Sizing a High Speed PM Generator for Green Energy Applications

This paper presents a step by step sizing procedure of High Speed Permanent Magnet Synchronous Generators (HSPMSGs) for green energy applications to be driven by micro-turbines. The final design offers significant reductions in both weight and volume in a power range of 5:500 Kw. A rotor length to diameter ratio is used as an important design parameter. The results are depicted by 3D plot figures for a number of machines sizing. The simulation of generators sizing is performed using MATLAB.

Keywords: Permanent Design, micro-turbines, High-speed, Permanent Magnet Generators, Size, Synchronous.

1. Introduction

Recently, more attention has been paid to the development of high speed PM generators driven by micro-turbines, as prime movers with local conversion at load points [1]. For high-speed applications, the rotor aspect ratio, defined as length-to-diameter, is a critical parameter. Stator core losses may be minimized by using laminated steel in stator construction and by not generating frequencies that are too high. The main applications of PMSG are for power generation as part of renewable energy resources and main generators for aircraft, etc. [2] – [9]. The sizing of HSPMSG design must address system topology for good power/volume, low cost, and superior efficiency [1-14]. The influence of the choice of stator lamination material on iron loss in a high speed, high power, permanent magnet generator is investigated. This paper presents general sizing of HSPMSGs for the power range of 5:500 Kw, with various tip speeds of 50:250 m/s.

2. Basic Selections

2.1. Permanent Magnets

The rare earth magnets, SmCo and NdFeB, have become popular because of their greater power density, high Coercivity, high flux densities, and the linearity of their demagnetization curves [13] and [15]. NdFeB is preferred because it is cheaper and more readily available. Therefore, NdFeB magnets are selected for use in PMG, with some conservatively assumed values [14].

2.2. Stator and Rotor Material

The rotor is usually built from the same material as the stator for ease of construction, but it can be made of any economical steel, provided it is strong enough for its function [13], [16]. TM19, 29 gauge electrical silicon steel is selected for the PMG because it is economical, its thin laminations minimize power losses due to the circulating eddy current.
and because it has a saturation flux density of about 1.8 T [13], [14].

3. Machine Design Parameters

3.1. Stator Mechanical Design

The stator is an important part of the machine because it serves as the main structural component, it provides the housing for the armature windings, and it completes the flux path for the magnetic circuit. Slotted stators are the traditional stator design and consist of openings around the stator for the armature windings, as shown in Figure 1 (a).

![Stator Design Diagram](image)

(a)

(b)

Fig. 1: (a) Slotted stator design; (b) and stator slot geometry.

In this paper, the slots are trapezoidal, but assumed to be approximately rectangular, as shown in Figure 1 (b). They contain form-wound windings so that the depression width is the same as the top slot width. Slotting is used because of its advantages, such as the achievement of a narrow air gap length to maximize the flux linkage, the increase in surface contact area between the windings, and a path of low thermal resistance, provided by stator steel for good heat conduction [14]. The initial design of the generator assumes a three-phase machine. Also, a 36 slots machine is chosen for the initial generator design [13].

3.2 Rotor Mechanical Design

The surface mounted permanent magnets in the rotor as shown in fig. 2 are selected here due to its suitability for high speed applications.
For high-speed applications, the rotor aspect ratio, defined as length-to-diameter (L/D), is a critical parameter. PM machines offer flexibility in selecting pole sizes, which allows for smaller diameters. A normal L/D ratio for a wound rotor machine is 0.5 – 1.0, compared to 1 – 3 for a PM machine [17]. So, here it is selected to be 2.5. The rotor radius and rotational speed also determine the tip speed of the machine, which is the surface velocity of the rotor.

$$v_{tip} = r \omega_m$$  \hspace{1cm} (i)

where $\omega_m = \text{angular speed (rad/sec)}$, $r = \text{rotor radius (m)}$

The upper limit on tip speed is between 100-250 m/s, depending on the design of the machine. In this design, a range of tip speed is taken to be (50:250).

### 3.3 Number of Poles and Magnets Pole Design

An even number of poles is always used, here $P = 3$, because this provides a balanced rotational design. Assuming a constant mechanical rotation speed, electrical frequency is given as.

$$N(2P) = 120f$$  \hspace{1cm} (2)

where $N = \text{speed (rpm)}$, $P = \text{pole pairs}$, $f = \text{electrical frequency (Hz)}$

If a PM generator is going to be the source of DC bus through a rectifier system, a high pole number is desirable because as the electrical frequency increases, support components such as filter capacitors and inductors can be much smaller. Therefore, for a given rotational speed, one cheap and efficient solution is to have a higher number of pole pairs and frequency. However, as the frequency increases, higher stator losses result because core losses are proportional to frequency squared. In addition, as the pole number gets larger, the number of slots per pole per phase decreases and can cause the voltage waveforms to become less sinusoidal so all factors must be considered. The pole arc of the magnets can also be varied. Magnets seldom span the full pole pitch because the flux at the transition between north and south poles leaks between poles without linking the coils in the stator. The gaps between the poles usually contain non – magnet pieces, such as soft iron, so that
no flux crosses over the air gap between magnets. All full pole arc is $\theta_{me} = 180^\circ$ and produces a full voltage waveform but has increased harmonic content. As the pole arc is reduced (up to 20 – 30 %) and those areas are filled in with soft – iron pieces, the resulting flux waveform is more sinusoidal and has fewer harmonics and therefore lower rotor losses. The magnet poles are sometimes skewed to reduce cogging torque and smooth out variations in air gap reluctance, flux, and voltage waveforms. Skewing of magnets occurs axially along the length of the rotor to provide a constant rotational torque and prevent pole pieces from exactly lining up with stator teeth. Magnet poles skew factor is selected to reduce cogging torque and smooth out variations in air gap reluctance, flux, and voltage waveforms.

$$k_m = \frac{\sin(n \theta_s)}{\theta_s/2}$$

where $\theta_s$: Skew angle, rad E; n: Harmonic number

### 3.4 Magnetic Dimensions

The magnetic dimensions that affect a PM machine are air gap and magnet height. The air gap flux density ($B_g$) can be represented by Equation 4. The radial air gap is made as small as possible to maximize the air gap flux density, minimize the flux leakage, and to produce a lower reluctance value.

$$B_g = \frac{h_m}{h_m + g} B_r$$

where $h_m$: Magnet height (mm); g: Air gap (mm); $B_r$: Magnet Remnant Flux Density (T)

Magnets losses are reduced, using smaller magnets. For uniform magnetic fields, the magnet height is usually larger than the air gap, by a factor 5 – 10.

### 3.5 Slots per Pole, Per Phase

Three-phase machines are typically used in this study as the standard choice for most motors and generators. Another important design parameter is the number of slots per pole, per phase (m), as in Equation 5.

$$m = \frac{N_s}{2 * p * q}$$

Varying the number of slots/pole/phase is used to produce a more sinusoidal voltage waveform and reduce machine harmonics.

### 3.6 Stator Windings

The pitch of a winding ($\alpha$) refers to the angular displacement between the sides of a coil. The breadth of a stator winding results from the coils occupying a distribution of slots within a phase belt. In smaller machines, coils are composed of round insulated wires that are placed in the stator slot, along with insulation material. A slot fill factor ($f_s$) is used to determine how much of the slot’s cross-sectional area is occupied by winding material, as in Equation 6.
\[ \lambda_s = \frac{\text{Winding Area}}{\text{Total Slot Area}} \]  

Typically, machines contain two coils sides per slot, making the winding a double-layer design [13]. Overall, slot fill factors vary in value from 0.3 – 0.7, depending on the number and size of the conductors in the slots, as well as the amount of labor utilized. In this paper, a slot fill factor of 0.5 is assumed. Almost all machines use series, wye – connected windings because they provide the safest alternative. Therefore, wye series connected windings are selected for use in the designs in this study.

3.7 Machine Calculated Parameters

Each phase of the machine is modeled, as shown in Figure 3.

![Diagram of a per phase electrical model](image)

Fig. 3: A Per Phase electrical model.

where: \( R_a \): Armature resistance; \( L_a \): Synchronous inductance; \( E_a \): Back e.m.f voltage and \( V_a \): Terminal voltage.

3.8. Winding Resistances

The resistance of copper phase windings is calculated in Equation 7

\[ R_a = \frac{l}{\sigma * A_{ac}} \]  

where \( l \): length of conductor; \( \sigma \): winding conductivity; \( A_{ac} \): winding cross – sectional area

\[ A_{ac} = \frac{A_s * \lambda_s}{2 * N_C} \]  

where \( A_s \): slot Area, \( N_C \): turns per coil

But the above stator resistance equation may be used as in low frequencies applications, so it has to be developed. Since the machine rotates at high speed, and high frequency and so the skin depth may be affected. In conductors that carry high frequency currents, skin effect can become an issue and affect the operation of the machine. Skin effect is caused by eddy currents in the windings themselves due to the changing magnetic field. These eddy currents force the current flowing in the conductor to crowd to the outer edges of the conductor. This in turn causes the current to flow through a smaller cross – sectional area and increase the resistance of the conductor.

It is well known that, when conductive material is exposed to an ac magnetic field, eddy currents are induced in the material in accordance with Lenz’s law. The power loss resulting from eddy currents which can be induced in the slot conductors appears as an
increased resistance in the winding. To understand this phenomenon, let us consider a rectangular conductor as shown in fig. 4. The average eddy current loss in the conductor due to a sinusoidal magnetic field in the y direction is given approximately by Hanselman [12].

![Rectangular conductor geometry](image)

Fig.4: Rectangular conductor geometry.

\[ P_{ec} = \frac{1}{12} \sigma L \omega_c h^3 \omega^2 u_0^2 H_m^2 \]  \hspace{1cm} (9)

where \( H_m \): the turn field intensity value; \( u_0 \): permeability of free space.

Since skin depth is defined as

\[ \delta = \frac{2}{\sqrt{\omega \mu_0 \sigma}} \] \hspace{1cm} (10)

Equation (9) can be written as

\[ P_{ec} = \frac{L \omega_c h^3}{6 \sigma \delta^4} H_m^2 \] \hspace{1cm} (11)

Using this expression it is possible to compute the ac resistance of the slot conductors. If the slot conductors are distributed uniformly in the slot, and substituting the field intensity into eq. (11) and summing over all ns conductors gives a total slot eddy current loss of

\[ P_e = \frac{d_s L h^2 n_s^2 I^2}{9 \sigma \delta^4 \omega} \] \hspace{1cm} (12)

where \( I \) is the rms conductor current; \( \omega_s \): Slot width(m); \( d_s \): Slot depth(m)

The slot resistance of a single slot containing \( n_s \) conductors connected in series is

\[ R_{st} = \frac{\rho n_s^2 L}{k_{sp} \omega_s d_s} \] \hspace{1cm} (13)

where L: the slot length; \( k_{sp} \): the conductor packing factor, is the ratio of cross sectional area occupied by conductors to the entire slot area and \( \rho \): electrical resistivity (\( \Omega \cdot m \)).

Using eq. (13), the total slot resistance can be written as

\[ R_{st} = R_{st} + R_{ec} = R_{st} (1 + \Delta_e) \] \hspace{1cm} (14)
In this equation, \( \Delta_e = R_{se}/R_{sl} \) is the frequency-dependent term. Using eq. (13) and eq. (12), this term simplifies to

\[
\Delta_e = \frac{R_{se}}{R_{sl}} = \frac{1}{9} \left( \frac{d_s}{\delta} \right)^2 \left( \frac{h}{\delta} \right)^2
\]

(15)

This result shows that the resistance increases not only as a function of the ratio of the conductor height to the skin depth but also as a function of the slot depth to the skin depth. Thus, to minimize ac losses, it is desirable to minimize the slot depth as well as the conductor dimension. For a fixed slot cross-sectional area, this implies that a wide but shallow slot is best.

3.9 Winding and Magnet Factors

Winding are short-pitched and have breadth associated with them. To account for these effects, a winding factor \((k_w)\) is utilized, as in Equation 16.

\[
k_{wn} = k_{pn} * k_{bn}
\]

(16)

Short-pitching is an important means for eliminating harmonics and improving the power quality of the machine. The pitch factor is shown in Equation 17.

\[
k_{pn} = \sin \left( \frac{n * \alpha}{2} \right) * \sin \left( \frac{n * \pi}{2} \right)
\]

(17)

The breadth factor explains the effect of the windings occupying a distribution or range of slots within a phase belt. The breadth factor is derived in Equation 18.

\[
k_{bn} = \frac{\sin \left( \frac{n * m * \gamma}{2} \right)}{m * \sin \left( \frac{n * \gamma}{2} \right)}
\]

(18)

where \(m\): slots per pole per phase; \(\gamma\): coil electrical angle

The magnetic flux factor equation [12], for the slotted stator and surface magnet configuration is shown in Equation 19.

\[
k_{pn} = \frac{R_i^{np-1}}{R_s^{2np} - R_s^{2np} \cdot \left( \frac{np}{np + 1} \right) \cdot (R_i^{np+1} - R_i^{np+1}) + \frac{np}{np - 1} \cdot R_s^{2np} \cdot (R_i^{1-np} - R_i^{1-np})}
\]

(19)

where \(R_s\): outer magnetic boundary, \(R_i\): outer boundary of magnet; \(R_c\): inner magnetic boundary, \(R_i\): inner boundary of magnet

3.10 Flux and Voltage

For useful voltage, only the fundamental components are used to determine the internal voltage (back e.m.f) of the generator, as shown in Equations 20, 21, and 22.

\[
E_a = \omega_0 \lambda
\]

(20)
\[ \lambda = \frac{2 \cdot R_s \cdot I_{st} \cdot N_a \cdot k_w \cdot k_z \cdot B_1}{P} \]  

(21)

\[ B_1 = \frac{4}{\pi} \cdot B_g \cdot k_s \cdot \sin \left( \frac{P \cdot \theta_m}{2} \right) \]  

(22)

where \( \theta_m \): magnet physical angle

\[ B_g = \frac{k_i C_\phi}{1 + k_r \cdot \frac{u_{rec}}{PC}} \]  

(23)

where \( u_{rec} \): recoil permeability; \( B_r \): remnant flux density

\[ PC = \frac{h_m}{g_e \cdot C_\phi} \]  

(24)

where \( PC \): permeance coeff.; \( C_\phi \): flux concentration factor (Am/Ag)

\[ N_a = 2 \cdot P \cdot N_c \]  

(25)

where \( N_c \): Turns per coil; \( N_a \): Number of armature turns (each slot has 2 half coils)

\[ \tau_s = w_s + w_t; g_e = k_e \cdot g \]  

(26)

where \( g_e \): effective air gap; \( w_s \): average slot width; \( w_t \): tooth width

Here, a leakage factor (K1 ~ 0.95) and a reluctance factor (Kr~1.05) are both used for surface magnets. The presence of the slots in the stator also affects the air gap flux density because of the difference in permeance caused by the slots. Carter’s coefficient (kc) is used to account for this effect [12].

\[ k_e = \left[ 1 - \frac{1}{\tau_s \cdot \left( 5 \cdot \frac{g}{w_s} + 1 \right)} \right]^{-1} \]  

(27)

The terminal voltage (\( V_a \)) is calculated from the internal voltage (\( E_a \)), and the synchronous reactance voltage drop. The armature resistance is usually ignored because it is much smaller than synchronous reactance. The voltage is found as a relation in output power (\( P_{wr} \)), e.m.f, and reactance from the resulting quadratic equation.

\[ V_a = \sqrt{\frac{-BB + \sqrt{BB^2 - 4CC}}{2}} \]  

(28)

where

\[ BB = \frac{2}{3} X_s P_{wr} - E_a^2 \]

\[ CC = \frac{2}{9} X_s^2 P_{wr}^2 \]
3.11 Machine Inductances

In a slotted PM machine, there are three distinct components of inductance: the largest, air gap inductance slot leakage inductance, and the smallest, end-turn inductance. The total inductance for the phase is the sum of the three inductances, ignoring other small factors.

\[ L_s = L_{og} + L_{slot} + L_e; \quad X_s = \omega_0 \times L_s \quad (29) \]

The air gap inductance is given by Eq. 30.

\[ L_{og} = \frac{\lambda}{i} = \frac{4}{2} \frac{u_0 \times R_s \times L_{st} \times N_a^2 \times k_{wn}^2}{n \pi \times n^2 \times P^2 \times (g + h_m)} \quad (30) \]

The slot leakage inductance is presented in Equation 31. Assume the slot is rectangular with slot depressions, as in Figure 1, and assume (m) slots per pole per phase, with a standard double layer winding.

\[ L_{slot} = L_{as} - L_{am}; \quad (3 \text{ phase}) \quad (31) \]

\[ L_{am} = 2 \times P \times L_{st} \times Perm \times N_{sp} \times N_c^2 \quad (32) \]

\[ L_{as} = 2 \times P \times L_{st} \times Perm \times [4 \times N_c^2 \times (m - N_{sp}) + 2 \times N_{sp} \times N_c^2] \quad (33) \]

A slot permeance per unit length is shown in Equation 34.

\[ Perm = \frac{1}{3} \times \frac{h_s}{w_{st}} + \frac{h_d}{w_d} \quad (34) \]

The end turn inductance is introduced in Equation 35, assuming the end turns are semi-circular, with a radius equal to one-half the mean coil pitch.

\[ L_e = \frac{u_0 \times N_c \times N_a^2 \times \tau_s \times \ln(\frac{\tau_s \times \pi}{\sqrt{2} \times A_s})}{2} \quad (35) \]

3.12 Basic Losses

Losses in a machine consist of core losses, conductor losses, friction and windage losses, and rotor losses. Rotor losses will be discussed later. Stator core losses, per weight, can be greater than normal in machines because of higher frequencies. These losses are minimized by using laminated steel in stator construction and by not generating frequencies that are too high. Core losses consist of hysteresis and eddy current losses. The best way to approximate core losses is to use empirical loss data. An exponential curve fitting is applied to the empirical data for M-19, 29 gauge material, in order to obtain an equation for estimating core losses, as in Equation 36, with constant values in [19].
\[ P_C = P_0 \ast \left( \frac{B}{B_0} \right)^{\delta B} \ast \left( \frac{f}{f_0} \right)^{\delta f} \]  

(36)

where \( P_0 \): Base power; \( B_0 \): Base flux density; \( \delta B \): Flux density exponent; \( f_0 \): Base frequency; \( \delta f \): Frequency exponent

The conductor losses are found, using Equation 37.

\[ P_a = q \ast I_a^2 \ast R_a \]  

(37)

For rotors operating at high speed, the friction and windage in air can cause losses which result in inefficiency and heat production. These losses are calculated, using the power necessary to overcome the drag resistance of a rotating cylinder, as given by Equation 38 [20].

\[ P_{wind} = C_f \ast \pi \ast \rho_{air} \ast \omega^3 \ast R^4 \ast L_{st} \]  

(38)

The coefficient of friction can be approximated by Equation 39.

\[ C_f \approx 0.0725 \ast R_{ey}^{-0.20} \]  

(39)

where \( R_{ey} \): Reynold’s Number

4. Sizing Results

4.1 Machine Initial Sizing

For the basic sizing calculations, an air-cooled generator is assumed with 10 psi [13], [21]. The machine power equation is utilized to derive the rotor radius and stack length of the machine, as in Equation 40.

\[ P_{wr} = 2 \ast \pi \ast r \ast L_{st} \ast v_{tip} \ast \tau \]  

(40)

where \( r \): rotor radius; \( L_{st} \): stack length; \( \tau \): shear stress (psi)

The L/D ratio is substituted for \( L_{st} \). Using shear stress, rotor tip speed, and machine power rating range, the power equation is calculated to obtain rotor radius and stack length, while matching the desired rotational speed of the machine with a L/D ratio equal to 2.5, as supposed here. Using a pole pair value of 3, a slot height of 10 mm, and a slot fill fraction of 0.5, the frequency is found.

The basic sizing parameters are illustrated in Figure 5 – 7.
4.2 Detailed Sizing

Once the basic sizing of the machine is complete, in-depth analysis is conducted to obtain the overall performance within the power range of 5: 500 KW generators. Using the equations presented in previous sections, all the detailed parameters can be obtained. The lengths, volumes, masses, and overall generator parameters are calculated, using basic geometric equations and relationships. A 15% service mass fraction is added to the total
mass estimate to account for the additional services associated with machines cooling [14], [21]. Once the mass of each of the stator parts is known, core losses are estimated in accordance with them. The calculation of lengths, volumes, and weights are presented. The mass of armature conductors, core mass, magnet mass, and shaft mass are calculated to give the total mass value. Finally, stator resistance, terminal voltage, current, loss types, input power, and efficiency are calculated. As it is shown in the following selected results figures; it is clear that as the speed grows up the performance is modified and it is easy to select the required sizing from the given ones in this range. Also, the tip speed of 50 m/s which considered being out of usual range for high speed machines (100: 250 m/s) to show the deal of improvement between entire range characteristics and one out it.

Fig. 8: Power with average slot width relations at various tip speeds

Fig. 9: Power with magnetic flux (Wb) relations at various tip speeds.

Fig. 10: Power with total inductance (H) relations at various tip speeds.
Fig. 11: Output power with stator resistance relations at various tip speeds.

Fig. 12: Power with terminal voltage relations at various tip speeds.

Fig. 13: Power with armature current relations at various tip speeds.

Fig. 14: Output power with total loss relations at various tip speeds.
Fig. 15: Output power with efficiency relations at various tip speeds.

Fig. 16: Power with overall machine length relations at tip speeds.

Fig. 17: Output power with overall diameter relations at various tip speeds.
5. Conclusions

The sizing method presented gives a step by step method for high speed PM generator design. This paper illustrates the benefits of HSPM generators, compared to the original PM synchronous generators, since it offers significant reductions in both weights and volumes. It discusses the electrical and magnetic sizing of HSPMGs, within the power range of 5:500 Kw and tip speed in the range of 50:250 m/s.

References


