong Malim, Perak, Malaysia there is only a population size of 450 students in this university are identified. This population is chosen because it is suitable related to research objectives that is to determine the level of mathematical thinking among mathematics pre-service teachers in UPSI. The students are also already exposed to the mathematics courses that are listed in the course structure. The cluster random sampling technique was applied by researchers in this study. This is due to the population and the desired sample size for this study is quite large. Thus, cluster random sampling was applied to divide the greater size of population into clusters which are several smaller groups. There are a total of 2 clusters divided in this study based on gender which are male and female. The samples of the study were randomly selected and resulted in 277 mathematics pre-service teachers; 67 of them were males and 210 were females.

The instrument of study is a tool that is used as a measuring tool to obtain data and be an intermediate between the researcher and the respondent of the study in a research topic (Lay & Khoo, 2016; Nor Azlina, 2014; Siti Nabila, 2019). The survey questionnaire was adopted and modified by Saoud Alhunaini, Kamisah and Naser Abdurab (2021). From the previous questionnaires, researchers will modify the items in the questionnaire to adapt with the mathematical thinking questionnaire. This survey questionnaire consists of two sections where Section A is the demographics of students such as gender, academic years and programme. Section B is the mathematical thinking test. In Section B, it comprises five dimensions of mathematical thinking which are problem solving, mathematical representation, reasoning and proof, mathematical communication and mathematical connection. Each dimension was composed of 5 items respectively. The total items in the questionnaire is 28 items.

Table 1: Distribution of Mathematical Thinking Questionnaire (MTQ)

Section	Construct Measurement	Number of Item
A	Student demographics	3
B	Mathematical thinking test	25
TOTAL		28

There are 25 items in Section B that will apply with the 5-point Likert scale that range from Scale 1 to Scale 5. If the respondents agree with each of the sub constructs, they will mark Scale 4 or 5 on the form. On the other hand, respondents can mark Scale 1 or 2 on the form if they did not agree with the sub construct. There is a neutral opinion which is 3 in this questionnaire.

Table 2: Four-point Likert scale of Mathematical Thinking Questionnaire (MTQ)

Scale	Indicator
1	Strongly disagree
2	Disagree
3	Neutral
4	Agree
5	Strongly agree

The data collection process is carried out through an online survey. The students selected through the sampling technique explained above are then identified through the MyGuru system which is an online learning system at UPSI. Students are also contacted via WhatsApp and email. The identified students were then given a survey link to answer the mathematical thinking questionnaire. The invitation to answer the questionnaire was carried out 3 times and took 3 months. Most responses are given a few days after the survey link is shared.

Descriptive analysis is used in this study to obtain the result from the data collection in research based on the first research question in this study. For this research, researchers only calculate and identify the means and standard deviation of collection data by using Statistical Package for the Social Science (SPSS) version 23.0. The purpose of identifying mean values and standard deviations for the study is to explore the level of mathematical thinking among mathematics pre-service teachers. An independent sample t-test is also used to test the differences in level of mathematical thinking between male mathematics pre-service teachers and female mathematics pre-service teachers. Finally, Pearson's correlation (r) measures the strength of the linear relationship between all the variables in this study.

Table 3: Guidelines for interpreting correlation coefficients

Correlation coefficient value (r)	Interpretation
0.90-1.00	Very high positive correlation
0.70-0.90	High positive correlation
0.50-0.70	Moderate positive correlation
0.30-0.50	Low positive correlation
0.00-0.30	Negligible correlation

RESULT & DISCUSSION

Table 4: Mean value and standard deviation for mathematical thinking abilities among pre-service mathematics teachers.

Mathematical Thinking Abilities	Mean	Standard deviation
Problem solving	4.580	0.511
Mathematical Representation	4.445	0.567
Reasoning and Proof	4.502	0.530
Mathematical Communication	4.490	0.543
Mathematical Connection	4.512	0.552
OVERALL	4.506	0.541

Table 5: Correlation coefficient value of Problem solving toward mathematical representation, reasoning, mathematical communication and mathematical connection among pre-service mathematics teachers.

	Problem solving	Mathematical Representation	Reasoning and Proof	Mathematical Communication	Mathematical Connection

Problem solving	Pearson Correlation Sig. (2-tailed) N	1.000 277	0.658 277	0.679 277	0.704 277	0.679 277
Mathematical Representation	Pearson Correlation Sig. (2-tailed) N	0.658 277	1.000 277	0.740 277	0.745 277	0.749 277
Reasoning and Proof	Pearson Correlation Sig. (2-tailed) N	0.676 277	0.740 277	1.000 271	0.764 277	0.729 277
Mathematical Communication	Pearson Correlation Sig. (2-tailed) N	0.704 277	0.745 277	0.764 277	1.000 271	0.717 277
Mathematical Connection	Pearson Correlation Sig. (2-tailed) N	0.679 277	0.749 277	0.729 277	0.717 277	1.000 271

Table 6: Difference in the level of mathematical thinking among pre-service mathematics teachers based on gender.

	t	**df**	**Sig.(2-tailed)**	**Mean Difference**
Male	-0.524	273	0.601	-0.03576
Female	-0.559	115.858	0.578	-0.03576

Based on the collected data, there a total of 277 pre-service teachers that taking mathematics either as their major or minor programme had been answered the questionnaire. The questionnaire was majority answer by female pre-service teachers which in a number of 210 while the remaining 67 respondents are male pre-service teachers. Table 4 show that the mean score and standard deviation for all items of all dimensions. Among these 5 mathematics skills, pre-service teachers had achieved a highest means value on problem solving (4.580) and followed by mathematical connection (4.512). As an alternative, the mean value of pre-service teachers in reasoning and mathematical communication are 4.502 and 4.490 respectively. However, research found out that pre-service teachers not well in mathematical representation compare to others mathematics skills which only achieve a mean value of 4.445. Based on the means value of these 5 mathematics skills, the mean value of mathematics thinking ability among pre-service teachers can be calculated and resulted in an average of 4.506. Since the mean values of all of the dimensions of mathematical thinking abilities are between 4.445 until 4.580, this indicate that pre-service teachers show a higher positive mathematical thinking. Besides, table 4 indicate that the correlation coefficient between all items and mathematics skills are more than 0.30 which are between 0.658 until 0.764. This revealed that there a significant relationship between problem-solving toward mathematical representation, reasoning, mathematical communication and mathematical connection.

Hence, it can be said that the results of this study are correspond with the findings of previous research studies. The effect of each mathematics skills toward the mathematical thinking skills able to well demonstrate the relationship between each other's. For instance, Nurzalena et al. (2019) had argued that the development of

mathematical thinking greatly promotes problem solving and reasoning skills as interrelated elements. This is because students can improve advanced mathematical thinking skills as they explore over problems (Susanti & Hartono, 2019). Therefore, it can be stated that there is an important relationship between students' mathematical thinking and their ability in solving problems that are related to mathematics (Yavuz & Cansz, 2019). This is because the problem solving has resulted in a significant influence on students' mathematical thinking due to it requiring mathematical critical thinking skills like comprehension, structuring, implementation, and evaluation of the solutions (Saoud, Kamisah& Naser, 2021). Moreover, NCII (2016) and Saoud, Kamisah and Naser (2021) revealed that mathematical representation assists students strengthen their knowledge of mathematical terms and problems by recording, structuring, and presenting mathematical ideas (NCII,2016; Saoud, Kamisah& Naser, 2021). In order words, students can apply various forms of mathematical presentation in order to enhance their understanding towards mathematics theory. In fact, the most significant talent of students to grow is proficiency in converting between various representations, and this is going to have a big impact on pupils studying mathematics.

Besides, the studies of Selden (2003) and Adamura and Susanti (2018) claimed that mathematical reasoning is particularly important for problem solving since it is a key component of the problem-solving process that makes use of students' critical thinking skills and reasoning skills in identifying alternate solutions to problems. As a result, it may be declared that taking advantage of problem-solving techniques, teachers can assess students' mathematical reasoning abilities (Susi Herawati & Puspa Amelia, 2020). For mathematical communication skills, Kamid et al. (2020) declared that it can be contributed to helping students comprehend more deeply toward mathematical concepts, problem solving, and made a correction toward misconceptions about mathematical ideas. This argument had been supported by Ismail, Siagian and Zul (2018) which stated that the ability of students to think critically and analyze the different connections between mathematical concepts is strengthened when mathematical ideas are communicated from a variety of perspectives. They also mentioned that students may structure and solidify their mathematical ideas through communication for the purpose to construct mathematical knowledge, strengthen problem solving and reasoning skills, promote confidence, and further develop interpersonal competencies. Last but not least, Ormand (2016) found out that mathematical connection skill may motivate students to establish a shared understanding of mathematics that increases simultaneously with their involvement in solving mathematical problems.

From the table 6, The p-value for male and female pre-service teachers is 0.578. p-value is greater than 0.05 which means that the null hypothesis is fail to rejected. So, it can be concluded that there is no significant difference in the mean of mathematical thinking level between male mathematics pre-service teachers and female mathematics pre-service teachers. This result of this study are contrary to the study by Hayal and Tolga in 2017 which shows that gender has a significant effect on mathematical reasoning skills. On the contrary, the scores of male pre-service teachers were slightly greater than female pre-service teachers on the mathematical reasoning skill test. However, the differences based on genders will decline among the pre-service teachers as its sample size grows.

CONCLUSION

In conclusion, it is essential to enhance the thinking ability of pre-service teachers in order to foster a more effective and impactful mathematics education system. A strong foundation in mathematical thinking is crucial for teachers as they play a significant role in shaping the mathematical skills and mind set of their students. By improving the thinking ability of these pre-service teachers, we can expect several positive outcomes like better classroom Instruction, increased student engagement, improved assessment and feedback, enhanced professional development, positive impact on student achievement.

To achieve these goals, teacher education programs and ongoing professional development should prioritize the development of mathematical thinking skills, problem-solving abilities, and a deep understanding of mathematical concepts. Additionally, mentorship and collaboration among experienced mathematics educators can provide valuable guidance and support for pre-service teachers to continuously enhance their teaching skills.

REFERENCE

[1] Abdullah, A., Fadil, S., Abdurahman, S., & Mohd, L. (2019). Emerging patterns and problems of higher order thinking skills (HOTS) mathematical problem- solving in the form-three assessment (PT3). *South African Journal of Education*, *29*(2), 1-18. https://doi.org/10.15700/saje.v39n2a1552.

[2] Acar-Erdol, T., & Yıldızl, H. (2018). Classroom assessment practices of teachers in Turkey. *International Journal of Instruction 11*(3), 587-602. Retrieved from https://doi.org/10.12973/iji.2018.11340a.

[3] Adamura, F., & Susanti, V. D. (2018). Penalaran Matematis Mahasiswa dalam Memecahkan Masalah Analisis Real Berdasarkan Kemampuan Berpikir Intuitif. Journal of Mathematics and Mathematics Education, 8(2), 156–172. https://doi.org/10.20961/jmme.v8i2.25852.

[4] Aljaberi, N., & Gheith, E. (2018). Ins-ervice mathematics teachers' beliefs about teaching, learning and nature of mathematics and their mathematics teaching practices. *Journal of Education and Learning,7*(5), 156. Retrieved from https://doi.org/10.5539/jel.v7n5p156.

[5] Aloisius Loka Son. (2022). The Students' Abilities on Mathematical Connections: A Comparative Study Based on Learning Models Intervention. *Mathematics Teaching Research Journal Summer 2022, 14*(2), 72-87. Retrieved from https://files.eric.ed.gov/fulltext/EJ1350655.pdf.

[6] Avni Yildiz, Serdal Baltaci & Büşra Kartal. (2020). Examining the Relationship between Pre-Service Mathematics Teachers' Mathematical Thinking Level and Attitude towards Mathematics Courses. *Acta Didactica Napocensia*, *13*(2), 256-270. Retrieved from https://eric.ed.gov/?id=EJ1280449.

[7] Banerjee, A., & Chaudhury, S. (2010). Statistics without tears: Populations and samples. *Industrial Psychiatry Journal*, 19(1), 60. https://dx.doi.org/10.4103/0972-6748.77642.

[8] Bergwall, A. (2021). *Proof-related reasoning in upper secondary mathematics textbooks: Characteristics, comparisons, and conceptualizations.* (Mälardalen University Press Dissertations). Retrieved from https://www.proquest.com/dissertations-theses/proof-related-reasoning-upper-secondary/docview/2706221486/se-2.

[9] Bjuland, R. (2007). Adult Students' Reasoning in Geometry: Teaching Mathematics through Collaborative Problem Solving in Teacher Education. *The mathematics enthusiast*, *4*(1), 1-30.

[10] Boesen, J., Lithner, J. & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educ Stud Math*, *75*, 89- 105. https://doi.org/10.1007/s10649-010-9242-9.

[11] Breen & O'Shea. (2010). Mathematical Thinking and Task Design. *Irish Math. Soc. Bulletin*, *66*, 39–49. Retrieved from https://www.maths.tcd.ie/pub/ims/bull66/ME6601.pdf.

[12] Celiki, H., & Ozdemir, F. (2020). Mathematical thinking as a predictor of critical thinking dispositions of pre-service mathematics teachers. *International Journal of Progressive Education*, *16*(4), 81-98. Retrieved from https://doi.org/10.29329/ijpe.2020.268.6.

[13] Coxford, A.F. (1995). "The Case for Connections", in Connecting Mathematics Across the Curriculum. Editor: House, P.A. and Coxford, A.F. Reston, Virginia: NCTM.

[14] Diah Nur Hafifah & Haninda Bharata. (2018). *The Importance of Mathematical Communication Skills for Students in Mathematics Learning*. Proceeding of the 3rd SHIELD International Conference of 2018. Retrieved from https://3rdshieldproceeding.uprci.org.

[15] Fiona, F., Cormac B., Mark, P. & Michael, C. (2020). Measuring the mathematical problem solving and procedural skills of students in an Irish higher education institution – A pilot study. *European Journal of Science and Mathematics Education.* 8(2), 2020, 92-106. Retrieved from https://files.eric.ed.gov/fulltext/EJ1252763.pdf.

[16] Fuat, Nusantara, T., Irawan, E. B., & Irawati, S. (2019). Students' mathematical conviction in mathematics proof. *IOP Conference Series.Earth and Environmental Science, 243*(1). doi:https://doi.org/10.1088/1755-1315/243/1/012133.

[17] García-García, J., & Dolores-Flores, C. (2018). Intramathematical connections made by high school students in performing calculus tasks. *International Journal of Mathematical Education in Science and Technology, 49*(2), 227-252. https://doi.org/10.1080 /0020739X.2017.1355994.

[18] Goldin, G. (2014). Mathematical Representations. Encyclopedia of Mathematics Education. 409-413. DOI:10.1007/978-94-007-4978-8_103.

[19] Goos, M., Kaya, S. (2020). Understanding and promoting students' mathematical thinking: a review of research published in *ESM*. *Educ Stud Math* 103, 7–25. Retrieved from https://doi.org/10.1007/s10649-019-09921-7.

[20] Hani Juita Sari. (2020). Analisis Kemampuan Representasi Matematis Siswa Dalam Memecahkan Masalah Geometri. J*urnal JPMI (jurnal Pendidikan Matematika Indonesia), 5*(2), Retrieved from http://etheses.uinmataram.ac.id/212/1/HaniI%20Juita%20Sari%20160103106 %20%282%29.pdf.

[21] Haradhan, M. (2021). Quantitative Research: A Successful Investigation in Natural and Social Sciences. *Journal of Economic Development, Environment and People. 9*. Retrieved from https://www.researchgate.net/publication/348237026_Quantitative_Research_A_Successful_Investigation_in_Natural_and_Social_Sciences .

[22] Hayal & Tolga. (2017). An Analysis of the Reasoning Skills of Pre-Service Teachers in the Context of Mathematical Thinking. *European Journal of Education Studies*, *3*(5), 225-254. doi: 10.5281/zenodo.495700.

[23] Indah P.F., Minarni A. & Surya E. (2019). Developing Learning Devices Based on GeoGebra Assisted Discovery Learning with SAVI Approach to Improve Motivation and Mathematical Communication of Senior High School Students MTs Aisyiyah Medan. *American Journal of Educational Research, 7*(12), 893-900. DOI:10.12691/education-7-12-1 .

[24] Ismail, P., Siagian, P., Zul Amr. (2018). The Differentiation of Improved Communication Mathematic and Disposition Skill Through Problem Based Learning and Realistic Mathematic Education. *Advances in Social Science, Education and Humanities Research, 20*. Retrieved from http://creativecommons.org/licenses/by-nc/4.0/.

[25] Ismel et al. (2020). Problem Solving Skill: What is the Difference between Practitioners and Experts? *Advances in Social Science, Education and Humanities Research, 513.* Retrieved from http://creativecommons.org/licenses/by-nc/4.0/.

[26] Jahring. (2020). Kemampuan Koneksi Matematis Pada Model Pembelajaran Connecting, Organizing, Reflecting, Extending Dan Numbered Head Together. *Jurnal AKSIOMA; Jurnal Program Studi Pendidikan Matematika, 9* (1), 182-189. Retrieved from https://www.researchgate.net/publication/340348534_KEMAMPUAN_KONEKSI_MATEMATIS_PADA_MODEL_PEMBELAJARAN_CORE_DAN_NHT.

[27] Jeinne Mumu & Benidiktus Tanujaya. (2019). Measure Reasoning Skill of Mathematics Students. *International Journal of Higher Education. 8*(6). doi:10.5430/ijhe.v8n6p85 .

[28] Julie Ponto. (2015). Understanding and Evaluating Survey Research. *Journal of the Advanced Practitioner in Oncology, 6* (2), 168-171. Retrieved from https://www.researchgate.net/publication/286445115_Understanding_and_Evaluating_Survey_Research.

[29] Kamid, K., Rusdi, M., Fitaloka, O., Basuki, F.R., Khairul Anwar. (2020). Mathematical communication skills based on cognitive styles and gender. *International Journal of Evaluation and Research in Education (IJERE)*. *9*(4), 847-856. DOI: 10.11591/ijere.v9i4.20497.

[30] Kenedi, A.K., Helsa, Y.,Ariani, Y., Zainil,M. & Hendri, S. (2019). Mathematical Connection of Elementary School Students to Solve Mathematical Problems. *Journal on Mathematics Education, 10*(1), 69–79.https://doi.org/10.22342/jme.10.1.5416.69-80 .

[31] Kheong, S. C. (2011). *Masalah pembelajaran matematik dalm tajuk kebarangkalian di kalangan pelajar tingkatan 5* [Tesis Sarjana].

[32] Lay, Y. F., Khoo, C. H., & Ley, C. M. (2016). Pengenalan Kepada Analisis Data Dengan IBM SPSS STATISTICS 19 Dalam Penyelidikan Sains Sosial. Kota Kinabalu: Penerbit Universiti Malaysia Sabah.

[33] Lestari, K.E. & Yudhanegara M.R. (2015). *Penelitian pendidikan matematika* (Bandung: PT Refika Aditama).

[34] Madia, R., Holimombo, N. (2022). Analysis of Mathematical Connection Ability When Solved Problem in 7th grade SMPN 9 Buru. *Matematika dan Pembelajaran, 10*(1), 38-51. DOI: http://dx.doi.org/10.33477/mp.v10i1.2689 .

[35] Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 9*(1),1-21. https://doi.org/10.46328/ijemst.1111 .

[36] Menanti, Hotmaria & Sinaga, Bornok & Siregar, Hasratuddin. (2018). Improve Mathematical Connections Skills with Realistic Mathematics Education Based Learning. *Advances in Social Science, Education and Humanities Research, 200*. doi: 10.2991/aisteel-18.2018.7.

[37] Minarni, A., & Napitupulu, E. E. (2017). Developing Instruction Materials Based on Joyful PBL to Improve Students Mathematical Representation Ability. *International Education Studies, 10*(9), 23. https://doi.org/10.5539/ies.v10n9p23.

[38] Murat Tezer & Meryem Cumhur (2017). *Mathematics through the 5E Instructional Model and Mathematical Modelling: The Geometrical Objects. EURASIA Journal of Mathematics Science and Technology Education, 13*(8), 4789-4804. DOI: 10.12973/eurasia.2017.00965a.

[39] NCII. (2016). *Principles for designing intervention in mathematics.* Office of Special Education, U.S. Department of Education.

[40] NCTM. (2000). *Principles and standards for school mathematics*. The National Council of Teachers of Mathematics, Inc.

[41] Nor Azlina, A. (2014). Kefahaman dan kesilapan pelajar tingkatan empat mengenai konsep-konsep asas kebarangkalian (Tesis sarjana, Universiti Pendidikan Sultan Idris, Tanjong Malim, Malaysia).

[42] Nurzalena, A., Susanti, E., Hapizah, H., Meryansumayeka, M., & Miswanto, A. (2019). Design of problem-solving questions for measuring mathematical thinking type mathematization. *Journal of Physics: Conference Series, 1318*(1). https://doi.org/10.1088/1742-6596/1318/1/012106.

[43] Ormond, C. (2016). Scaffolding the mathematical "connections": A new approach to preparing teachers for the teaching of lower secondary algebra. *Australian Journal of Teacher Education, 41*(6). https://doi.org/10.14221/ajte.2016v41n6.8.

[44] Palraj, S., DeWitt, D., & Alias, N. (2017). Teachers beliefs in problem solving in Rural Malaysian secondary schools. *Malaysian Online Journal of Educational Technology, 5*(4), 45-57.

[45] Polat, S. (2020). Multidimensional analysis of the teaching process of the critical thinking skills. *Research in Social Sciences and Technology (RESSAT), 5*(2), 134-157. doi.org/10.46303/ressat.05.02.8.

[46] Rahi, S, (2017). Research design and methods: A systematic review of resesrch paradigms, sampling issues and instruments development. I*nternational Journal of Economics& Management Sciences, 6*(2), 1-5.

[47] Reyes-Cedeno, C. C., Rivas-Cun, H. I., Espinoza Cevallos, C. E., & Rojas-Garcia, C. R. (2019). Assessment of the practices for early mathematics thinking in preschools of Pasaje city, Ecuador. *European Journal of Educational Research, 8*(4), 1063-1070. https://doi.org/10.12973/eu-jer.8.4.1063 .

[48] Saoud Alhunaini, Kamisah Osman & Naser Abdurab. (2021). The Development and Validation of Mathematical Thinking Beliefs (MTB) Instrument. *EURASIA Journal of Mathematics, Science and Technology Education,17*(11). Retrieved from https://doi.org/10.29333/ejmste/11213.

[49] Siemon, D., Day, L., Stephens, M., Horne, M., Callingham, R., Watson, J., Seah, R., & Siemon, D. (2017). *Reframing mathematical futures: Using learning progressions to support mathematical thinking in the middle years.* Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia, In A. Downton, S. Livy, & J. Hall (Eds.).

[50] Siregar, Nenta & Surya, Edy. (2017). Analysis of Students' Junior High School Mathematical Connection Ability. *International Journal of Sciences: Basic and Applied Research (IJSBAR), 33,* 309-320. Retrieved from https://core.ac.uk/reader/249335820.

[51] Siti Nabila. (2019). *Pembangunan dan penilaian modul pengajaran STEM dalam bidang Statistik dan kebarangkalian dalam KSSM matematik tingkatan dua E* (tesis, Universiti Pendidikan Sultan Idris, Tanjong Malim, Malaysia).

[52] Solikhah, I. & Budiharso, T. (2019). Investigating the Learning Outcomes of an INQF Based English Language Teaching Curriculum in Indonesia. *JSSER: Journal of Social Studies Education Research, 10*(4), 153- 175. www.jsser.org.

[53] Stylianides, G. J., & Stylianides, A. J. (2017). Research-based interventions in the area of proof: the past, the present, and the future. *Educational Studies in Mathematics, 96*(2), 119–127. https://doi.org/10.1007/s10649-017-9782-3 .

[54] Susanti, E., & Hartono. (2019). An analysis mathematical problem solving and mathematical critical thinking skills of junior high school students. *Journal of Physics: Conference Series, 1320*(1). https://doi.org/10.1088/1742-6596/1320/1/012071.

[55] Susi, H. & Puspa,A. (2020). Analyzing of Student's Mathematical Reasoning Ability in Solving Mathematical Problem Based on Higher Order Thinking Skill (HOTS). *Advances in Social Science, Education and Humanities Research. 550*. Retrieved from http://creativecommons.org/licenses/by-nc/4.0/.

[56] Utomo, D.P., & Syarifah, D.L. (2021). Examining mathematical representation to solve problems in trends in mathematics and science study: Voices from Indonesian secondary school students. I*nternational Journal of Education in Mathematics, Science, and Technology (IJEMST), 9*(3), 540-556. https://doi.org/10.46328/ijemst.1685.

[57] Uyangör, S. M. (2019). Investigation of the mathematical thinking processes of students in mathematics education supported with graph theory. *Universal Journal of Educational Research, 7*(1), 1-9. https://doi.org/10.13189/ujer.2019.070101.

[58] Xenofontos, C., & Kyriakou, A. (2017). Prospective elementary teachers' beliefs about collaborative problem solving and dialogue in mathematics. *Mathematics Teacher Education and Development, 19*(2), 142-158.

[59] Yadav, S. (2018). Correlation analysis in biological studies. *Journal of the Practice of Cardiovascular Sciences, 4*(2), 116. https://doi.org/10.4103/jpcs.jpcs_31_18

[60] Yavuz Mumcu, H., & Cansız Aktaş, M. (2019). The effect of the use of mathematics course in terms of prospective teachers' attitudes toward mathematics and their beliefs on the nature, teaching, and learning of mathematics. *Pegem Eğitim ve Öğretim Dergisi, 9*(3), 697-728. https://doi.org/10.14527/pegegog.2019.023 .

[61] Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 2*, 179-217.

[62] Zulfa, A., Kartono, K. & Cahyono, A.N. (2021). Mathematical Communication Skills in Terms of Self Efficacy Through Guided Discovery Learning Assisted Immediate Feedback. *Unnes Journal of Mathematics Education Research (UJMER), 10*(1), 75-82. Retrieved from https://journal.unnes.ac.id/sju/index.php/ujmer/article/view/34665/14375 .

[63] Zulkarnain, I., Kusumawati, E., & Mawaddah, S. (2021). Mathematical communication skills of students in mathematics learning using discovery learning model. *Journal of Physics: Conference Series*, 1760(1). https://doi.org/10.1088/1742-6596/1760/1/012045.