Application of Inverse Problem Methodology in Design Optimization of a Permanent Magnet Synchronous Motor (PMSM)

Application of modern mathematical optimization has become an important tool for the design permanent magnet machines. Inverse problems have been found in numerous research areas, such as medical imaging, non destructive testing and geophysical prospecting. The inverse problem can be considered as a large-scale nonlinear programming problem. This paper describes the optimal design of a permanent magnet synchronous motor based on inverse problem methodology. In the optimization problem, we choose the increase of torque as objective. The magnet thickness and arc magnet pole are selected for the design. The performance of the designed motor are then studied using finite element method (FEM) calculations and analytical models. The simulation results show the usefulness and effectiveness of the proposed strategy.

Keywords: Surface Permanent Magnet Synchronous Motor, Inverse Problem, Finite Element Method, Optimal Design.

1. INTRODUCTION

Permanent magnet ac motors are classified into sinusoidally fed PM synchronous motors (PMSM) and rectangular fed brushless permanent magnet motors (BLDC). The magnets are either mounted on the surface of the rotor called surface mount permanent magnet (IPM), or placed inside the rotor called interior permanent magnet (SPM) motors. Computational optimization is considered very important for electrical machine design [1-3]. Different optimization techniques have been developed for electric machine design to check iteratively the changes of the design variables, which more in the direction of improving the objective function [2], [4], [5]. However, it is difficult to carry out optimization work on PMSM structures that can provide good performance, both in constant torque [2],[3],[6] and flux weakening regions[4],[7]with acceptable computational time because of the complicated rotor configurations and the complex influence of magnetic saturation.
Finite element numerical computation can be used to compute magnetic fields of this machine in order to calculate the machine parameters, such as flux linkage, direct axis inductance $L_d$, and quadrature axis inductance $L_q$. These parameters are essential in the optimization of machine performance. Since these parameters are directly related to the geometry of the rotor, the optimization requires the optimum design of the rotor geometry [1]. There are many geometrical variables in the rotor design. Optimization of all the variables is impractical and may not be necessary. A selected critical number of variables such as Magnet thickness ($h_m$) and Magnet pole ($\alpha$) can be optimized to achieve an economic solution [8-11]. Recently, there has been an increased interest in the development of the methods for solving electromagnetic inverse problems that is, reconstructing the material properties of unknown objects [12]. These methods have been applied to numerous areas, including medical imaging, non-destructive testing, where the objective is to estimate the spatial distribution of complex permittivity from a set of scattered field measurements. These inverse problems are among the most challenging optimization problems in computational science and engineering. The crucial point in inverse problem is the efficiency of the process by which the solution is arrived at [12]. Ill-conditioned inverse problems require regularization to prevent the solutions from being excessively sensitive to noise in the data. While efficient algorithms exist for computing inverses, the role of regularization in increasing the cost of the computations has been well considered. The regularization techniques that are widely employed are Tikhonov’s, Levenberg’s…etc. This paper proposes an optimization process of a PM motor. Both finite element and electrical models of the motor are used to define an optimization problem by developing
an objective function. The problem is then solved by a numerical method to achieve optimum PM pole dimensions. The simulation results are also shown to confirm the validity of the proposed optimization method.

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer radius of stator</td>
<td>50 mm</td>
</tr>
<tr>
<td>Stator yoke thickness</td>
<td>7.77 mm</td>
</tr>
<tr>
<td>Half tooth</td>
<td>1.9 mm</td>
</tr>
<tr>
<td>Slot high</td>
<td>11.43 mm</td>
</tr>
<tr>
<td>Thickness of the tooth head</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Open angular tooth</td>
<td>2 °</td>
</tr>
<tr>
<td>Air-gap length</td>
<td>1 mm</td>
</tr>
<tr>
<td>Magnet thickness</td>
<td>8.7 mm</td>
</tr>
<tr>
<td>Shaft radius</td>
<td>7.52 mm</td>
</tr>
<tr>
<td>Open angular magnet</td>
<td>44.5 °</td>
</tr>
<tr>
<td>PM material</td>
<td>Ferrite</td>
</tr>
<tr>
<td>Coercive force of magnet</td>
<td>250KA/m</td>
</tr>
<tr>
<td>Residual flux density of magnet</td>
<td>0.37 T</td>
</tr>
</tbody>
</table>

### 2. MACHINE MODELS

Fig. 1 shows the initial configuration of the prototype surface permanent magnet synchronous motor (SPMSM). The stator has 24 slots and the rotor is built of four tiles of radial magnetic ferrite bonded on a soft iron ring. Table I summarizes the design specification of this low power motor.

#### 2.1. Finite Element Modeling

The governing equation for the magnetic field is represented in two-dimensional rectangular coordinates as

\[
\frac{\partial}{\partial x} \left( \nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu \frac{\partial A}{\partial y} \right) = -J_s - \nu \left( \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y} \right)
\]

Where \( A \) is the \( Z \) component of magnetic vector potential, \( J_s \) is the stator winding current density, \( M_x, M_y \) are the \( x \) and \( y \) components of the magnetization \( M \) respectively, \( \nu \) is the reluctivity.

#### 2.1. Electric Model

Fig. 2 shows the vector diagram of a PM motor. The steady-state voltage equations in the reference frame can be expressed as follows
\[
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} =
\begin{bmatrix}
R + pL_d & -\omega L_q \\
\omega L_d & R + pL_q
\end{bmatrix}
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  
(2)

Where \( V_d \) and \( V_q \) are the state d- and q-axis voltages, respectively, \( i_d \) and \( i_q \) are the stator axis currents respectively, \( R, \omega, L_d, L_q \) and \( \psi_m \) are the stator resistance, the electrical angular frequency, the d-axis inductance, the q-axis inductance, and the PM flux linkage, respectively, \( p = d/dt \) is heaviside notation for the time derivative operator.

The motor torque equation can be expressed as

\[
T_e = \frac{3}{2} P_n \left[ \psi_m i_q + (L_d - L_q) i_d i_q \right]
\]  
(3)

Where \( T_e \) and \( P_n \) are the generated torque and the number of pole pairs respectively.

The first term in equation (3) represents the synchronous torque generated from interlinkage flux of the permanent magnet the second term represents the reluctance torque generated by the differences between d-axis and q-axis inductances. The SPMSM uses one type of these torques which is the synchronous torque.

3. DESIGN OPTIMIZATION

According to the produced power \( P \) of the motor in steady state described by the following equation

\[
P = \frac{3E_0 V}{X_d} \sin \delta + \frac{3V^2}{2} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) \sin 2\delta
\]  
(4)

The expression of developed torque is
\[ T_d = \frac{3(2P_n)E_0V}{\omega X_d} \sin \delta + \frac{3(2P_n)V^2}{2\omega} \left( \frac{1}{X_d} - \frac{1}{X_q} \right) \sin 2\delta \]  

(5)

Seeing from (5), the factors about the PMSM such as the main voltage \( V \), number of pole pairs, load angle, field winding electromotive \( E_0 \) in no-load, synchronous electric speed, stator reactance in d- and q- axis (\( X_d \) and \( X_q \) respectively), the ratio \( X_q/X_d \) all can influence the maximum torque of the motor. In the case of an SPMSM (\( X_d=X_q \)), the load angle that generates the maximum torque is \( \delta=90^\circ \). The design objective in this paper is an increase in motor developed torque. The torque of a PM motor depends on d- and q-axis inductances and the maximum PM flux linkage, \( L_d \) and \( \psi_m \) respectively. The parameters depend on the dimensions and location of PM poles on the rotor [1]. Therefore, the magnet thickness \( h_m \), and the magnet pole angle are regarded as the variables to be optimized. Fig. 1 shows the initial configuration of rotor with two shape design variables (\( h_m \), \( \alpha \)). Ill conditioned inverse problems require regularization to prevent the solutions from being excessively sensitive to noise in the data arrived at [12-13]. A conventional d-q electrical model of the motor in synchronously reference frame can be used in design optimization and evaluation where d-axis passes through the centre of a magnet pole. In the model flux distribution in the air gap is assumed to be sinusoidal and the iron loss and magnetic saturation are not considered. The maximum value of PM flux linkage is obtained as

\[ \psi_m = \frac{4DL}{8\pi} \left( \frac{K_{w1}N_{ph}}{2P_n} \right) B_g \sin(a) \]  

(5a)

Where \( K_{w1} \) is the winding factor, \( N_{ph} \) the winding turns per phase, \( D \) is the inner diameter of the stator and \( a \) is one half of magnet pole-arc-angle. The d- and q-axis inductances are given by

\[ L_d = \frac{3\mu_0DL}{g} \left( \frac{K_{w1}N_{ph}}{2P_n} \right)^2 \pi K_d \]  

(6)

\[ L_q = \frac{3\mu_0DL}{g} \left( \frac{K_{w1}N_{ph}}{2P_n} \right)^2 \pi K_q \]  

(7)

Where \( K_d \) and \( K_q \) are the reaction factors of the PM motor defined as

\[ K_d = \left( \alpha - \sin(\alpha\pi) \right) + g \left( 1 - \alpha + \sin(\alpha\pi) \right) \]  

(8)

\[ K_q = \left( \alpha + \sin(\alpha\pi) \right) + g \left( 1 - \alpha - \sin(\alpha\pi) \right) \]  

(9)

Where \( g \) is the effective air gap given by
\[ g_e = \frac{l_m}{\mu_r} + k_c g \] (10)

For a high-performance PM drive to achieve fast transient response and to have high-efficiency operation are the two important concerns. Hence, the maximum torque per ampere (MTPA) control strategy was widely adopted in the constant torque-limit operation range. The design optimization is carried out under the condition of maximum torque per ampere control. The resulting stator current components are obtained as follows:

\[
i_d = \frac{\Psi_m}{4L_d} (\zeta - 1) - \sqrt{\left(\frac{\Psi_m}{4L_d} (\zeta - 1)\right)^2 + \frac{I_s^2}{2}}
\] (11)

\[
i_q = \sqrt{I_s^2 - i_d^2}
\] (12)

Where \( \zeta = L_q / L_d \)

The constrained optimization problem can be expressed mathematically as follows

\[
\text{Min } F(x_i) \quad i = 0,1,\ldots,k
\] (13)

Subject to

\[
G_j(x_i) = 0 \quad (\leq 0, \geq 0) \quad j = 0,1,\ldots,m \quad x_{iL} \leq x_i \leq x_{iU}
\] (14)

The function \( F(x_i) \) is the objective function, \( G_j(x_i) \) is a set of constrained functions with the dimensions of \( m \). \( x_i \) are \( k \) design variables with lower and upper bounds of \( x_{iL} \) and \( x_{iU} \) respectively. In this paper the objective function is defined as:

\[
F(X) = \frac{1}{2} \sum_{i=1}^{n} \left( B_{FEM}^i - B_d^i \right)^2
\] (15)

Where \( B_{FEM}^i \) and \( B_d^i \) are the computed flux density using finite element method and the desired flux density respectively at the \( i \)-th point on a path of the air gap PM motor.

\( X = \{h_m, \alpha\} \) is the vector of parameter design (magnet thickness and arc pole angle).

The sensitivity of the objective function \( F \) as to the design variable \( X \) can be written as follows

\[
\frac{\partial F}{\partial X} = \sum_{i=1}^{n} \frac{\partial F}{\partial B_{FEM}^i} \cdot \frac{\partial B_{FEM}^i}{\partial X} = \sum_{i=1}^{n} (B_{FEM}^i - B_d^i) \cdot \frac{\partial B_{FEM}^i}{\partial X}
\] (16)

The finite difference method is employed to approximate the gradient of the objective function.
\[ \frac{\partial B^{i}_{\text{FEM}}}{\partial X} = \frac{B^{i}_{\text{FEM}}(X + \delta X) - B^{i}_{\text{FEM}}(X)}{\partial X} \tag{17} \]

where \( \delta X \) represents a small perturbation of the corresponding parameter \( X \).

Where the optimization problem is ill-posed that is when existence or unicity of the solution with respect to experimental data is not verified, it is common to use a regularization methods, in order to limit the space parameter [12]. The most commonly used methods are Tikhonov’s, Lenvenberg’s and Levenberg-Marquardt’s. All of these introduce a regularization term \( F_r \) representing more or less, the least-squared difference between the calculated parameter vector \( X \) (Lenvenberg) and the initial guessed one \( X^0 \) (Tikhonov) or the previous calculated one.

\[ F^* = (1 - \lambda)F + \lambda F_r \tag{18} \]

Where \( \lambda \in [0, 1] \) is the regularization parameter.

The regularization term for Tikhonov method is of the form (19), where \( X^0_i \) is the initial set of parameters and \( X^k_i \) is the current set of parameters.

\[ F_r = \sum_i (X^k_i - X^0_i)^2 \tag{19} \]

Levenberg’s method is of the same kind, but \( X^0_i = X^{k-1}_i \) is the set of parameters solved by the Gauss-Newton algorithm at previous iteration \( (k-1) \). The convergence criterion in our design is based on the variation of the objective function value. If the differences of the objective function value in two subsequent iterations is less than a specified positive number \( \varepsilon \) (Eq.20).

\[ \left| F(X^{k+1}) - F(X^k) \right| < \varepsilon \tag{20} \]

The optimization process will stop and the final optimization is achieved.

4. RESULTS AND DISCUSSION

As mentioned above, we try to optimize the magnet shape of 90 W, 3000 rpm, four- pole and three- phase PM synchronous motor to achieve a high torque. The optimization problem can be formulated in the following way: find the set \( (h_m, \alpha)_{\text{opt}} \) that minimizes the objective function in Eq (15) under the geometrical constraints \( 25^\circ \leq \alpha \leq 45^\circ \), and \( 1 \text{ mm} \leq h_m \leq 8.75 \text{ mm} \), the torque constraint \( T_{\text{max}} \geq 0.27 \text{ N.m} \). Table I summarizes the design specification of this low power motor. The design optimization in this work is carried out based on the finite element model (FEM). The corresponding FEM numerical results are used to calculate the motor parameters and torque. Using a direct search method and a regularization parameter \( \lambda=10^{-9} \), the process of optimization with inverse problem method converge after 19 iterations.
Table II shows the comparison of the optimally designed motor with the prototype. It can be seen that the optimization reduces the PM volume and increases the torque. The FEM results for radial and tangential components of the air gap flux density $B$ are respectively shown in Fig. 3 (a) and (b) for the initial structure and the optimized one. Throughout these figures the proposed structure model has a more sinusoidal flux density waveform. The harmonic spectrum of the air gap flux density produced by the magnets only is shown in Fig. 4. The dominant harmonics $3^{rd}$, $7^{th}$ are almost decreased by 58.82% and
50% respectively, the 5th is eliminated while the fundamental component of the optimized PM motor decreases by 2.48%. Fig. 5 shows the comparison of torque versus load angle characteristic of the prototype permanent magnet synchronous motor and the optimized one. We were able to confirm that the optimized motor had 7.62% higher torque than the original one. Fig. 6 shows the flux and the field distributions of the optimized PM motor model using 2D finite element method (FEM).

Fig. 4 Spectrum of the air gap flux density.

Fig. 5. Electromagnetic torque versus load angle.

\[ T_{\text{max, opt}} = 0.7652 \text{ N.m} \]
\[ T_{\text{max, init}} = 0.7068 \text{ N.m} \]

Fig. 6. Flux density distribution and Equipotential flux lines distribution (open-circuit condition).
5. CONCLUSION

This paper deals with the design optimization of a surface permanent magnet synchronous motor (SPMSM) via inverse problem methodology in order to maximize the developed torque. This proposed procedure, regarding optimized rotor pole design of PMSM is a suitable optimization method that can easily be extended for other machines. The PM motor designed by the proposed optimization method is analyzed by finite element method to evaluate the accuracy of the motor model employed in the design optimization results.

REFERENCES


