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Three Persons Satisfactory Roommates Problem with Ties, Ties and Incomplete List

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Abstract: - Solving the Satisfactory Roommates Problem (SFRP) involves determining a satisfactory match between any two roommates. Every member of the set of even cardinality $2n$ ranks the other $2n - 1$ members in order of preference in the SFRP. The division of the set into $2n/2$ roommate pairs according to each person's satisfactory level is known as satisfactory matching. The satisfactory roommates' problem (SFRP) is generalized to a new level where preference lists with ties, ties, and incomplete lists are present. There are 3n individuals in this Three person Satisfactory Roommates Problem (TPSRP), and each individual has preference lists for their two companions. Less than $3n - 1$ with ties is preferred by certain people. We use the TPSR algorithm to identify suitable matching, which is based on preference value and helps determine satisfactory matching of roommates problem with ties (TPSRT), ties and incomplete list (TPSRTI). A new, complex algorithm for finding perfect triples in rooms is presented in this study.

Key words: *Satisfactory value matrix, preference value, three person rooms, ties, ties, and incomplete lists, modified satisfactory value matrix.*

I. INTRODUCTION

Stable roommates' problems are an extension of stable marriage problems. A non-bipartite graph $G = (V, E)$ is an example of the stable roommates problem with ties and incomplete lists (SRTI). For each vertex in the adjacency list, there is a list of ties that is linearly ordered and consists of subsets of vertices that are equally excellent for that particular vertex. Ties can have one vertex and are disjoint.

The well-known Stable Roommates problem was explored by Irving and Manlove [1], who allowed participants to indicate ties in their preference lists. It was further demonstrated that the weakly stable matching could vary in size and that, for a given Stable Roommates instance with ties and incomplete lists, finding a maximum cardinality weakly stable matching is NP-hard, although approximable within a factor of 2. In the presence of ties and incomplete lists, Adil et al. [2] examined the parameterized complexity of NP-hard optimization variants of stable roommates and stable matching. As a result, Stable Matching is fixed-parameter tractable (FPT) concerning solution size, while Stable Roommates is FPT concerning a structural parameter. Subsequently, the parameterized complexity analysis of the NP-hard Stable Roommates with Ties and Incomplete Lists problem was established by Bredereck et al. [3], which reinforced previous findings in terms of parameterized hardness and fixed-parameter tractability.

While computing a maximum-sized relaxed stable matching proves to be NP-hard, Nasre et al. [4] demonstrated that a crucial matching that is relaxed stable always exists in the present situation. The significant contribution for the stable marriage problem with two-sided ties and critical vertices is a $3/2$ approximation to the maximum-sized critical relaxed stable matching. New integer linear programming models and preprocessing procedures for Stable Marriage Problem with Ties and Incomplete lists (SMTI) with ties on both sides were introduced by Delorme et al. [5]. The stable marriage problem with ties and incomplete lists (SMTI problem) was first explicitly solved by Gent and Prosser [6] using a constraint programming encoding of the problem.

The set of all highly stable matching can be represented by a partial order with O(m) members, as demonstrated by Kunysz [7]. Also provided an O(nm) strategy for creating this kind of representation. The algorithms are predicated on a straightforward reduction to the problem's bipartite form. The efficient algorithms were introduced

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by Fleiner et al. [8] for a wide range of problems involving the identification of different kinds of stable matching. These comprise enumerating all super-stable matching, locating egalitarian and minimum regret super-stable matching, and locating all super-stable pairs. In order to solve the stable marriage problem with ties and incomplete lists, Viet et al. [9] created the well-known Gale-Shapley method. Also additionally developed a heuristic repair function to enhance the stable matching that was obtained in terms of maximum size.

By extending Stable Roommate to three-person rooms, Iwama et al. [10] developed a system known as 3D-SR (3-Dimensional Stable Roommates). The three-person satisfactory roommate problem is made up of three distinct individuals (3D-SR) that each have a list of preferences over the other three persons $(3n-1)$. Every individual has a completely ordered preference list that includes every other person ranked from 1 to $3n$ based on his preferences. There are now n disconnected triples in a matching. For a given instance, 3D-SR inquires as to whether a stable matching is there. The TPSR technique is utilized to identify an ideal match for the roommate's problem in incomplete lists and incomplete with ties, which is an extension of the three-person roommate problem.

Ramachandran et al. [11] developed preference lists containing ties, ties, and incomplete lists using the SMAR algorithm as a generalization of the satisfactory roommates problem (SFRP). This will aid in determining the satisfactory matching of roommates problem with ties (SFRPT), ties, and incomplete list (SFRPTI) based on preference value, using the assignment method. Logapriya and Ramachandran [12] proposed three persons satisfactory roommates problem using Hungarian algorithm.

The primary objective is to identify a three person satisfactory matching of roommates' problem. This study uses a novel complicated TPSR method, which is based on preference value and assists in determining Three person satisfactory matching of roommates problem with ties (TPSRT), ties, and incomplete list (TPSRTI), to identify suitable matching for finding perfect triples in rooms.

II. LITERATURE SURVEY

Satisfactory Roommates problem with ties lists (SFRPT) is a natural generalization of Satisfactory Roommates problem (SFRP) and is closely connected to Stable roommates problem with ties (SRT). The circumstance comes up when nobody feels compelled to rank every member of his group in a rigorous order. Preference lists may contain ties since some of those may be neutral towards specific group members. SFRPT stands for the SFRP variation where ties are allowed on preference lists. (Henceforth, we will assume that a tie is at least two lengths.) An instance of SFRPT becomes an instance of SFRP when the ties are unilaterally broken.A traditional Satisfactory Roommate's Problem (SFRP) where each group member will receive their full preference list. The term SFRPI refers to the Satisfactory Roommates Problem, which is characterised by an incomplete preference list. In other words, some members of the group of $2n$ members are preferred above members of $2n-1$. The SMAR procedure, which is detailed in the SFRPI, can be used to produce excellent matching for this kind of SFRPI problem. Participant p is acceptable to participant q if it appears on q's preference list, and unacceptable otherwise. In addition, if the person prefers p and q as same place in the list then the problem is said to be SFRPTI.

III. METHODOLOGY

A. Three Person Satisfactory Matching (TPSM)

In this paper we propose an algorithm to find triple roommates from the group of 3n person based on their preference lists and lists may involve ties, and incomplete list with ties. This algorithm provides 3n set of triple roommates based on the individual satisfactory level.

Preference value (TPSRP) is defined as the value assigned to the members in the preference list according to the order of preference with respect to the persons by considering the first person as $\frac{3n-2}{3n-2}$, the second person as $\frac{3n-2}{3n-2}$, the third person as $\frac{3n-3}{3n-2}$, the fourth person as $\frac{3n-4}{3n-2}$ and so on. If the person i prefer j & k, in same place in the list then j^{th} & k^{th} place in i^{th} list must be considered as the same value. In addition If the person i did not prefer j, then j^{th} place in i^{th} list must be considered as infinity.

P: $[P_{ij}]$ Defines the preference value matrix.

$$P_{ij} = \begin{cases} \frac{3n-2}{3n-2} & \text{if } j \text{ is the first or second preference of } i \\ \frac{3n-k}{3n-2} & \text{if } j \text{ is } k^{\text{th}} \text{ preference of } i \text{ and } k = 3,4,5,6 \ldots\ldots \text{upto } (3\text{n}-1); \\ & \text{also if there are no ties in the preference of } i \\ \frac{3n-k}{3n-2} & \text{If } k^{\text{th}} \text{ and } k+1^{\text{th}} \text{ preferences of } i \text{ are ties} \\ - & \text{if } i = j \end{cases}$$

The modified preference value matrix is defined as follows:

$$m_{(ij),k} = \begin{cases} - & \text{if k} = \text{i and k} = \text{j} \\ P_{ik} + P_{ji} & \text{k} = 1,2, \ldots .3\text{n} \neq \text{i, j} \end{cases}$$

B. Algorithm (TPSMA)

1. Get the preference lists from each person.

2. Form a preference value matrix based on their preference lists.

3. Considering the preference value matrix as a Maximization assignment problem.

4. Construct a Minimized Preference Value Matrix and applying Hungarian algorithm for that matrix.

5. The Resultant pairs must be the optimum pairs like *(i,j), (k,l), (m,n)* and so on obtained.

6. Construct the Modified Preference Value matrix by considering the pairs *(i,j), (k,l), (m,n)* … as rows and 1,2,….3n members as column. By using MPVM definition which is given above.

7. Considering the Modified preference value matrix as a minimized assignment problem and apply Hungarian Algorithm for getting the optimum triples.

8. List out all the triples and let it be *(i,j,k), (l,m,n), (i,l,n)*…..

9. From the obtained triples, choose one by one and find the satisfactory value of each member of a group.

10. If *(i,j,k)* be the first triples, find the preference value of *i* with respect to *j* and *k*, then adding the preference values. Now we get the overall preference value and multiply the value by 50. It gives a satisfactory value of *i* with respect to *j* and *k.* similarly find the satisfactory value of *j* w.r.to *i & k*, also for *k* w.r.to *i & j*.

11. After getting these three satisfactory values, find overall satisfaction for the triple *(i,j,k).* In the same manner repeat the process for all the remaining triples.

12. Now choose mutually exclusive and exhaustive triples which achieves the maximum level of satisfaction that is the optimum triples.

IV. NUMERICAL ILLUSTRATION

Example 1. Consider the problem instance of TPSRP based on order of preference and calculate the following

1	3 5 6 (2 4)
2	1 4 5 6 3
3	4 2 5 (1 6)
4	5 3 2 6 1
5	2 6 3 1 4
6	5 4 3 (2 1)

Solution:

The given preference list can be constructed as a preference value matrix by considering first person as $\frac{3n-2}{3n-2}$, second person as $\frac{3n-2}{3n-2}$, third person as $\frac{3n-3}{3n-2}$, fourth person as $\frac{3n-4}{3n-2}$, fifth person as $\frac{3n-5}{3n-2}$. The preference value of the persons 3,5,6,2,4 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{2}{4}$ with respect to person 1. The preference value of persons 1,4,5,6,3 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{1}{4}$with respect to person 2. The preference value for persons 4,2,5,1,6 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{2}{4}$ with respect to person 3. The preference value for persons 5,3,2,6,1 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{1}{4}$with respect to person 4. The preference value for persons 2,6,3,1,4 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}$, $\frac{2}{4}$ and $\frac{1}{4}$ with respect to person 5. The preference value for members 5,4,3,2,1 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{2}{4}$ with respect to person 6. The preference values are presented in the form of matrix which is given below.

The preference value matrix

$$PVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ - & \frac{2}{4} & \frac{4}{4} & \frac{2}{4} & \frac{4}{4} & \frac{3}{4} \\ \frac{4}{4} & - & \frac{1}{4} & \frac{4}{4} & \frac{3}{4} & \frac{2}{4} \\ \frac{1}{4} & \frac{4}{4} & - & \frac{4}{4} & \frac{3}{4} & \frac{2}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{4}{4} & - & \frac{4}{4} & \frac{2}{4} \\ \frac{2}{4} & \frac{4}{4} & \frac{3}{4} & \frac{1}{4} & - & \frac{4}{4} \\ \frac{2}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{4}{4} & - \end{array}$$

From the preference value matrix to construct the minimized preference value matrix by subtracting all the elements in the matrix from the highest element in the matrix. Then the Minimized preference value matrix given below

The minimized preference value matrix

$$mPVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ - & \frac{2}{4} & 0 & \frac{2}{4} & 0 & \frac{1}{4} \\ 0 & - & \frac{3}{4} & 0 & \frac{1}{4} & \frac{2}{4} \\ \frac{3}{4} & 0 & - & 0 & \frac{1}{4} & \frac{2}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 & - & 0 & \frac{2}{4} \\ \frac{2}{4} & 0 & \frac{1}{4} & \frac{3}{4} & - & 0 \\ \frac{2}{4} & \frac{2}{4} & \frac{1}{4} & 0 & 0 & - \end{array}$$

Considering the mPVM as the assignment problem and applying Hungarian algorithm for finding the optimum pairs.

(1,3), (2,1), (3,2), (4,5), (5,6), (6,4) be the optimum pairs.

To find the optimum triples, from the mPVM choose (1,3) pair, the first row third column corresponding element 0 and add with entire first row and assign that particular element place with -. Likewise, choose the next pair (2,1), the second row first column element 0 and add with entire second row and assign that particular element place with -. Similarly, repeat the process for all optimum pairs. Then the resultant matrix will be a Modified Minimum Preference Value Matrix (MmPVM).

The Modified minimized preference value matrix

$$MmPVM = \begin{array}{c} \\ (1,3) \\ (2,1) \\ (3,2) \\ (4,5) \\ (5,6) \\ (6,4) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & \frac{2}{4} & - & \frac{2}{4} & 0 & \frac{1}{4} \\ - & - & \frac{3}{4} & 0 & \frac{1}{4} & \frac{2}{4} \\ \frac{3}{4} & - & - & 0 & \frac{1}{4} & \frac{2}{4} \\ \frac{3}{4} & \frac{1}{4} & 0 & - & - & \frac{2}{4} \\ \frac{2}{4} & 0 & \frac{1}{4} & \frac{3}{4} & - & - \\ \frac{2}{4} & \frac{2}{4} & \frac{1}{4} & - & 0 & - \end{pmatrix} \end{array}$$

Considering this above matrix as the assignment problem and apply Hungarian algorithm then get the optimum triples.

The triples are (1,3,5), (2,1,6), (3,2,4) (4,5,3), (5,6,2), (6,4,1).

Here the mutually exclusive and exhaustive triples are (2,1,6) and (4,5,3).

An individual preference value and overall satisfaction percentages are given below

(2,1,6)	2 → 1 & 6 = 75%.	**62.5%**
	1→ 2 & 6 = 62.5%.	
	6 → 1 & 2 = 50%.	
(4,5,3)	4 → 5 & 3 = 100%.	**79%**
	5 → 4 & 3 = 50%.	
	3 → 4 & 5 = 87.5%.	

Example 2. Consider the problem instance of TPSRP based on order of preference and calculate the following

1 4 5 3 6

2 6 3 4 5

3 2 6 1 (5 4)

4	1 5 3 2 6
5	4 1 6 (2 3)
6	3 2 5 1 4

Solution:

The given preference list can be constructed as a preference value matrix by considering first person as $\frac{3n-2}{3n-2}$, second person as $\frac{3n-2}{3n-2}$, third person as $\frac{3n-3}{3n-2}$, fourth person as $\frac{3n-4}{3n-2}$, fifth person as $\frac{3n-5}{3n-2}$. The preference value of the persons 4,5,3,6, are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ with respect to person 1. The preference value of persons 6,3,4,5 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ with respect to person 2. The preference value for persons 2,6,1,5,4, are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{2}{4}$ with respect to person 3. The preference value for persons 1,5,3,2,6, are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{1}{4}$ with respect to person 4. The preference value for persons 4,1,6,2,3 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{2}{4}$ with respect to person 5. The preference value for members 3,2,5,1,4 are $\frac{4}{4}, \frac{4}{4}, \frac{3}{4}, \frac{2}{4}$ and $\frac{1}{4}$ with respect to person 6. The preference values are presented in the form of matrix which is given below

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$$PVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & - & \frac{3}{4} & \frac{4}{4} & \frac{4}{4} & \frac{2}{4} \\ - & - & \frac{4}{4} & \frac{3}{4} & \frac{2}{4} & \frac{4}{4} \\ \frac{3}{4} & \frac{4}{4} & - & \frac{2}{4} & \frac{2}{4} & \frac{4}{4} \\ \frac{4}{4} & \frac{2}{4} & \frac{3}{4} & - & \frac{4}{4} & \frac{1}{4} \\ \frac{4}{4} & \frac{2}{4} & \frac{2}{4} & \frac{4}{4} & - & \frac{3}{4} \\ \frac{2}{4} & \frac{4}{4} & \frac{4}{4} & \frac{1}{4} & \frac{3}{4} & - \end{pmatrix} \end{array}$$

From the preference value matrix to construct the minimized preference value matrix by subtracting all the elements in the matrix from the highest element in the matrix. Then the Minimized preference value matrix given below

The minimized preference value matrix

$$mPVM = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & - & \frac{1}{4} & 0 & 0 & \frac{2}{4} \\ - & - & 0 & \frac{1}{4} & \frac{2}{4} & 0 \\ \frac{1}{4} & 0 & - & \frac{2}{4} & \frac{2}{4} & 0 \\ 0 & \frac{2}{4} & \frac{1}{4} & - & 0 & \frac{3}{4} \\ 0 & \frac{2}{4} & \frac{2}{4} & 0 & - & \frac{1}{4} \\ \frac{2}{4} & 0 & 0 & \frac{3}{4} & \frac{1}{4} & - \end{pmatrix} \end{array}$$

Considering the mPVM as the assignment problem and applying Hungarian algorithm for finding the optimum pairs.

(1,4),(2,3),(3,6),(4,5),(5,1),(6,2) be the optimum pairs.

To find the optimum triples, from the mPVM choose (1,4) pair, the first row fourth column corresponding element 0 and add with entire first row and assign that particular element place with -. Likewise, choose the next pair (2,3), the second row third column element 0 and add with entire second row and assign that particular element place with -. Similarly, repeat the process for all optimum pairs. Then the resultant matrix will be a Modified Minimum Preference Value Matrix (MmPVM).

The Modified minimized preference value matrix

$$MmPVM = \begin{array}{c} \\ (1,4) \\ (2,3) \\ (3,6) \\ (4,5) \\ (5,1) \\ (6,2) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} - & - & \frac{1}{4} & - & 0 & \frac{2}{4} \\ - & - & - & \frac{1}{4} & \frac{2}{4} & 0 \\ \frac{1}{4} & 0 & - & \frac{2}{4} & \frac{2}{4} & - \\ 0 & \frac{2}{4} & \frac{1}{4} & - & - & \frac{3}{4} \\ - & \frac{2}{4} & \frac{2}{4} & 0 & - & \frac{1}{4} \\ \frac{2}{4} & - & 0 & \frac{3}{4} & \frac{1}{4} & - \end{pmatrix} \end{array}$$

Considering this above matrix as the assignment problem and apply Hungarian algorithm then get the optimum triples.

The triples are (1,4,5), (2,3,6), (3,6,2) (4,5,1), (5,1,4), (6,2,3).

Hence the best triple as a roommates are **(1,4,5) and (2,3,6).**

An individual preference value and overall satisfaction percentages are given below

(1,4,5)	1 → 4 & 5 = 100%.	**100%**
	4 → 1 & 5 = 100%.	
	5→ 1 & 4 = 100%.	
(2,3,6)	2 → 3 & 6 = 100%.	**100%**
	3 → 2 & 6 = 100%.	
	6 → 2 & 3 = 100%.	

V. CONCLUSION

In this paper, we proposed a new complex TPSR algorithm for $3n$ individuals to identify suitable matching for finding perfect triples in rooms, which is based on preference value and helps determine satisfactory matching of roommates problem with ties (TPSRT), ties and incomplete list (TPSRTI). Implementing the Hungarian algorithm to identify the optimum pairs while considering the preference value matrix as the assignment problem. The optimum triples were ultimately determined to be the exclusive and exhaustive triples that achieve the greatest level of satisfaction.

REFERENCES

[1] Irving, R. W., & Manlove, D. F. (2002). The stable roommates problem with ties. *Journal of Algorithms*, *43*(1), 85-105.

[2] Adil, D., Gupta, S., Roy, S., Saurabh, S., & Zehavi, M. (2018). Parameterized algorithms for stable matching with ties and incomplete lists. *Theoretical Computer Science*, *723*, 1-10.

[3] Bredereck, R., Heeger, K., Knop, D., & Niedermeier, R. (2022). Parameterized complexity of stable roommates with ties and incomplete lists through the lens of graph parameters. *Information and Computation*, *289*, 104943.

[4] Nasre, M., Nimbhorkar, P., & Ranjan, K. (2023). Critical Relaxed Stable Matchings with Two-Sided Ties. *arXiv preprint arXiv:2303.12325*.

[5] Delorme, M., García, S., Gondzio, J., Kalcsics, J., Manlove, D., & Pettersson, W. (2019). Mathematical models for stable matching problems with ties and incomplete lists. *European Journal of Operational Research*, *277*(2), 426-441.

[6] Gent, I. P., & Prosser, P. (2002, July). An empirical study of the stable marriage problem with ties and incomplete lists. In *ECAI* (pp. 141-145).

[7] Kunysz, A. (2016). The strongly stable roommates problem. In *24th Annual European Symposium on Algorithms (ESA 2016)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik.

[8] Fleiner, T., Irving, R. W., & Manlove, D. F. (2007). Efficient algorithms for generalized stable marriage and roommates problems. *Theoretical computer science*, *381*(1-3), 162-176.

[9] Viet, H. H., Uyen, N. T., Cao, S. T., & Chung, T. (2022, February). A Heuristic Repair Algorithm for the Maximum Stable Marriage Problem with Ties and Incomplete Lists. In *Australasian Joint Conference on Artificial Intelligence* (pp. 494-506). Cham: Springer International Publishing.

[10] Iwama, K., Miyazaki, S., & Okamoto, K. (2007). Stable roommates problem with triple rooms. *Proceedings of Workshop on Algorithms and Computation (WAAC)*, *7*, 105-112.

[11] Ramachandran, T., Udhayakumar, D., & Selvakumar, T. (2014). Satisfactory Roommates problem with Ties, Ties and incomplete list. International Journal of Scientific Research, ISSN 2277 – 8179, *3*(10).

[12] Logapriya, N., & Ramachandran, T. (2023). Three Persons Satisfactory Roommates Problem. *Gradiva Review Journal*, ISSN- 0363 8257, *9*(8).