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Using Simplified Distflow-Based Mixed-Integer Quadratically Constrained Programming Formulation for Optimum Selection of Conductor Size in Electrical Distribution Networks

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Abstract: *-* Determining conductor size is a subproblem that plays a vital role in the planning process of distribution systems. The problem of selecting the conductor cross-sectional area is usually made a model of mixed-integer non-linear programming (MINLP). It is important to note, however, that the MINLP model is not always capable of ensuring convergence to a solution that is globally optimum. In this research, a mixed-integer quadratically constrained programming (MIQCP) formulation is developed as a method for determining the conductor cross-sectional area in power distribution networks in an optimal manner. The goal function aims at minimizing the lifetime cost of lines, including initial capital cost together with operational expenses. The suggested optimization model's constraints consist of power balance equations, thermal loading capacity of branches, nodal voltage restrictions, budget limitations for investment, and the need to have the same wire size in the main feeder. By using the simplified DistFlow approach for electrical distribution networks and precisely linearizing the product of a binary variable and a continuous variable, the MIQCP formulation is derived from the MINLP model. It is likely possible to solve the MIQCP formulation efficiently in the GAMS environment by using available commercial solvers like CPLEX. The developed MIQCP model is validated using three medium voltage distribution grids of IEEE 33 buses, IEEE 85 nodes, and the Vietnamese real 102 buses. The calculation results revealed the accuracy along with the efficacy of the proposed optimization procedure.

Keywords: Electrical Distribution Grids, Lifetime Cost (LTC), Mixed-Integer Quadratically Constrained Programming (MIQCP), Selection of Conductor Size.

I. INTRODUCTION

Distribution grid planning is one of the important research topics in the field of electrical engineering. Electrical distribution systems often operate with a radial configuration to simplify protection methods and operating procedures [1]. Due to the low nominal voltage, power loss on this grid accounts for a large proportion of the overall power loss of the entire power system. Therefore, accurate and efficient distribution network planning helps to reduce power loss in the power system. Distribution grid planning is a complex optimization problem in which the objective function is to minimize the total cost while considering operational limitations, reliability of power supply, and power quality [2]. The process of power distribution grid planning consists of various subproblems, one of which is the determination of conductor size [3].

The problem pertaining to the choosing of conductor size has been extensively examined in a variety of scholarly publications. The methodology outlined in reference [3] elucidates the process of determining the optimal conductor cross-sectional area for the distribution grid, with the objective of minimizing both the overall capital investment and operational expenses. The process for determining conductor size to minimize power losses and enhance voltage quality is outlined in the study conducted by the authors [4]. This approach utilizes power flow calculations as a means of achieving these objectives. A simple method for selecting cable cross-sectional area under 1000V to minimize thermal aging of cable insulation is described in [5]. However, the aforementioned studies [3]-[5] do not describe the problem of conductor size selection as an optimization formulation, hence potentially yielding a solution that could not be globally optimal. A model called mixed-integer linear programming (MILP) was proposed by the authors of [6] as a solution to the problem of size choice of conductor and reconductoring in radial power distribution grids. The MILP model in [6] is constructed from a mixed-integer non-linear programming (MINLP) formulation by utilization of the piecewise linearization technique. The study [7] presented an economic current density-based method and a heuristic index-directed method to effectively determine the cross-sectional area for conductors. The approach described in [7] does not necessitate the utilization

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of intricate optimization techniques and is straightforward to execute. It is, nevertheless, important to note that this method may not yield a globally optimal solution. The authors of [8] introduced the Nondominated Sorting Improved Harmony Search Algorithm-II as a means to address the multi-objective optimization problem in distribution grid expansion planning, specifically taking into account the presence of Distributed Generation (DG). The paper [9] presented an algorithm on the basis of Salp swarm optimization to select the optimum conductor size for the Egyptian power grid during construction and reinforcement, taking into account the increasing penetration level of DG. In addition, in [9], the feeder reinforcement index (FRI) was proposed as a way of identifying branches that need to be improved, thus helping to reduce the solution space and improve computational efficiency. The work [10] developed a MINLP model for the simultaneous optimization of conductor size and shunt capacitor. This optimization model considers the dynamic characteristics of the load and accurately represents the expense and location of the shunt capacitor as integer variables. However, this MINLP formulation in [10] is solved directly, making it challenging to find and achieve a globally optimal solution. Similarly, authors in [11] proposed the MINLP model and employed the DICOPT non-linear solver in conjunction with the GAMS programming language for determining the best conductor size. The harmony search method coupled with a different operator in the form of a meta-heuristic optimization technique is described in [12] for the purpose of conductor cross-section selection.

Table 1: Comparison between the presented studies in the literature and the approach constructed in this paper

Reference Number	Power Flow Model	Test System	Objective Function	Solving Method
[3]	-	18-bus	Capital investment and operational expense	Analysis
[4]	-	11-bus	Power loss and voltage quality	Analysis
[5]	-	12-bus	Thermal aging of cable insulation	Analysis
[6]	Linearized power flow model	50-bus, 200-bus and 600-bus	Total investment and operational cost	MILP
[7]	Linearized power flow model	33-bus	Total investment and operational cost	Economic current density-based method and heuristic index-directed method
[8]	-	33-bus and 69-bus	Cost of energy loss and electricity purchase	NSIHSA-II
[9]	-	9-bus and 69-bus	Total investment and energy loss cost	Salp swarm optimization
[10]	Non-linear power flow model	117-bus	Investment cost of conductors, capacitors, and power loss	MINLP
[11]	Non-linear power flow model	8-bus and 27-bus	Cost of capital investment and energy loss	MINLP
[12]	-	16-bus and 85-bus	Energy loss cost	Harmony search method
This paper	Simplified DistFlow (SD) model	33-bus, 85-bus, and Vietnamese real 102-bus	Lifetime cost, including investment cost, maintenance cost, and energy loss cost	MIQCP

The research papers [3]-[12] demonstrate that the determination of the most suitable conductor size for the distribution grid is primarily addressed through the application of heuristic, meta-heuristic, and MINLP optimization methods. The main disadvantage of these optimization techniques is that they are not guaranteed to find a globally optimal solution. Table 1 presents an overview of the works in the literature associated with the optimal selection of conductor size for electrical distribution grids.

As a result, this paper aims to develop a mixed-integer quadratically constrained programming (MIQCP) model for the problem of conductor size selection. Through the use of the simplified DistFlow (SD) approach [13] and the precisely linear form of the product of a binary variable and a continuous variable, the MINLP formulation is converted into the MIQCP model. The MIQCP model ensures the attainment of a solution that is globally optimum, and it may be efficiently solved by using widely available commercial software tools like CPLEX/GAMS. The primary contributions of this research paper encompass:

- Proposing the MINLP model for addressing the conductor cross-sectional area selection problem of the power distribution network, in which the goal function is the lifetime cost (LTC) to be minimized, and the constraints include the power balance equations, voltage magnitude bounds, thermal limits on branches, and the requirement for the primary feeder to have the same conductor size;
- Developing the MIQCP model from the proposed MINLP model by making use of the SD-based distribution power flow approach and the precise linearization of non-linear elements related to a binary variable multiplying by a continuous variable;
- Validating the developed MIQCP formulation by selecting the conductor size for power distribution grids of IEEE 33 buses, IEEE 85 buses, and Vietnamese real-world 102 nodes.

The remainder of this paper will be presented in three Sections. Section II presents mathematical optimization models of the problem of conductor size selection. Section III describes the results and discussions, while conclusions are given in Section IV.

II. METHODOLOGY

This section presents the mixed-integer non-linear programming (MINLP) model and the mixed-integer quadratically constrained programming (MIQCP) formulation with the aim of determining the conductor cross-sectional area in power distribution systems optimally.

A. *Mixed-Integer Non-Linear Programming-Based Formulation*

The MINLP model, which represents the mathematical formulations of the conductor size selection problem, is described below:

$$\begin{aligned}\min \mathrm{LTC} = &\sum_{ij\in\Phi_\mathrm{L}}\sum_{k\in\Phi_\mathrm{S}} K_{0,k}\times u_{ij,k}\times L_{ij}\\ &+\sum_{t=1}^{N}\left[\begin{aligned}&a_\mathrm{MC}\times\left(\sum_{ij\in\Phi_\mathrm{L}}\sum_{k\in\Phi_\mathrm{S}} K_{0,k}\times u_{ij,k}\times L_{ij}\right)\\ &+\sum_{ij\in\Phi_\mathrm{L}}\sum_{k\in\Phi_\mathrm{S}} R_{0,k}\times L_{ij}\times\frac{\left(u_{ij,k}P_{ij,k}\right)^2+\left(u_{ij,k}Q_{ij,k}\right)^2}{m_{ij}\times U_i^2}\times S_\mathrm{base}\times\mathrm{LsF}\times 8760\times c_{\Delta\mathrm{A}}\end{aligned}\right]\times\frac{1}{(1+r)^t}\end{aligned} \quad (1)$$

subject to:

$$P_{hi} = \sum_{j\in\Phi_i, j\neq h} P_{ij} + R_{hi}\times\frac{(P_{hi})^2+(Q_{hi})^2}{U_h^2} - P_i;\quad \forall i\in\Phi_\mathrm{N} \quad (2)$$

$$Q_{hi} = \sum_{j\in\Phi_i, j\neq h} Q_{ij} + X_{hi}\times\frac{(P_{hi})^2+(Q_{hi})^2}{U_h^2} - Q_i;\quad \forall i\in\Phi_\mathrm{N} \quad (3)$$

$$U_i^2 - U_j^2 - 2\times\left(R_{ij}P_{ij}+X_{ij}Q_{ij}\right)+\left(R_{ij}^2+X_{ij}^2\right)\frac{P_{ij}^2+Q_{ij}^2}{U_i^2}=0;\quad \forall ij\in\Phi_\mathrm{L} \quad (4)$$

$$P_i = P_{\mathrm{G}i} - P_{\mathrm{D}i};\quad \forall i\in\Phi_\mathrm{N} \quad (5)$$

$$Q_i = Q_{\mathrm{G}i} - Q_{\mathrm{D}i};\quad \forall i\in\Phi_\mathrm{N} \quad (6)$$

$$R_{ij} = \frac{L_{ij}}{m_{ij}}\times\sum_{k\in\Phi_\mathrm{S}} u_{ij,k}R_{0,k};\quad \forall ij\in\Phi_\mathrm{L} \quad (7)$$

$$X_{ij} = \frac{L_{ij}}{m_{ij}}\times\sum_{k\in\Phi_\mathrm{S}} u_{ij,k}X_{0,k};\quad \forall ij\in\Phi_\mathrm{L} \quad (8)$$

$$P_{ij} = \sum_{k \in \Phi_S} u_{ij,k} P_{ij,k}; \quad \forall ij \in \Phi_L \tag{9}$$

$$Q_{ij} = \sum_{k \in \Phi_S} u_{ij,k} Q_{ij,k}; \quad \forall ij \in \Phi_L \tag{10}$$

$$I_{ij}^2 = \frac{P_{ij}^2 + Q_{ij}^2}{U_i^2} \le \sum_{k \in \Phi_S} m_{ij} \times I_{\max,k}^2 \times u_{ij,k}; \quad \forall ij \in \Phi_L \tag{11}$$

$$U_i^{\min} \le U_i \le U_i^{\max}; \quad \forall i \in \Phi_N \tag{12}$$

$$\sum_{\forall ij \in \Phi_T} u_{ij,k} = N_{\Phi_T} u_k; \quad \forall k \in \Phi_S \tag{13}$$

$$\sum_{k \in \Phi_S} u_{ij,k} = 1; \quad \forall ij \in \Phi_L \tag{14}$$

$$\sum_{ij \in \Phi_L} \sum_{k \in \Phi_S} K_{0,k} \times u_{ij,k} \times L_{ij} \le K \tag{15}$$

$$\begin{aligned} & u_{ij,k} \in \{0,1\}; \quad \forall ij \in \Phi_L; \forall k \in \Phi_S \\ & u_k \in \{0,1\}; \quad \forall k \in \Phi_S \end{aligned} \tag{16}$$

where:

- Φ_i is set of all buses directly connected to bus *i*;
- Φ_L is set of all branches;
- Φ_N is set of all buses;
- Φ_S is set of all used standard conductor sizes;
- Φ_T is set of all branches on the main feeder;
- a_{MC} denotes the operational cost-related factor;
- $c_{\Delta A}$ denotes the marginal cost of energy losses ($/MWh);
- $I_{\max,k}$ denotes current ratings of conductor with type *k* (p.u.);
- $K_{0,k}$ denotes initial capital cost per kilometer of conductor with the type *k* ($/km);
- *K* denotes investment budget ($);
- L_{ij} denotes the length of branch *ij* (km);
- LsF denotes the loss factor;
- m_{ij} denotes the number of line circuits;
- *N* denotes the line lifespan in years;
- N_{Φ_T} denotes the number of branches belonging to the primary feeder;
- P_{Di} denotes the active power of load at bus *i* (p.u.);
- P_{Gi} denotes the active power output of the generating unit at bus *i* (p.u.);
- $P_{ij,k}$ represents the active power flow on branch *ij* with conductor type *k* (p.u.);
- P_{ij} represents the active power flow on branch *ij* (p.u.);
- Q_{Di} denotes the reactive power of load at bus *i* (p.u.);
- Q_{Gi} denotes the reactive power output of the generating unit at bus *i* (p.u.);
- $Q_{ij,k}$ represents the reactive power flow on branch *ij* with conductor type *k* (p.u.);
- Q_{ij} represents the reactive power flow on branch *ij* (p.u.);
- *r* denotes the discount rate;
- $R_{0,k}$ denotes resistance per kilometer of conductor with type *k* (p.u./km);
- R_{ij} represents series resistance of branch *ij* (p.u.);
- $X_{0,k}$ denotes reactance per kilometer of conductor with type *k* (p.u./km);
- X_{ij} represents series reactance of branch *ij* (p.u.);

- S_{base} denotes the base power (MVA);
- t denotes the index of time (year);
- U_i denotes the voltage magnitude at bus *i* (p.u.);
- $U_i^{\min}$ denotes the lower limit of voltage at bus *i* (p.u.);
- $U_i^{\max}$ denotes the upper limit of voltage at bus *i* (p.u.);
- $u_{ij,k}$ is binary variable, $u_{ij,k} = 1$ when conductor type *k* is selected for the branch *ij*; otherwise $u_{ij,k} = 0$;
- u_k is binary variable representing the wire size of the main feeder to be the same.

The objective function (1) is to minimize the lifetime cost of the distribution network, including capital investment cost, maintenance cost, and the expense incurred due to network energy loss. Formula (2)-(3) are power flow equations on the basis of the power summation method for electrical distribution networks with the radial topology. Equations (4) present the voltage relationship between nodes of branch *ij*. The real and reactive power injected at bus *i* are given by (5) and (6). The resistance and series reactance of branch *ij*, corresponding to conductor type *k*, are described by (7) and (8). Equations (9) and (10) are deployed to calculate power flow on branch *ij* with different conductor sizes. Branch thermal limits are described by (11). Equation (12) presents voltage magnitude bounds. Constraints of the same conductor cross-sectional area for the main feeder are represented by (13). Constraints (14) illustrate that each line is restricted to merely utilizing a single type of conductor size. The investment budget confinement is enforced by (15). Constraints (16) define binary variables.

Mathematically, the aforementioned proposed MINLP formulation is classified as non-linear due to the following reasons:

- The objective function and power flow equations are non-linear;
- The existence of a product between a binary variable and a continuous variable can be observed.

The drawbacks of the MINLP formulation consist of a high degree of computational complexity, the challenging attainment of a globally optimal solution, and a significant amount of computational time. The next section of this paper formulates the MIQCP model of the problem of conductor size choice.

B. Mixed-Integer Quadratically Constrained Programming-Based Formulation

1) Simplified DistFlow method

In this section, the simplified DistFlow method [13]-[14] is applied to formulate the MIQCP model of the problem (1)-(16).

In steady-state operating conditions, the nodal voltage magnitudes are approximately equal to 1. The small changes in nodal voltages are attained:

$$1-\varepsilon \le U_i \le 1+\varepsilon; \quad \forall i \tag{17}$$

Based on the operational standard, the parameter ε is chosen as 0.05. Therefore, we achieve:

$$\left(U_i - 1\right)^2 \approx U_i^2 - 2U_i + 1 \approx 0 \tag{18}$$

and:

$$U_i^2 \approx 2U_i - 1 \tag{19}$$

Substituting (17)-(19) into (1)-(4) and (11), the following expressions are achieved:

$$\begin{aligned}\min \text{LTC} = & \sum_{ij\in\Phi_{\text{L}}} \sum_{k\in\Phi_{\text{S}}} K_{0,k} \times u_{ij,k} \times L_{ij} \\ & + \sum_{t=1}^{N} \left[\begin{aligned} & a_{\text{MC}} \times \left(\sum_{ij\in\Phi_{\text{L}}} \sum_{k\in\Phi_{\text{S}}} K_{0,k} \times u_{ij,k} \times L_{ij} \right) \\ & + \sum_{ij\in\Phi_{\text{L}}} \sum_{k\in\Phi_{\text{S}}} R_{0,k} \times L_{ij} \times \frac{\left(u_{ij,k}P_{ij,k}\right)^2 + \left(u_{ij,k}Q_{ij,k}\right)^2}{m_{ij}} \times S_{\text{base}} \times \text{LsF} \times 8760 \times c_{\Delta\text{A}} \end{aligned} \right] \times \frac{1}{\left(1+r\right)^t}\end{aligned} \tag{20}$$

$$P_{hi} = \sum_{j\in\Phi_i,\, j\ne h} P_{ij} + R_{hi} \times \left(P_{hi}^2 + Q_{hi}^2\right) - P_i; \quad \forall i \in \Phi_{\text{N}} \tag{21}$$

$$Q_{hi} = \sum_{j\in\Phi_i,\, j\ne h} Q_{ij} + X_{hi} \times \left(P_{hi}^2 + Q_{hi}^2\right) - Q_i; \quad \forall i \in \Phi_{\text{N}} \tag{22}$$

$$\left(2U_i - 1\right) - \left(2U_j - 1\right) - 2\times\left(R_{ij}P_{ij} + X_{ij}Q_{ij}\right) + \left(R_{ij}^2 + X_{ij}^2\right)\left(P_{ij}^2 + Q_{ij}^2\right) = 0; \forall ij \in \Phi_{\text{L}} \tag{23}$$

$$I_{ij}^2 = P_{ij}^2 + Q_{ij}^2 \le \sum_{k\in\Phi_S} m_{ij} \times I_{\max,k}^2 \times z_{ij,k}; \quad \forall ij \in \Phi_L \tag{24}$$

Furthermore, for each branch *ij*, the power loss is by far lower than the power flow. Consequently, we can neglect the element of branch power loss. Constraints (21)-(23) can be rewritten as follows:

$$P_{hi} = -P_i + \sum_{j\in\Phi_i, j\ne h} P_{ij}; \quad \forall i \in \Phi_N \tag{25}$$

$$Q_{hi} = -Q_i + \sum_{j\in\Phi_i, j\ne h} Q_{ij}; \quad \forall i \in \Phi_N \tag{26}$$

$$U_j = U_i - \left(R_{ij}P_{ij} + X_{ij}Q_{ij}\right); \quad \forall ij \in \Phi_L \tag{27}$$

2) The precisely linear form of the product of a binary variable and a continuous variable

The objective function (20) and constraints (9)-(10) contain elements involving the product of a binary variable and a continuous variable. According to the method given in [15]-[16], the non-linear terms can be exactly linearized as follows.

Let:

$$\bar{P}_{ij,k} = u_{ij,k} \times P_{ij,k}; \quad \bar{Q}_{ij,k} = u_{ij,k} \times Q_{ij,k} \tag{28}$$

then,

$$\bar{P}_{ij,k} = \begin{cases} 0 & \text{if } u_{ij,k} = 0 \\ P_{ij,k} & \text{if } u_{ij,k} = 1 \end{cases} \text{ and } \bar{Q}_{ij,k} = \begin{cases} 0 & \text{if } u_{ij,k} = 0 \\ Q_{ij,k} & \text{if } u_{ij,k} = 1 \end{cases} \tag{29}$$

The linearization of terms $\bar{P}_{ij,k}$ and $\bar{Q}_{ij,k}$ is expressed as in

$$\bar{P}_{ij,k} = P_{ij,k} - x_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{30}$$

$$-H \times u_{ij,k} \le \bar{P}_{ij,k} \le H \times u_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{31}$$

$$-H\left(1-u_{ij,k}\right) \le x_{ij,k} \le H\left(1-u_{ij,k}\right); \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{32}$$

$$\bar{Q}_{ij,k} = Q_{ij,k} - y_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{33}$$

$$-H \times u_{ij,k} \le \bar{Q}_{ij,k} \le H \times u_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{34}$$

$$-H\left(1-u_{ij,k}\right) \le y_{ij,k} \le H\left(1-u_{ij,k}\right); \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{35}$$

where:

- $x_{ij,k}, y_{ij,k}, \bar{P}_{ij,k}, \bar{Q}_{ij,k}$ are auxiliary variables involving making the linear form of the product of a binary variable and a continuous variable;
- *H* is a sufficiently large positive constant set to 20 in this study.

3) Mixed-integer quadratically constrained programming-based formulation

Based on the linear power flow model in subsection 1) and the precise linearization of the product of a binary variable and a continuous variable in subsection 2), the proposed MIQCP model is derived below:

$$\begin{aligned} \min \text{LTC} = & \sum_{ij\in\Phi_L}\sum_{k\in\Phi_S} K_{0,k} \times u_{ij,k} \times L_{ij} \\ & + \sum_{t=1}^{N} \left[\begin{aligned} & a_{MC} \times \left(\sum_{ij\in\Phi_L}\sum_{k\in\Phi_S} K_{0,k} \times u_{ij,k} \times L_{ij} \right) \\ & + \sum_{ij\in\Phi_L}\sum_{k\in\Phi_S} R_{0,k} \times L_{ij} \times \frac{\bar{P}_{ij,k}^2 + \bar{Q}_{ij,k}^2}{m_{ij}} \times S_{\text{base}} \times \text{LsF} \times 8760 \times c_{\Delta A} \end{aligned} \right] \times \frac{1}{\left(1+r\right)^t} \end{aligned} \tag{36}$$

$$P_{hi} = -P_i + \sum_{j\in\Phi_i, j\ne h} P_{ij}; \quad \forall i \in \Phi_N \tag{37}$$

$$Q_{hi} = -Q_i + \sum_{j\in\Phi_i, j\ne h} Q_{ij}; \quad \forall i \in \Phi_N \tag{38}$$

$$U_i - U_j = \frac{L_{ij}}{m_{ij}} \sum_{k\in\Phi_S} \left(R_{0,k}\bar{P}_{ij,k} + X_{0,k}\bar{Q}_{ij,k}\right); \quad \forall ij \in \Phi_L \tag{39}$$

$$P_i = P_{Gi} - P_{Di}; \quad \forall i \in \Phi_N \tag{40}$$

$$Q_i = Q_{Gi} - Q_{Di}; \quad \forall i \in \Phi_N \tag{41}$$

$$P_{ij} = \sum_{k \in \Phi_S} \bar{P}_{ij,k}; \quad \forall ij \in \Phi_L \tag{42}$$

$$Q_{ij} = \sum_{k \in \Phi_S} \bar{Q}_{ij,k}; \quad \forall ij \in \Phi_L \tag{43}$$

$$\bar{P}_{ij,k} = P_{ij,k} - x_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{44}$$

$$-H \times u_{ij,k} \le \bar{P}_{ij,k} \le H \times u_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{45}$$

$$-H\left(1-u_{ij,k}\right) \le x_{ij,k} \le H\left(1-u_{ij,k}\right); \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{46}$$

$$\bar{Q}_{ij,k} = Q_{ij,k} - y_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{47}$$

$$-H \times u_{ij,k} \le \bar{Q}_{ij,k} \le H \times u_{ij,k}; \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{48}$$

$$-H\left(1-u_{ij,k}\right) \le y_{ij,k} \le H\left(1-u_{ij,k}\right); \quad \forall ij \in \Phi_L, \forall k \in \Phi_S \tag{49}$$

$$P_{ij}^2 + Q_{ij}^2 \le \sum_{k \in \Phi_S} m_{ij} \times I_{\max,k}^2 \times u_{ij,k}; \quad \forall ij \in \Phi_L \tag{50}$$

$$U_i^{\min} \le U_i \le U_i^{\max}; \quad \forall i \in \Phi_N \tag{51}$$

$$\sum_{\forall ij \in \Phi_T} u_{ij,k} = N_{\Phi_T} u_k; \quad \forall k \in \Phi_S \tag{52}$$

$$\sum_{k \in \Phi_S} u_{ij,k} = 1; \quad \forall ij \in \Phi_L \tag{53}$$

$$\sum_{ij \in \Phi_L} \sum_{k \in \Phi_S} K_{0,k} \times u_{ij,k} \times L_{ij} \le K \tag{54}$$

$$\begin{aligned} &u_{ij,k} \in \{0,1\}; \quad \forall ij \in \Phi_L; \forall k \in \Phi_S \\ &u_k \in \{0,1\}; \quad \forall k \in \Phi_S \end{aligned} \tag{55}$$

III. RESULTS AND DISCUSSIONS

This section applies the proposed MIQCP model to optimally select conductor cross-sectional area for the IEEE 33-bus distribution grid, the IEEE 85-bus distribution grid, and the Vietnamese real-world 102-bus distribution network. The developed MIQCP model is implemented by leveraging the available commercial solver CPLEX [17] with the GAMS programming language on a 2.6-GHz Intel Core i7 8850H computer and 16 GB of RAM. The optimal gap representative of the solution accuracy is set equal to 0%. Ten conductor sizes are considered in this study. Characteristics of these conductor sizes are shown in Table 2. The numerical values of parameters utilized for case studies are presented in Table 3.

A. IEEE 33-bus Distribution Grid

The structure of the IEEE 33-bus distribution grid [18] is depicted in Figure. 1, in which the main feeder is drawn in red. The calculation time of the proposed optimization model for this electrical distribution system is 3 minutes and 48 seconds.

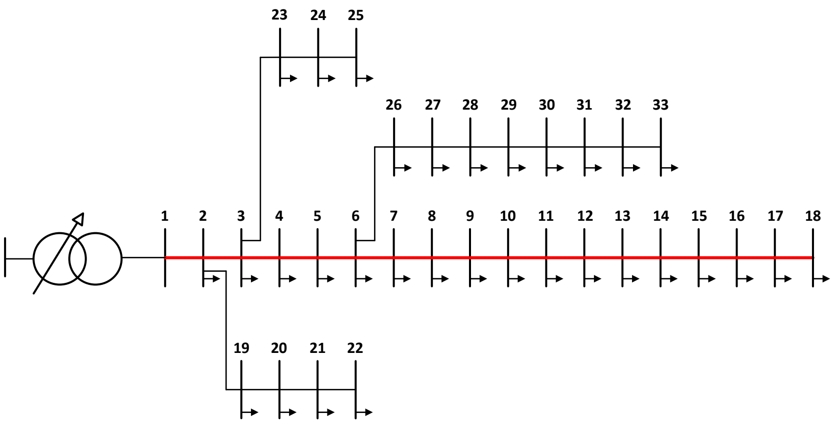


Figure. 1: IEEE 33-bus power network with main feeder

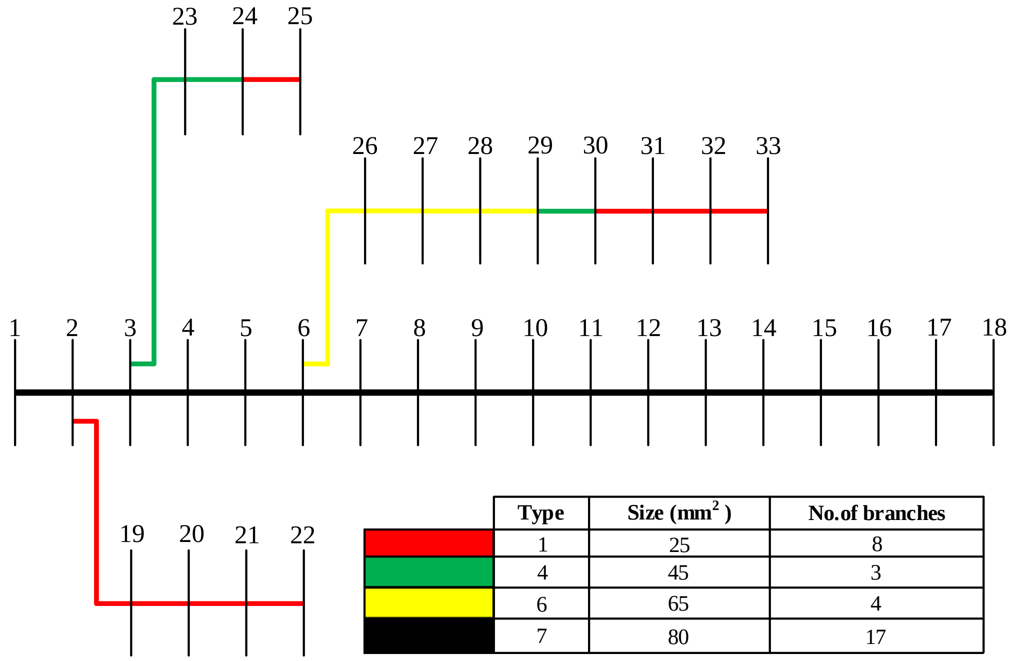


Figure 2: Optimal conductor size for IEEE 33-bus system

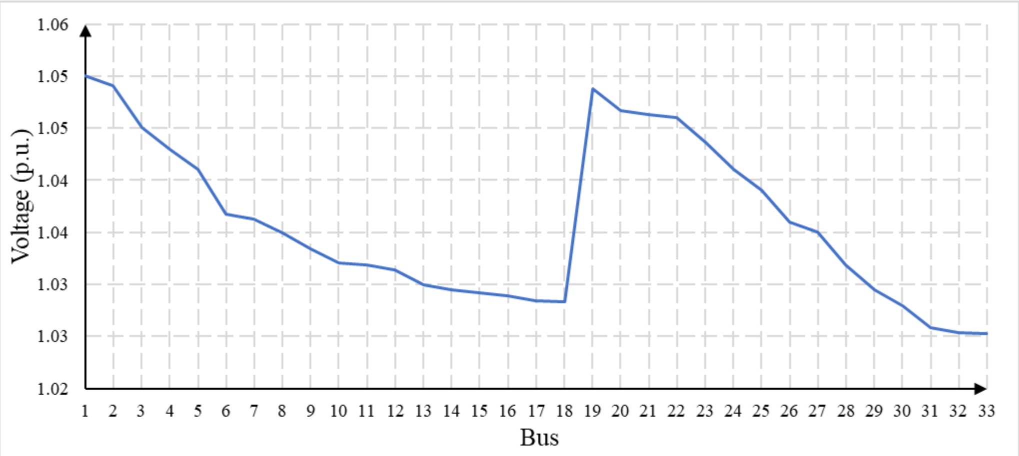


Figure. 3: Bus voltage profile for IEEE 33-bus system

Table 2: Attributes of considered conductor types [19]

Conductor type	Size (mm^2)	R_0 (Ω/km)	X_0 (Ω/km)	I_{max} (A)	K_0 ($/km)
1	25	0.6795	0.339	175	340
2	30	0.5449	0.335	200	420
3	40	0.4565	0.353	250	500
4	45	0.3841	0.327	257	590
5	50	0.3434	0.328	270	770
6	65	0.2745	0.315	305	820
7	80	0.2193	0.282	395	1010
8	95	0.1844	0.266	425	1370
9	110	0.1589	0.261	470	1590
10	130	0.1375	0.256	510	1840

Table 3: Values for the tested parameters

Parameter	Value
Loss factor	0.2
The marginal cost of energy loss ($/MWh)	29
Line lifespan (years)	20
Discount rate (%)	7
Maintenance cost-related factor (%)	7
Lower bound of voltage (p.u.)	0.95
Upper bound of voltage (p.u.)	1.05
Operating voltage at main substation node (p.u.)	1.05

The optimal solution of the developed MIQCP formula for the IEEE 33-node network is:

- Minimal voltage is equal to 1.02526 p.u. (at bus 33);
- Maximal voltage is equal to 1.05 p.u. (at bus 1);
- Average voltage equals 1.035984 p.u.;
- The overall power loss is 42.9812 kW, accounting for approximately 1.157% of the total power demand;
- The initial capital expenditure is 15,822 $;
- The maintenance cost is 11,733.3 $;
- The energy loss-related expense is 23,135.1 $;
- The lifetime cost of the evaluated electricity network is 50,690.4$;
- The optimal cross-sectional areas of conductors are depicted in Figure 2;
- The nodal voltage profile is sketched in Figure. 3.

B. The IEEE 85-bus Distribution System

The structure of the IEEE 85-bus distribution grid [20] is depicted in Figure 4, in which the primary feeder is drawn in red. The calculation time of the proposed optimization model for this distribution grid is 15 minutes and 29 seconds.

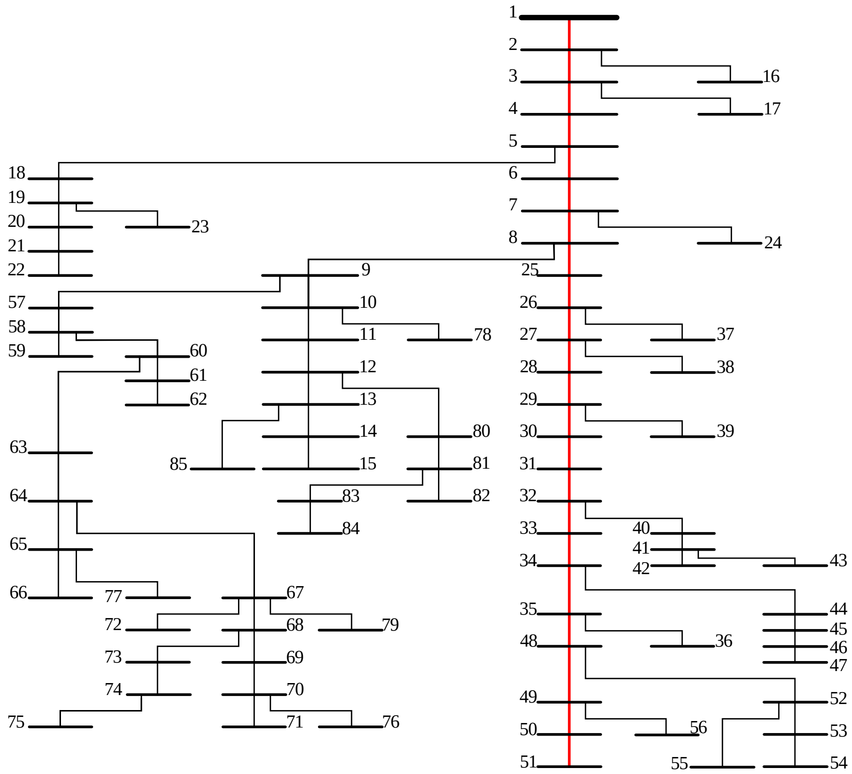
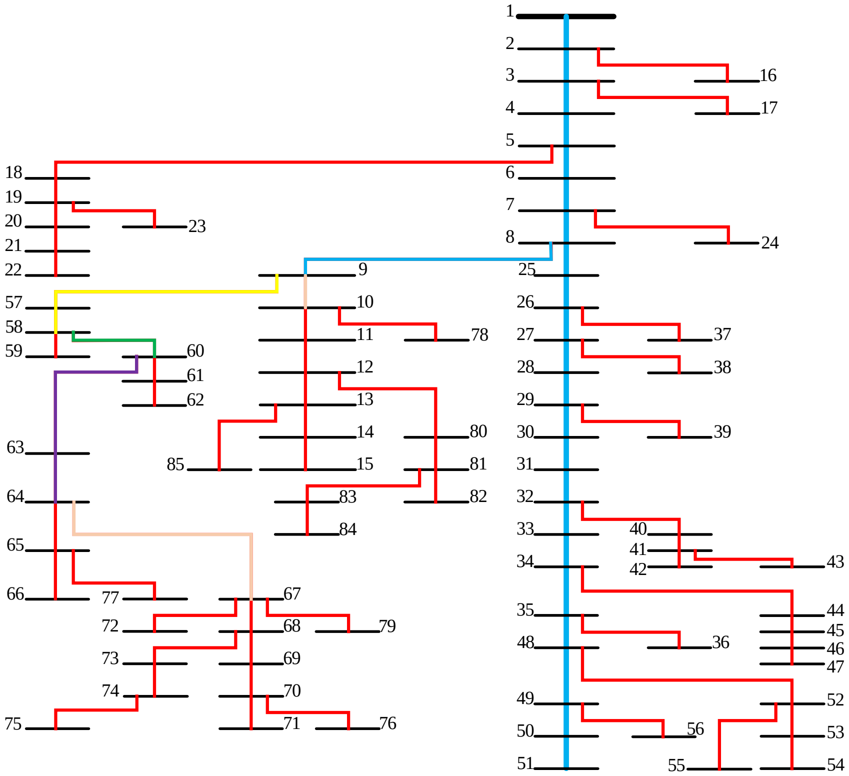


Figure 4: IEEE 85-bus power network with main feeder

The optimal solution of the developed MIQCP formula for the IEEE 85-node network is:

- Minimal voltage is equal to 0.991043 p.u. (at bus 75);
- Maximal voltage is equal to 1.05 p.u. (at bus 1);
- Average voltage equals 1.004548 p.u.;
- The overall power loss is 84.6227 kW, accounting for approximately 3.222% of the total power demand;
- The initial capital expenditure is 31,385.6 $;
- The maintenance cost is 23,275 $;
- The energy loss-related expense is 45,549.1 $;
- The lifetime cost of the evaluated electricity network is 100,210 $;
- The optimal cross-sectional areas of conductors are depicted in Figure 5;
- The nodal voltage profile is sketched in Figure 6.



Type	Size (mm^2)	No.of branches
1	25	54
2	30	2
3	40	2
4	45	1
6	65	2
10	130	23

Figure 5: Optimal conductor size for IEEE 85-bus system

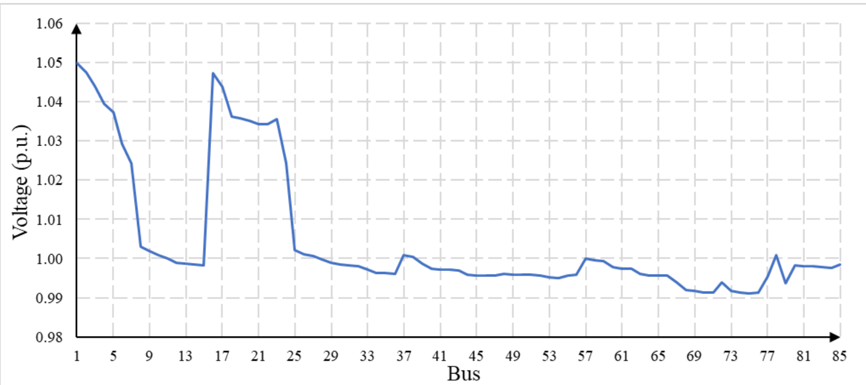


Figure 6: Bus voltage profile for IEEE 85-bus system

C. The Real-World 102-bus Distribution System in Vietnam

The structure of the real 102-bus distribution grid in Vietnam is depicted in Figure 7, in which the main feeder is drawn in red. The data for this real grid is described below:

- The nominal voltage is 35 kV;
- The total power of demand is 13.219 + *j*6.401 MVA;
- The line and load data are given in the Appendix.

The calculation time of the proposed optimization model for this realistic distribution network is 28 minutes and 46 seconds.

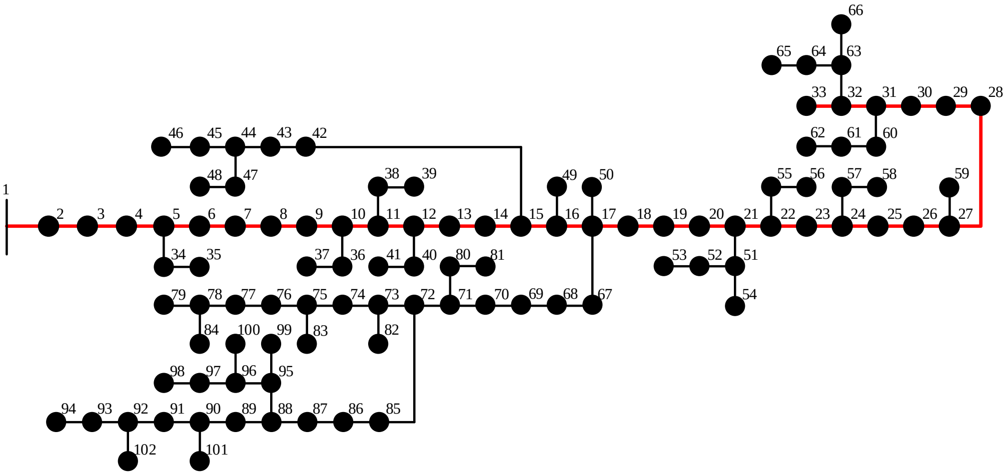


Figure 7: Realistic 102-bus power network with the primary feeder in Vietnam

The optimal solution of the developed MIQCP formula for the real 102-node power network is:

- Minimum voltage is equal to 1.02529 p.u. (at bus 94);
- Maximum voltage is equal to 1.05 p.u. (at bus 1);
- Average voltage equals 1.032029 p.u.;
- The overall power loss is 144.131 kW, accounting for approximately 1.09% of the total power demand;
- The initial capital expenditure is 56,054.8 $;
- The maintenance cost is 41,569.2 $;
- The energy loss-related expense is 77,580.1 $;
- The lifetime cost of the evaluated electricity network is 175,204 $;
- The optimal cross-sectional areas of conductors are depicted in Figure 8;
- The nodal voltage profile is sketched in Figure 9.

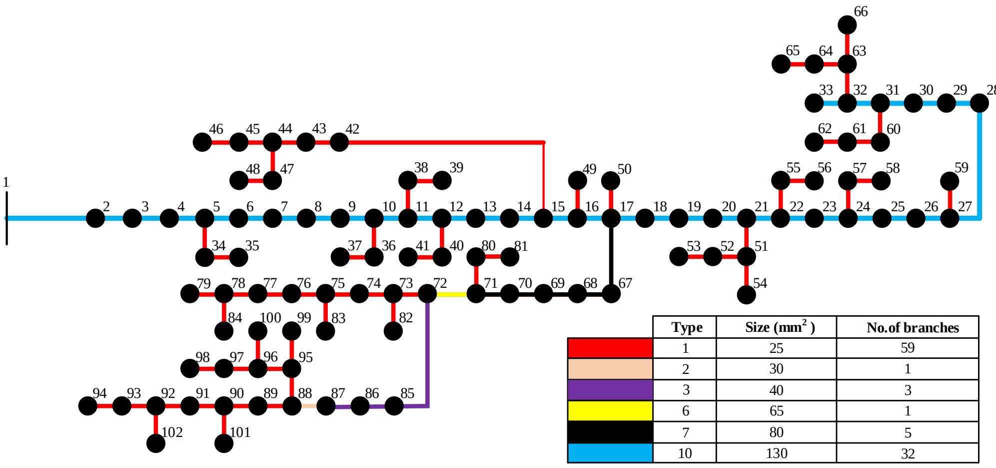


Figure 8: Optimal conductor size for Vietnamese real 102-bus system

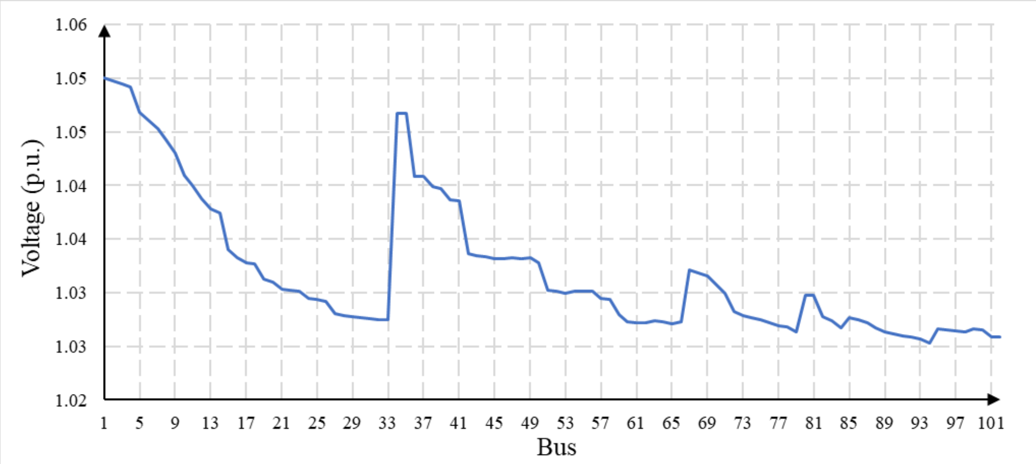


Figure 9: Bus voltage profile for the real 102-bus system

IV. CONCLUSION

This paper proposes the MIQCP formulation that is transformed from the MINLP model, with the aim of optimum selection of the conductor cross-sectional area for power distribution systems. This optimization model aims to minimize the lifetime cost of the electrical network while satisfying power balance equations, voltage magnitude bounds, branch current ratings, the constraint of the main feeder with the same conductor size, and the investment budget constraint. The globally optimal solution of the constructed MIQCP model can be effectively attained using available commercial solvers. The three power distribution systems with different sizes, including 33 buses, 85 buses, and Vietnamese real-world 102 nodes, are deployed to evaluate the established optimization model. The calculation results show that the proposed MIQCP model in this study is both effective and well-suited for practical applications.

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APPENDIX

See Table 4.

Table 4: Data for branches and loads of the realistic 102-bus distribution grid in Vietnam

No.	From bus	To bus	Length (m)	Demand at receiving node	
				P (kW)	Q (kVAr)
1	1	2	100	405	196.1
2	2	3	100	129.6	62.8
3	3	4	100	291.6	141.2
4	4	5	900	0	0
5	5	6	300	81	39.2
6	6	7	300	453.6	219.6
7	7	8	400	145.8	70.6
8	8	9	500	145.8	70.6
9	9	10	900	0	0
10	10	11	400	0	0
11	11	12	600	0	0
12	12	13	500	60.75	29.4
13	13	14	200	0	0
14	14	15	1800	0	0
15	15	16	460	0	0
16	16	17	340	0	0
17	17	18	180	145.8	70.6
18	18	19	2300	145.8	70.6
19	19	20	650	60.75	29.4
20	20	21	1100	0	0
21	21	22	250	0	0
22	22	23	300	81	39.2
23	23	24	1500	0	0
24	24	25	180	0	0
25	25	26	650	259.2	125.5
26	26	27	3500	145.8	70.6
27	27	28	400	60.75	29.4
28	28	29	500	60.75	29.4
29	29	30	450	202.5	98.1
30	30	31	200	0	0
31	31	32	900	0	0
32	32	33	500	202.5	98.1
33	5	34	400	145.8	70.6
34	34	35	100	202.5	98.1

No.	From bus	To bus	Length (m)	Demand at receiving node	
				P (kW)	Q (kVAr)
35	10	36	100	453.6	219.6
36	36	37	100	453.6	219.7
37	11	38	520	145.8	70.6
38	38	39	620	324	156.9
39	12	40	200	510.3	247.1
40	40	41	100	453.6	219.6
41	15	42	500	145.8	70.6
42	42	43	300	202.5	98.1
43	43	44	100	0	0
44	44	45	900	202.5	98.1
45	45	46	100	202.5	98.1
46	44	47	600	202.5	98.1
47	47	48	500	202.5	98.1
48	16	49	300	259.2	125.5
49	17	50	400	81	39.2
50	21	51	800	0	0
51	51	52	800	0	0
52	52	53	900	202.5	98.1
53	51	54	400	81	39.2
54	22	55	800	145.8	70.6
55	55	56	300	81	39.2
56	24	57	200	81	39.2
57	57	58	2200	40.5	19.6
58	27	59	500	145.8	70.6
59	31	60	1700	129.6	62.7
60	60	61	700	60.75	29.4
61	61	62	1100	81	39.2
62	32	63	300	0	0
63	63	64	700	60.75	29.4
64	64	65	950	202.5	98.1
65	63	66	800	202.5	98.1
66	17	67	500	145.8	70.6
67	67	68	200	145.8	70.6
68	68	69	300	324	156.9
69	69	70	760	81	39.2
70	70	71	700	0	0
71	71	72	1500	0	0
72	72	73	500	0	0
73	73	74	200	129.6	62.8
74	74	75	400	0	0
75	75	76	400	60.75	29.4
76	76	77	550	324	156.9
77	77	78	600	0	0
78	78	79	3300	202.5	98.1
79	71	80	300	60.75	29.4
80	80	81	100	526.5	254.9
81	73	82	600	145.8	70.6
82	75	83	300	129.6	62.8
83	78	84	700	202.5	98.1
84	72	85	600	145.8	70.6
85	85	86	200	0	0
86	86	87	300	202.5	98.1
87	87	88	500	0	0
88	88	89	700	81	39.2
89	89	90	300	0	0
90	90	91	500	24.3	11.8
91	91	92	300	0	0

No.	From bus	To bus	Length (m)	Demand at receiving node	
				P (kW)	Q (kVAr)
92	92	93	620	60.75	29.4
93	93	94	1900	324	156.9
94	88	95	100	0	0
95	95	96	360	0	0
96	96	97	500	81	39.2
97	97	98	650	202.5	98.1
98	95	99	300	324	156.9
99	96	100	400	129.6	62.8
100	90	101	1000	324	156.8
101	92	102	1000	60.75	29.4