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Hybrid Multi Kernel Polar Code Construction and Optimized Decoding Algorithm for 5G Mobile Communications

Abstract: - 5G mobile communications faces difficulty in channel coding to satisfy user requirements. This paper evaluates flexibility, reliability and complexity for different channel coding schemes(Turbo codes, Low Density Parity check codes, mother polar codes, Multi Kernel-polar codes) on regular channel AWGN with BPSK modulation of code rate R=1/2. The proposed hybrid Multi Kernel-polar codes will have greater flexibility and reliability for short and large code words. The optimized decoding of Multi Kernel polar codes reduces the number of computations by using repeated LLR's of sub-kernel matrices.

Keywords: polar codes, Multi kernel polar codes, hybrid construction of polar codes, optimized decoding of polar codes.

I. INTRODUCTION:

Consumer-demand surges in technology development. The basic requirements in 5th generation are maximum transmission of data with highest speed, greater reliability and minimum latency. As a consequence, 5G technologies can be used in applications such as smart homes, buildings and cities, 3D video, virtual and augmented reality. All these applications come under i) Ultra Reliable Low Latency (URLL) ii) enhanced Mobile Broad Band (eMBB) iii) massive Machine Type Communications (mMmtc).

In 5G coding technology, the over view of challenges and solutions are given in [1]. The best method to achieve aforementioned requirements is by using Polar codes at the channel coding in communication systems [2]. Different channel coding schemes like LDPC, Turbo Codes, Convolutuional codes and Polar codes through AWGN channel with BPSK modulation of code rate ½ is performed to analyze optimum channel coding technique for short length message transmission for machine type communication in 5G technology [3] and for LDPC, QC-LDPC, SC-LDPC, Polar codes, NBLDPC-Polar codes are shown in [4].

Multi kernel polar code construction was introduced in[5], they proposed any transformation matrix can be generated by using the sub-kernel's of size 2×2 and 3×3. Different sizes of block lengths are possible using multi kernel construction. By using density evolution, generation of 2×2 and 3×3 sub kernels made greater reliability for multi kernel polar code construction and it is checked for SC decoding algorithm [6]. Conventional reliability design can also be used to design sub kernels of different sizes. We can extend the sub kernel size to 5×5.

The development of systemic and non-systemic polar codes have made in this paper [7] by using only 3×3 sub-kernel, the code word length is multiples of 3 only. For other length we can elaborate by using 2×2, 3×3 and 5×5. In the generation of 5×5 sub-kernel, reliability design and distance designs are used, results a Hybrid MK-Polar construction.

Mother polar codes are decoded by using successive cancelation (SC) decoding algorithm and Successive cancelation list (SCL) algorithms. These algorithms use hard decision and soft decisions to get polarization. The received bits are decoded by computing Log Likelihood Ratio's. To reduce the computational complexity we propose a new decoding algorithm called optimized decoding algorithm. The optimized decoding algorithms have reduced computational complexity.

II. POLAR CODES:

The channel coding techniques which are used in 3G and 4G are turbo codes and Low Density Parity Check (LDPC) codes are not suitable for many 5G applications. One of the power full methods in channel coding to capacity achieve and Forward Error Correcting codes in future generation codes are Polar codes. Polar codes,

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proposed by Arikan are channel specific designs. That is one polar code for a channel may differ from another polar code for other channel. These polar codes show symmetric capacity for B-DMC. This results in controllability of the channels while data channels for the LDPC codes. The channel coding design for URLLC is to obtain BER 10^{-5} (reliability of 99.999%) with low latency of less than 1ms, no error floor and low complexity decoding method is polar codes.

III. POLAR CODES ENCODING:

Polar Codes, the best Error correction technique, implemented recently by Arikan are the vital code that could provide the top-notch capacity for more number of channels. Building of a polar code with length $N = 2^n$ is totally dependent on binary matrix's T_2 recursive concatenation that implies transformation of kernel. This entire method results in Matrix transaction of $T_N = T^{\otimes n}$, which is provided with T^2 Kronecker produce of n-fold and also that turns physical channel into synthetic virtual N channels that are given either by low or very high reliability. The polarization of channel also leads the way to full portion of reliable channels which in turn gives the binary-input memory less channels capacity, mainly at the time of infinity code length.

MK- polar code structure

MK-polar codes are derived from the basic arikans polar code structures[8] In MK-polar code structure, the transformation matrix can be constructed by using binary kernels of different sizes. $N \times N$ Transformation matrix can be defined as, a MK-polar code of length N and information bits K. The notation for the transformation matrix of order $N \times N$ is $T_N = T_{p_1} \otimes\ T_{p_2} \otimes \ldots\ldots T_{p_s}$ and N = $p_1\ p_2 \ldots\ldots p_s$, information set is defined as I , the number of message bits in code word is K and its complementary set is known as frozen set. Frozen set is defined as $= \frac{N}{I}$ [4]. Now the code rate is defined as the ratio of number of message bits transmitted to total number of bits transmitted i.e; $R = \frac{K}{N}$.

Construction of MK-Polar codes:

The conceptual model of MK-polar code is shown above. The input data U^N is linearly transformed into X^N by G_N *as* $X^N = U^N G_N$. Then X^N is transmitted over Binary- Discrete Memoryless Channel. The received signal Y^N is decoded using SC decoding algorithm. The resultant data is represented as $\widehat{U}^N$, and it is the estimated value of U^N [9].

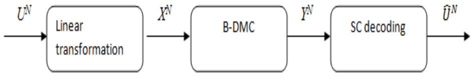

Figure.1 model of MK polar codes

In the construction of MK-polar codes, we are using kronecker products with different Kernel sizes. $G_N = B_N$. $G_N = B_N \cdot F_{l_1} \otimes F_{l_2} \otimes \ldots . \otimes F_{l_n}$ Where $F_{l_\lambda} (\lambda = 1,2 \ldots n)$ denotes the λ^{th} kernel with a dimensions of l_λ and B_N is a permutation matrix. The tanner graph for MK-polar code is constructed by using the equations and the graph is shown in fig2.

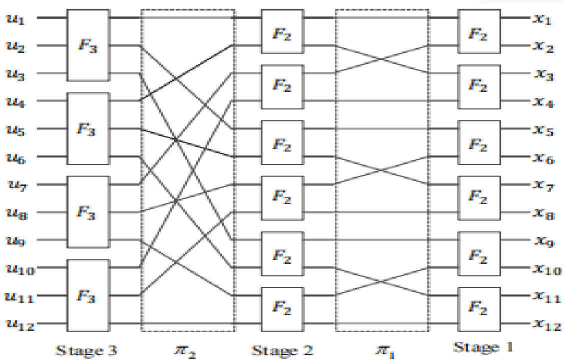

Fig 2: tanner graph for 12-bit polar code

The tanner graph for MK-polar code is constructed by using the equations and the graph is shown above. For the function F_3 the size of the kernel is 3×3 is represented as $F_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, for 2×2 it is $F_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

In the same manner 5×5 can be designed as $F_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

The MK-polar code structure can be shown (depicted) by tanner graph as shown in fig 2 and transposed matrix shown in fig 3.From the figure it is clear that, the N number of input bits u_i and N number of output bits x_i are related by using transformation matrixT_N. In the kroneker product, each stage is counted from left to right in the transformation matrixT_N. The vertically distributed, each stage i has $\frac{N}{p_i}$ number of boxes. Edge permutation vectors describe the input and output of consecutive stages, which are connected by edges [10, 11]. From the structure of basic polar codes, edge permutations are defined by the Kroneker product. Stage i and $i-1$ are connected by π_i , where $i = 2{,}3 \dots s$. Stage -1 ‘s output j is connected to the input $\pi_i(j)$ of stage i. For example for the first stage, from right to left i.e; π_1 connects the encoded bits at the stage 1. Code word at each stage is defined as $N_i = \prod_{j=1}^{i-1} p_j \ where \ i = 2{,}3 \dots s$, assumed that for N_{i+1}entries, $\frac{N}{N_{i+1}}$blocks for each π_i stage. At each stage, each block is permutated independently and these permutations are calculated from LLR’s. These permutations are defined by $\rho_i$with respect to $N_i \times p_i$ block interleaver. From [3], it is noted that$\pi_s = p_s$. The entries of each block π_i is shifted by multiples of N_{i+1}. for the first stage permutations are obtained by interleaving the product of other permutations and these are represented as $\pi_1 = (\pi_2 \dots \pi_s)^{-1}$ and it is just like bit reversal permutations of polar codes.

$$T_{12} = \left(\begin{array}{ccc:ccc|ccc:ccc}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hdashline
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
\hdashline
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1
\end{array}\right),$$

Figure 3: Transposed matrix of size 12X12

Hybrid Polar code Construction:

Reliability: The channel coding scheme used in 5G technology should reliable for variable code word lengths. Reliability can be measured in BER. The suitable channel coding in 4G technology is Low Density Parity Check [LDPC] codes. LDPC codes are perfect channel coding for large code words. In URLL scenarios, polar codes have greater reliability when compared with LDPC codes.

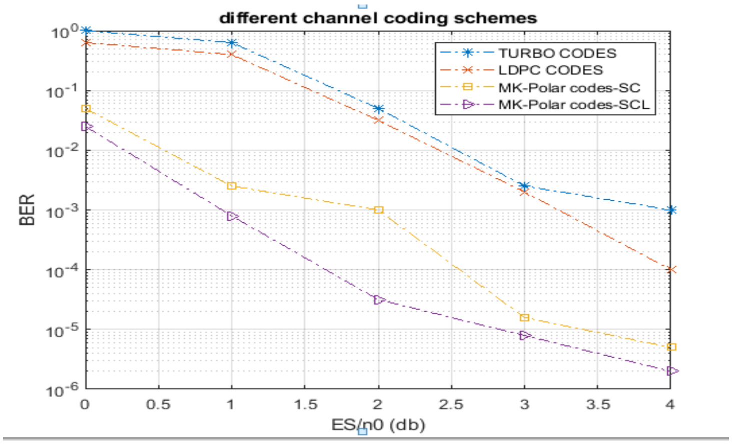

Figure 4.different channel coding schemes with N=512 and R=1/2

The above figure shows, the comparison of different channel coding schemes with code word length N=512. It is evident from the above that, turbo codes doesn't have error floor capacity for large length. LDPC codes have BER at 10^{-4} at R=1/2. Here, the channel used for all the channel coding schemes is AWGN channel, code rate R=1/2, code word length N=512 and the modulation scheme used was BPSK.

Reliability of the channel coding scheme can be improved by using multi kernel polar codes. MK-Polar codes have BER 10^{-5} to 10^{-6} for SC and SCL decoding algorithms. This reliability further can be improved by changing the positions of sub-kernel matrices.

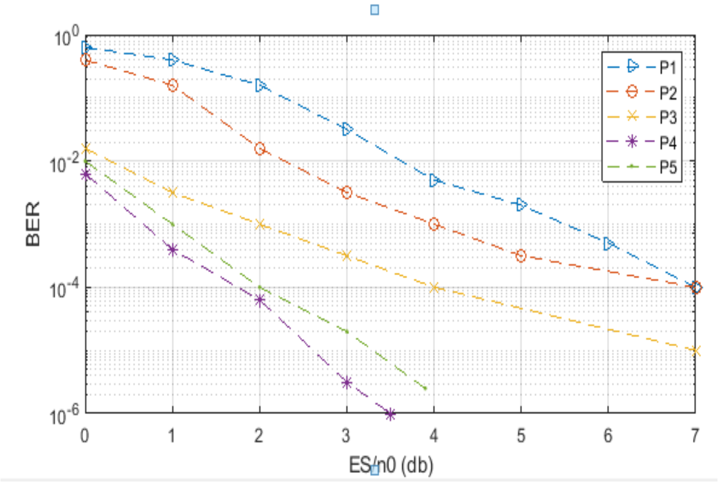

Figure 5. Reliability in the design of polar codes of N=640

The above figure shows that the **reliability design** of polar codes, for code word length N=640, sub matrices 5×5, 2×2, 2×2, 2×2, 2×2, 2×2, 2×2 and 2×2. The position of 5×5 matrix is varied from 1st position to 5th position. It is observed that, for initial positions are applicable for weaker signal strength, where as tail end positions are preferable for strong signal strengths.

Distance design can be used to make the positions of 0's and 1's in sub-kernel matrices. After deciding the positions of 0's and 1's and the order of sub-kernels, hybrid design of polar codes will be constructed. That is the combination of reliability design and distance design are called as **hybrid design of polar codes**.

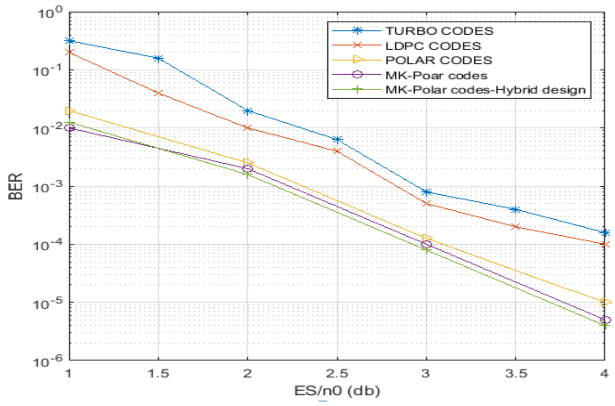

Figure 6: BER performance of hybrid design polar codes

The analysis of hybrid design of polar codes with different channel coding was presented in the above figure. Hybrid design polar code with code word length N=640, R=1/2 have BER performance of 10^{-5} to 10^{-6} was achieved.

Flexibility: Services and applications of 5G technologies require variable data block lengths in the transmission. It yields; the channel code should support multiple code rates. With respect to different code words, different sixes of sub-kernel matrices are generated. As an example, code word length=30, the size of transformation matrix is 30, which is generated by sub-kernels of sizes 2×2, 3×3 and 5×5.

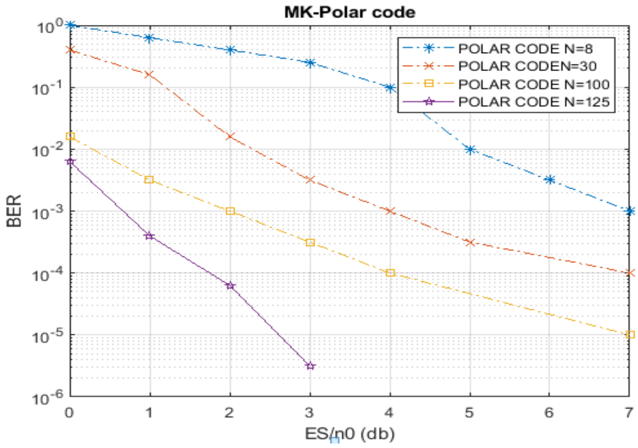

Figure 7: Generation of MK-polar code for different N values

Any code word will be generated by using 2×2, 3×3 and 5×5 matrices. There is flexibility in the generation of variable code words. If the length of the code word is multiples of 5×5, then achieved BER is 10^{-5} to 10^{-6}. Here Arikan proposed basic successive cancelation algorithm was used.

IV. POLAR CODES DECODING

Successive Cancelation (SC) algorithm makes an assumption that its previous bit decoded successfully. Let a channel have input and outputs are X and Y respectively with input vector U. the bit u_i can be estimated by using log likely hood ratio (LLR) as $L_{(n)}^{i} \triangleq \ln(\frac{W_n{}^{(i)}\left(y, \widehat{u_0}{}^{i-1}\middle|0\right)}{W_n{}^{(i)}(y,\widehat{u_0}{}^{i-1}|1}$ $i \in N$. The LLRs are recursively updated in each nodes of the factor graph. The LLRs for the different nodes are determined by $L_s{}^{(2i)} = f(L_{s-1}{}^{(2i-i \bmod 2^{s-1}}, L_{s-1}{}^{2^s+2i-(\bmod 2^{s-1}})$

$L_s{}^{(2i+1)} = g(L_{s-1}{}^{(2i-i \bmod 2^{s-1}}, L_{s-1}{}^{2^s+2i-(\bmod 2^{s-1}}, U_s{}^{2i})$ s=n ,n-1……0

Optimized polar code decoding :

To decode MK-Polar codes Successive Cancelation (SC) decoding algorithm will be employed. The same is applied for kernel substitution of MK polar codes. Decoding procedure starts with the end of encoding of signal flow graph. In the kernel substitution method we used repeated sub-kernels speed up the encoding method. The same procedure can also be applied for the decoding as well. The number of LLR's is reduced to a large sub-kernel matrix order. Hence the complexity in decoding gets reduced, which reduces the LLR Storage. Here initial, intermediate, final LLR calculations are not needed. The complexity can be determined from $RL\{N\,log_2{}^{(N)}$ + (N-1)}+K.2L $log_2{}^{(2L)}$.For the kernel of size 2× 2, the decoding equations are

$$\boldsymbol{f_0{}^2: \lambda_0 = L_0 \oplus L_1} \quad , f_1{}^2: \lambda_1 = (-1)^{\widehat{u_0}}$$

For the kernel of size 3× 3, the decoding equations are

$$f_0{}^3: \lambda_0 = L_0 \oplus L_1 \oplus L_2$$

$$f_1{}^3: \lambda_1 = (-1)^{\widehat{u_0}} L_0 + L_1 \oplus L_2$$

$$f_2{}^3: \lambda_2 = (-1)^{\widehat{u_0}} L_1 + (-1)^{\widehat{u_0+u_1}} L_2$$

For the kernel size 5 × 5 the decoding equations as follows

$$f_0{}^5: \lambda_0 = L_1 \oplus L_2 \oplus L_4$$

$$f_1{}^5: \lambda_1 = (-1)^{\widehat{u_0}}. L_0 \oplus L_3 \oplus (L_2 \oplus (L_1 \oplus L_4))$$

$$f_2{}^5: \lambda_2 = (-1)^{\widehat{u_1}} (L_0 \oplus L_1) + (L_3 + L_4)$$

$$f_3{}^5: \lambda_3 = (-1)^{u_0 \widehat{\oplus u_1} \oplus u_2} L_0 + (-1)^{\widehat{u_0}} L_1 \oplus (L_2 \oplus (L_3 \oplus L_4))$$

$$f_4{}^5: \lambda_4 = (-1)^{\widehat{u_0 \oplus u_3}}. L_2 + (-1)^{\widehat{u_0 \oplus u_2}} L_3 \oplus (-1)^{\widehat{u_0}} L_4$$

Complexity:

In 5G technology, the advanced channel coding technique with robust error protection and low complexity in encoding and decoding preferred. Turbo codes used in 3G technology have less encoding complexity and high decoding complexity. Turbo codes uses I_{max}.16.N.S number of iterations to decode code word of length N. S is number of memories required. In 4G technology LDPC codes are used with high complexity encoder and low complexity decoder. LDPC codes uses Imax .(2.N. d_v +2J) number of iterations to encode a code word of length N, minimum distance d_v , J number of parity bits. In 5G technology polar codes are used at channel coding. MK-polar code with hybrid design gives effective BER performance around 10^{-6} with low complexity in encoding. To decode polar codes using successive cancelation algorithm, it uses N.$\log_2$ (N) number of computations.

To reduce decoding complexity in polar codes, optimized coding technique proposed. In this algorithm Log Likely-hood Ratio's [LLR's] are computed once for each sub-kernel. Their values were repeatedly used for the remaining bits. Hence, the number of LLR's for decoding of bits is reduced. The number of computations used to decode optimized polar odes is $N_5\ \log_2 N_5 + N_2\ \log_2 N_2 + N_3\ \log_2 N_3$. Computational complexity in optimized polar codes is reduced greatly.

V. SUMMARY:

the hybrid code construction of MK-polar codes has great flexibility in the construction of short and large messages. The hybrid construction method also gives BER of 10^{-5} to 10^{-6} achieved. The optimized decoding algorithm uses a reduced number of LLR's compared with mother polar codes.

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