${ }^{1, *}$ Lei Wang<br>${ }^{2}$ Jianfeng Ren<br>\title{ Big Data Scheduling with Order Acceptance Consideration in Supply Chain }




#### Abstract

The efficient job scheduling schemes can make full use of resources, and then achieve different goals, such as maximizing efficiency, minimizing cost and saving energy. In supply chain, there are a lot of members and enormous data. Therefore, a suitable scheduling scheme has become the most common and effective method to optimize the execution of big data in supply chain. A scheduling problem with production and delivery consideration, which is usually in supply chain has been considered. For the reason of highly time emergency and random coming orders in quick production and delivery system, the effective algorithms for order acceptance scheduling problem are required. This paper addressed the production and delivery problem which has one manufacturer and multiple customers. There is single machine for order production at the manufacturer. For the consideration of order acceptance or not, the manufacturer need to choose order set to be accepted for processing. The paper aims at finding a balance between orders profit, delivery cost and time-based cost to maximize the total revenue. We consider three main objective functions in scheduling theory, analyze the problem complexity. The complexity of two of the problems are weakly NP hard, and the other one is strongly NP hard. For three weakly NP hard problems, we give pseudo-polynomial time optimal algorithms.


Keywords: Scheduling, Big Data, Order Acceptance, Supply Chain.

## I. INTRODUCTION

In this age of information explosion, data from supply chain, medical care, transportation and other fields is growing exponentially. Efficient data management is important for enterprise and supply chain. Scheduling is the most common and effective method to optimize the execution of big data in supply chain. Different scheduling schemes focus on different aspects, including finding the optimal job arrangement, minimizing job execution time, minimizing cost under some constraints. There are some research about job scheduling approach for big data applications. Shao et al. [1] proposed an Energy-aware Fair Scheduling framework based on YARN, which can effectively reduce energy consumption while meet the required Service Level Agreements .Zheng et al. [2] focused on deadline constraints and tried to minimize the execution cost of big data jobs, they proposed three scheduling heuristic algorithms for this problem. In supply chain management, production and batch delivery are two important operational factors. It has been well known that integrated production and transportation scheduling significantly reduce operation cost, and improve customer service level in supply chain.

Order acceptance scheduling is an operation management problem for manufacturing industry. In some situations, because of production capacity constraint and tight delivery time request, the decision maker should decide accept which orders with batch delivery. There are some research which is on the integration decisions of order acceptance scheduling with transportation. Nobibon et al. [3] have studied a general problem of order acceptance scheduling, they give two effective algorithms to solve this problem. A survey of the OAS problem is the research of Slotnick [4]. In this paper, they summarize the models into many kinds and give a summary for the optimal scheduling and algorithms. Ou [5] consider some order acceptance scheduling problems with two conflict criteria, algorithm is provided to solve this problem. Geramipour [6] studied objective of maximization of the total revenue, and a new heuristic algorithms has been given. Sarvestani [7] studied a scheduling for revenue maximization problem, which contain supplier selection and order acceptance scheduling with multiple customers. Noroozi [8] consider a scheduling with integration of order acceptance and delivery problem. They give two exact algorithms to solve this problem. Noroozi [9] give the research of batch delivery problem with third party logistics. The delivery with round trip mode are considered. They presented heuristic algorithms for this problem. Wang [10] give the research of scheduling problem of maximizing the profit minus weighted tardiness with unrelated machines environment. Kong [11] studied green order acceptance scheduling with machine use cost, there are some constraints in machine launch and energy consumption. Khalili [12] studied the OAS problem with package delivery, and give an efficient competitive algorithm. Lu [13] study a general OAS problem with delivery, they provide some polynomial time optimal algorithms for them. Aminzadegan [14] studied scheduling problem with

[^0]resource allocation and order acceptance. They provide two heuristic approaches, which is tabu search and genetic algorithm, to solve this problem. Xue [15] consider order acceptance scheduling in the just in time distribution system, and design some heuristic algorithms for this model. The OAS problem and batch delivery scheduling problems are studied a lot separately, but there are little studies about their integration.

The remainder of the paper is organized as follows. We define notations of these three problems, and show the optimal properties of three problems in section 2 . In section 3, we give complexity results and optimal algorithms for the problems. Finally, we give some conclusion and further research topic in section 4.

## II. Materials and Methods

We studied a supply chain scheduling problem, and there is one manufacturer and many customers. There are $m$ customers which are at different locations in a supply chain. At time 0 , the manufacturer receives $n_{i}$ orders from customer $i$ that is for processing. $\omega_{i}$ denotes the weight of customer $i$. Let $n_{1}+n_{2}+\cdots+n_{m}=n$ be the total number of orders. Only one machine is at the manufacturer for processing. For each order $J_{i j}$, it's processing time is $p_{i j}$, due date is $d_{i j}$, and revenue is $R_{i j}$.

When the orders are completed, they are delivered to the customers in batches. Because of the arrive time request, and each customer located at a different place, so we adopt direct shipping mode from the manufacturer to every customer. So, only the orders come from the same customer can be batched together. Each delivery incurs a delivery cost and time. The cost and time from the manufacturer to customer $i$ are $f_{i}$ and $t_{i}$. The delivery batch has capacity limit, we define $B$ as the batch capacity of orders that can be shipped together in a batch. For a given schedule scheme, we define time-based objective functions in table 1.

Table 1: The Time-Based Objective Functions

| $C_{i j}$ | the processing completion time of $J_{i j}$ |
| :---: | :---: |
| $D_{i j}$ | the arrive time of order $J_{i j}$ |
| $L_{i j}=D_{i j}-d_{i j}$ | the lateness of order $J_{i j}$. If $L_{i j}>0$, then $U_{i j}=1 ;$ otherwise $U_{i j}=0$ |
| $T_{i j}=\max \left(D_{i j}-d_{i j}, 0\right)$ | the tardiness of order $J_{i j}$ |
| $\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} D_{i j}$ | total weighted arrive time of the accept orders |
| $\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} U_{i j}$ | total weighted number of late orders |
| $\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} T_{i j}$ | total weighted tardiness of accept orders |

The objective function is to maximize the total profit, that is the accepted orders revenue minus batch delivery cost and time-based objective functions.

There are three decisions to be made for these problems:
(1) Choose which orders should be accepted and delivery;
(2) Decide the scheduling sequence and when to start processing for accepted orders;
(3) Decide to choose which orders in a same delivery batch and when to leave.

Based on the three-field notation, these three problems denoted as follows:
P1: $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} D_{i j}-T C$
P2: $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} U_{i j}-T C$
P3: $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} T_{i j}-T C$
In these notations, OA means order acceptance, bd means batch delivery. R is total revenues of the accepted orders, TC is total batch delivery cost.

It satisfies the following properties for an optimal schedule:
(1) When the orders are processed, there is no idle time between orders on the machine;
(2) All orders in a same delivery batch are processed one by one; the batch starting time is completion time of last order in it.

## III. ReSUlts and Discussion

## A. Total Weighted Arrive Time Problem

Lemma 1. There exists an optimal schedule that accepted orders are sequenced in SPT sequence for each customer on the machine for problem P1.

We give optimal algorithm for this problem. It is formally described as follows.
We arrange the orders in SPT order, which is $p_{i 1} \leq p_{i 2} \leq \cdots \leq p_{i n_{i}}, i=1,2, \cdots, m$. Let $F\left(l_{1}, \cdots, l_{m} ; T, b, i\right)$ as the optimal solution of the objective, and satisfying three cases:

The numbers of arranged orders is $l_{1}+\cdots+l_{m}$, and $l_{u}$ jobs are from the SPT sequence of customer $u$;
(2) The makespan of the accepted orders is $T$;
(3) The last scheduled order in the last batch is for customer $i(1 \leq i \leq m)$, and the last batch has b orders .

Algorithm DP1
The boundary condition: $F(0, \cdots, 0 ; 0,0,0)=0$
Optimal value: $\max F\left(n_{1}, \cdots, n_{m} ; T, b, i\right)$
Recurrence relations: $F\left(l_{1}, \cdots, l_{m} ; T, b, i\right)$

$$
=\max \left\{\begin{array}{c}
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T, b, i\right) ;  \tag{1}\\
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T-p_{i i_{l}}, b-1, i\right)+R_{i l_{i}}-\omega_{i}\left(T+t_{i}\right)-\omega_{i}(b-1) p_{i_{i}} ; \\
\max _{1 \leq k \leq m}\left\{F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T-p_{i i_{i}}, h, k\right)+R_{i l_{i}}-\omega_{i}\left(T+t_{i}\right)-f_{i}\right\} ;
\end{array}\right.
$$

The first stage of recurrence relation is to assign the order $J_{i l_{i}}$ as a not accept order, so there is no cost change in objection function. The second stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and deliver with other orders, so the orders revenue and time-based objective functions are changed. The third stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and delivery with a new batch, so the accepted orders revenue, batch delivery cost and time-based objective functions are changed. Algorithm DP1 gives an optimal schedule for $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} D_{i j}-T C$ in $O\left(m^{2} n^{m} P\right)$ time, which is pseudo-polynomial time optimal algorithm, so the complexity of P1 is weakly NP-hard.

## B. Total Weighted Number of Late Orders

Lemma 2. There exists an optimal schedule that accepted and on time arrive orders are sequenced in EDD sequence for each customer on the machine for problem P2.

We arrange the orders in EDD order, which is $d_{i 1} \leq d_{i 2} \leq \cdots \leq d_{i n_{i}}, i=1,2, \cdots, m$. Let $F\left(l_{1}, \cdots, l_{m} ; u_{1}, \cdots, u_{m} ; T, i, g\right)$ as the optimal solution of the objective, and satisfying three cases:
(1) The numbers of arranged orders is $l_{1}+\cdots+l_{m}$, and $l_{k}$ jobs are from the EDD sequence of customer $k$;
(2) The makespan of the accepted orders is $T$;
(3) The last scheduled order in the last batch is for customer $i(1 \leq i \leq m)$, and the last batch has b orders.

Algorithm DP2
The boundary condition: $F(0, \cdots, 0 ; 0, \cdots, 0 ; 0,0,0)=0$
Optimal value: $\max \left\{F\left(n_{1}, \cdots, n_{m} ; u_{1}, \cdots, u_{m} ; T, i, g\right)-\sum_{i=1}^{m}\left[\frac{u_{i}}{B}\right\rceil f_{i}\right\}$
Recurrence relations: $F\left(l_{1}, \cdots, l_{m} ; u_{1}, \cdots, u_{m} ; T, i, g\right)$

$$
=\max \left\{\begin{array}{cc}
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; u_{1}, \cdots, u_{m} ; T, i, g\right) ; &  \tag{2}\\
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; u_{1}, \cdots, u_{m} ; T-p_{i i_{i}}, i, g\right)+R_{i l_{i}} ; & T+t_{i} \leq d_{i g} \\
\max _{1 \leq k \leq m}\left\{\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; u_{1}, \cdots, u_{m} ; T-p_{i i_{i}}, k, h\right)+R_{i_{i}}-f_{i} ;\right. & T+t_{i} \leq d_{i i_{i}} \\
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; u_{1}, \cdots u_{i}-1, \cdots u_{m} ; T, i, g\right)+R_{i_{i}}-\omega_{i} ; & T+t_{i}>d_{i l_{i}},
\end{array}\right.
$$

The first stage of recurrence relation is to assign the order $J_{i l_{i}}$ as not accept order, so there is no cost change in objection function. The second stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and deliver with other orders on time, so the accepted orders revenue is changed. The third stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and deliver with a new batch on time, so the accepted orders revenue, batch delivery cost are changed. The last stage of recurrence relation is to assign the order $J_{i l_{i}}$ as a accept but tardy order, so the accepted orders revenue and time-based objective functions are changed. Algorithm DP2 gives an optimal schedule for $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} U_{i j}-T C$ in $O\left(n^{2 m+1} m^{2} P\right)$ time, which is pseudo-polynomial time optimal algorithm, so the complexity of problem P2 is weakly NP-hard.

## C. Total Weighted Tardiness Problem

For problem P3: $1|O A, b d| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} T_{i j}-T C$, even each customer has only one order, and delivery cost is not under consideration, problem P3 is still strongly NP-hard. The reason is that $1|O A| R-\sum_{i=1}^{m} \omega_{i} T_{i}$ is strongly NP-hard which is point in [7]. We now give a special case of P3 that due dates of orders for the same customer are same, i.e. $d_{i j}=d_{i}, j=1,2, \cdots, n_{i}$. This new problem can be denoted as P4:1 $\left|O A, b d, d_{i j}=d_{i}\right| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} T_{i j}-T C$.

Lemma 3. There exists an optimal schedule that accepted orders are sequenced in SPT sequence for each customer on the machine for problem P4.

We arrange the orders in SPT order, which is $p_{i 1} \leq p_{i 2} \leq \cdots \leq p_{i n_{i}}, i=1,2, \cdots, m$. Let $F\left(l_{1}, \cdots, l_{m} ; T, i, h\right)$ as the optimal solution of the objective, and satisfying three cases:
(1) The numbers of arranged orders is $l_{1}+\cdots+l_{m}$, and $l_{u}$ jobs are from the SPT sequence of customer $u$;
(2) The makespan of the accepted orders is $T$;
(3) The last scheduled order is forming customer $i(1 \leq i \leq m)$, and the last batch has $h$ orders.

Algorithm DP3
The boundary condition: $F(0, \cdots, 0 ; 0,0,0)=0$
Optimal value: $\max F\left(n_{1}, \cdots, n_{m} ; T, i, h\right)$
Recurrence relations: $F\left(l_{1}, \cdots, l_{m} ; T, i, h\right)$

$$
=\max \left\{\begin{array}{c}
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T, i, h\right)  \tag{3}\\
\max _{1 \leq k \leq m}\left\{F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T-p_{i l_{i}}, k, g\right)+R_{i l_{i}}-\omega_{i} \max \left(0, T+t_{i}-d_{i}\right)-f_{i}\right\} \\
F\left(l_{1}, \cdots l_{i}-1, \cdots l_{m} ; T-p_{i l_{i}}, i, h-1\right)+R_{i l_{i}}-\left[\omega_{i} \cdot h \cdot \max \left(0, T+t_{i}-d_{i}\right)\right. \\
\left.-\omega_{i}(h-1) \max \left(0, T-p_{i l_{i}}+t_{i}-d_{i}\right)\right]
\end{array}\right.
$$

The first stage of recurrence relation is to assign the order $J_{i l_{i}}$ as not accept order, so there is no cost change in objection function. The second stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and deliver with a new batch, so the accepted orders revenue, batch delivery cost and time-based objective functions are changed. The third stage of recurrence relation is to assign the order $J_{i l_{i}}$ as an accept order and deliver with other orders, so the accepted orders revenue and time-based objective functions are changed.. Algorithm DP3 gives an
optimal schedule for $1\left|O A, b d, d_{i j}=d_{i}\right| R-\sum_{i=1}^{m} \sum_{j=1}^{n_{i}} \omega_{i} T_{i j}-T C$ in $O\left(n^{m+1} m^{2} P\right)$ time, which is pseudo-polynomial time optimal algorithm, so the complexity of problem P4 is weakly NP-hard.

## IV. Conclusions

We studied a production scheduling and batch delivery with order acceptance consideration. The mentioned problems are very important in many real-life companies and manufacturing industries. We consider three main objective functions in scheduling theory, analyze the problem complexity. Several optimal properties are given for the scheduling scheme. We give dynamic programming algorithms to decide the order acceptance and batch delivery to solve these problems. In our paper, the weakness of our work is a static point, and sometimes with limitation. Because all of the customers' orders are known at the beginning in production. But in realistic, orders come at random. This setting is not considered, and some important finding may lead to. Another topic is to find the optimal properties of our new models. We will study on some realistic settings for production and batch delivery scheduling, such as dynamic processing time, uniform machines, and other objective functions. It is important to design efficient algorithms, for example, artificial algorithms and heuristic algorithms for future research direction.

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[^0]:    ${ }^{1}$ School of Management, Qufu Normal University, Rizhao, Shandong, China
    ${ }^{2}$ School of Management, Qufu Normal University, Rizhao, Shandong, China
    *Corresponding author: Lei Wang
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