Implicit Fault Tolerant Control Technique Based Backstepping: Application to Induction Motor

The aim of this paper is to apply a resent implicit fault tolerant control (FTC) technique based backstepping to induction motor (IM). Mathematical models of the motor, the disturbances as well as the faults signals have been developed. Using these models, the dynamic behavior of motor is studied. A backstepping control is then synthesized and applied to the system. Based on the simulation results, it is evident that this control law is robust to the unknown torque load and parametric variations but is not to the rejection of the faults effect. In order to detect and compensate this later, a recent FTC technique is then studied and applied to the motor model. The performances of the applied technique are highlighted by simulations.

about the consequent benefits as for EMC of this drive are also presented.

Keywords: Fault Tolerant Control, Backstepping Control, Mathematical Model, Induction Motor, Fault Detection and Isolation.

NOMENCLATURE

\( s, r \) Stator, rotor index.
\( d, q \) Synchronous reference frame indexes.
\( \alpha, \beta \) Fixed stator reference frame indexes.
\( V, i, \Phi \) Voltage, current, flux.
\( L_s, L_r \) Stator, rotor inductance.
\( R_s, R_r \) Stator, rotor resistance.
\( T_s, T_r \) Stator, rotor time constant.
\( M \) Mutual inductance.
\( \Omega \) Rotor speed.
\( \theta_s \) Rotor flux angular position.
1. INTRODUCTION

Fault tolerance is an issue that has been addressed by many authors. Comprehensive introduction to this area can for example be found in [1] and [2]. Research on fault tolerant control systems has received increased attention recently due to significantly increasing demand for reliability, maintainability and survivability of safety-critical systems, such as automotive and aerospace systems, nuclear power [3]. Many efforts have recently been devoted to study fault tolerant control systems [4].

The induction motor is definitely one of the most used electric machines in the world. When supplied by a symmetrical and balanced sinusoidal three-phase voltage and operating according to manufacturers’ instructions namely regarding the environment and load type’ the induction motor is a very robust machine [5].

A breakdown in the motors can cause the stop of the production facility or require the use of redundant equipment to circumvent the problem. Several failures can affect electrical motor drives [6] and can appear on the level of rotor or stator of induction motor [7]. They can be electric, mechanical or magnetic. Their causes very varied. Indeed, of the studies showed that each faults revealed harmonics at specific frequencies in the currents of the machine [8], [9] and [10]. These frequencies depend on the motor characteristics.

The objective of this paper is to design a robust control (backstepping) [11], [12] compared to the torque of load and to the parametric disturbances but don’t present an insufficiency as for the rejection of the faults effect. Into this case, we introduce the internal model which generates an additive term that we adds to the nominal control to compensate the faults effect on the system and which reproduces the unknown state exactly a priori exogenos system simulating the faults [13].

2. INDUCTION MOTOR DISTURBANCE AND FAULTS MODELISATION

2.1. Induction Motor Modeling

The setting in the state form of the induction motor model allows the simulation of the latter, in a stationary reference (αβ) frame choosing us like states variables the stator currents, rotor flux and the mechanic speed Ω [11], [12]. And like control vector the stator tensions. The model of the IM is given by the state form:

\[ \dot{x} = f(x) + Bu + dT_L \]  \hspace{1cm} (1)

The state vector, the entry matrix B and the d vector are given by:
With the following expression of field vector $f(x)$:

\[
\begin{align*}
\begin{bmatrix}
\end{align*}
\end{equation}
\]

The components of this vector are expressed according to the parameters of the IM as follows:

\[
\begin{align*}
a_1 &= a_4 = \left(\frac{1}{T_s \sigma} + \frac{1 - \sigma}{T_r \sigma}\right) ; \\
a_2 &= a_6 = \frac{1 - \sigma}{T_r \sigma} M \sigma ; \\
a_3 &= -a_5 = \frac{n_p (1 - \sigma)}{M \sigma} ; \\
a_7 &= a_{10} = \frac{M}{T_r} ; \\
a_8 &= a_{12} = -\frac{1}{T_r} ; \\
a_9 &= -a_{11} = -n_p ; \\
a_{13} &= -a_{14} = -\frac{n_p M}{J L_r} ; \\
a_{15} &= \frac{f}{J} ; \\
b_1 &= b_2 = \frac{1}{\sigma L_s} ; \\
d_1 &= \frac{1}{J} ;
\end{align*}
\]

With: \( \sigma = 1 - \frac{M^2}{L_r L_s} \), \( T_r = \frac{L_r}{R_r} \) et \( T_s = \frac{L_r}{R_s} \).

### 2.2. Disturbance Modeling

An unknown term \( \Delta(x, \Delta a_i) \) but all the time limited is added in the model (1) which takes the form:

\[
\dot{x} = f(x) + Bu + dT_L + \Delta(x, \Delta a_i)
\] (3)

A way of simulating the parametric variations effect on the behavior of the motor is to cause at a given moment, a random change in the system (2) coefficients:

\[
\begin{align*}
a_i &\rightarrow a_i^0 + \Delta a_i, & b_i &\rightarrow b_i^0 + \Delta b_i \text{ and } d_1 &\rightarrow d_1^0 + \Delta d_1
\end{align*}
\]

The disturbances \( \Delta(x, \Delta a_i) \) will take the following form:
2.3. Faults modeling

In this section we underline the change of the IM model in the presence of faults. A term $V$ representing an unknown disturbance but all the time limited [13] -which results from the presence of one or more faults in the motor- is added in model (1) which written in the form:

$$\dot{x} = f(x) + Bu + dT_L + \sigma V$$

(5)

In absence of faults, $V$ is identically invalid and we have:

$$\sigma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$$

and $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

(6)

A way of introducing the harmonics which affecting the stator currents in the state system (1) is to use a stable linear differential system represented in this form:

$$\dot{z} = Sz$$

(7)

The dynamic matrix $S$ being the only known characteristic of the system, it is consisted of the pulsations $\omega_i = 2\pi f_i$ where $f_i$ presents the frequency characteristic of the faults:

$$S = \text{diag}(S_i)$$

$$S_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$$

(8)

We can write the faults disturbances $V$ in this form:

$$V = \begin{bmatrix} a_1 Q_d + Q_d S \\ a_4 Q_q + Q_q S \end{bmatrix} z = -\Gamma z$$

(9)

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \begin{bmatrix} a_1 Q_d + Q_d S \\ a_4 Q_q + Q_q S \end{bmatrix}$$

Finally in the presence of faults and parametric disturbances the model of the IM becomes:

$$\dot{x} = f(x) + Bu + dT_L + \Delta(x, \Delta a_i) + \sigma V$$

(10)
3. BACKSTEPPING CONTROL BASED ON THE PRINCIPLE OF FIELD-ORIENTED

3.1. Principle of Field-Oriented Control:

The objective of field oriented control is to have an electromagnetic torque proportional to the stator current of the motor (as in the case of a DC machine) with an aim of controlling the electromagnetic torque and consequently the mechanical speed of the motor [12]. This method consists in orienting rotor flux according to the direction of the revolving axis (d), which makes it possible to transform the motor model given by (2) in the reference mark turning \( dq \) [3], [5].

In these new coordinates. The rotor position is defined by the angle \( \theta_s \) as follows:

\[
\theta_s = \arctan \left( \frac{\Phi_r \beta}{\Phi_r \alpha} \right) \tag{11}
\]

The transformation \( \alpha \beta \rightarrow dq \) is done as follows:

\[
\begin{bmatrix}
  x_{sd} \\
  x_{sq}
\end{bmatrix}
= \begin{bmatrix}
  \cos(\theta_s) & \sin(\theta_s) \\
  -\sin(\theta_s) & \cos(\theta_s)
\end{bmatrix}
\begin{bmatrix}
  x_{s\alpha} \\
  x_{s\beta}
\end{bmatrix} \tag{12}
\]

With \( x \) can be used for the current, flux and tension.

The new model of machine in \( dq \) reference is given by:

\[
\begin{align*}
  \dot{x}_1 &= a_1 x_1 + \dot{\theta}_s x_2 + a_2 x_3 + bu_1 \\
  \dot{x}_2 &= -\dot{\theta}_s x_1 + a_1 x_2 + a_5 x_3 x_5 + bu_2 \\
  \dot{x}_3 &= a_8 x_3 + a_{10} x_1 \\
  \dot{x}_4 &= 0 \\
  \dot{x}_5 &= a_{14} x_2 x_3 + a_{15} x_5 + d_1 T_L
\end{align*} \tag{13}
\]

With: \( x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T = [i_{sd} \ i_{sq} \ \Phi_d \ \Phi_{rq} \ \Omega]^T \)

\[
\Phi_d = \Phi_{rd} = \sqrt{\Phi_{r\alpha}^2 + \Phi_{r\beta}^2} \quad ; \quad \dot{\theta}_s = \frac{d}{dt} \frac{x_2}{x_3}
\]

3.1. Backstepping Control

The backstepping control technique is a method of synthesis into nonlinear when it is difficult to apply the direct method of Lyapunov. It is a question of choosing at the start a Lyapunov function for the first subsystem and of increasing it as we stability the various successive subsystems, to lead finally
to a total Lyapunov function which stabilizes the total system. The backstepping design procedure consists of the following two steps [11], [12].

**Step 1:** This first step consists to identify the errors $\varepsilon_1$ and $\varepsilon_2$ which respectively represent the error between real speed $\Omega$ and reference $\Omega^d$ as well as the module of flux $\Phi_d$ and that of reference $\Phi_d^d$.

\[
\varepsilon_1 = x_5^d - x_5 \\
\varepsilon_2 = x_3^d - x_3
\] (14)

The error derivative is given by:

\[
\dot{\varepsilon}_1 = x_5^{d'} - \dot{x}_5 = x_5^d - a_{14}x_2x_3 - a_{15}x_5 - d_1T_L \\
\dot{\varepsilon}_2 = x_3^{d'} - \dot{x}_3 = x_3^d - a_8x_3 - a_{10}x_1
\] (15)

The first Lyapunov function is defined by:

\[
v_1 = \frac{1}{2}(\varepsilon_1^2 + \varepsilon_2^2)
\] (16)

We selected stabilizing functions as follows:

\[
(x_2)_{\text{ref}} = \frac{1}{a_{14}x_3}(k_1\varepsilon_1 + \dot{x}_5^d) - a_{15}x_5 - d_1T_L
\]

\[
(x_1)_{\text{ref}} = \frac{1}{a_{10}}(k_2\varepsilon_2 + \dot{x}_3^d - a_8x_3)
\] (17)

Then the errors dynamics is given by:

\[
\dot{\varepsilon}_1 = -k_1\varepsilon_1 \\
\dot{\varepsilon}_2 = -k_2\varepsilon_2
\] (18)

The derivative of (16) compared to time is:

\[
\dot{v}_1 = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2 < 0
\] (19)

With $k_1$ and $k_2$ are positive design constants. So the control in (17) is asymptotically stabilizing.

**Step 2:** In this step, we definite two new errors of the components of the stator current given by:

\[
\varepsilon_3 = (x_2)_{\text{ref}} - x_2 = \frac{1}{a_{14}x_3}(k_1\varepsilon_1 + \dot{x}_5^d) - a_{15}x_5 - d_1T_L - x_2
\]

Then the equation

\[
\varepsilon_4 = (x_1)_{\text{ref}} - x_1 = \frac{1}{a_{10}}(k_2\varepsilon_2 + \dot{x}_3^d - a_8x_3) - x_1
\]

(15) will be:

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\[ \dot{e}_1 = -k_1 e_1 + a_{14} e_3 \]
\[ \dot{e}_2 = -k_2 e_2 + a_{10} e_4 \]  

The derivative of (19) gives us:
\[ \dot{e}_3 = (\dot{x}_2)_{ref} - \dot{x}_2 = (\dot{x}_2)_{ref} - f_2(x) + bu_2 \]
\[ \dot{e}_4 = (\dot{x}_1)_{ref} - \dot{x}_1 = (\dot{x}_1)_{ref} - f_1(x) + bu_1 \]  

Where:
\[ f_1(x) = a_1 x_1 + \dot{\theta}_s x_2 + a_2 x_3 \]
\[ f_2(x) = -\dot{\theta}_s x_1 + a_1 x_2 + a_5 x_3 x_5 \]

To define the control laws, we adopt a new Lyapunov function described by the following expression:
\[ v_2 = \frac{1}{2} \left( e_1^2 + e_2^2 + e_3^2 + e_4^2 \right) \]  

Thus the derivative of the final Lyapunov function is:
\[ \dot{v}_2 = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 \]

Its derivative is given by:
\[ \dot{v}_2 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_3 \left( k_3 e_3 + (\dot{x}_2)_{ref} - f_2(x) - bu_2 \right) \]
\[ + e_4 \left( k_4 e_4 + (\dot{x}_1)_{ref} - f_1(x) - bu_1 \right) \]

We choose \( k_3 > 0 \) and \( k_4 > 0 \). Finely to make \( \dot{v}_2 < 0 \) the derivative of the complete Lyapunov function be negative definite, the voltage control input is chosen as follows:
\[ u_1 = V_{sd} = \frac{1}{b} \left( (\dot{x}_1)_{ref} + k_4 e_4 - f_1(x) \right) \]
\[ u_2 = V_{sq} = \frac{1}{b} \left( (\dot{x}_2)_{ref} + k_3 e_3 - f_2(x) \right) \]  

The error derivative \( e_3 \) and \( e_4 \) will be as follows:
\[ \dot{e}_3 = -k_3 e_3 - a_{14} x_3 e_1 \]
\[ \dot{e}_4 = -a_{10} e_2 - k_4 e_4 \]  

4. A RECENT IMPLICIT FAULTS TOLERANT CONTROL

The idea behind implicit FTC is that of designing a control unit able to automatically offset the faults effect, without need of an explicit FDI process and consequent explicit reconfiguration [13]. This objective will be pursued for the IM by means of the control scheme sketched in Fig.1.
By supposing that the faults effects on the system can be suitably modeled by an exogenous signal resulting from a stable autonomous system called "exosystem". An additive control is added to the nominal control and setting to compensate the faults effect (aspect FTC). This additive control results from the internal model whose role is to reproduce the signal representing the faults effect (aspect FDI).

The torque of load and the disturbances parametric are compensated by the nominal control. For this (10) becomes:

\[ \dot{x} = f(x) + Bu + \sigma V \]  

(25)

The new control is expressed by:

\[
\begin{align*}
    u &= u_{nom} + u_{ad} \\
    u &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_{1nom} \\ u_{2nom} \end{bmatrix} + \begin{bmatrix} u_{1ad} \\ u_{2ad} \end{bmatrix}
\end{align*}
\]  

(26)

On the basis of which we calculate the unknown term \( u_{ad} \) with the expression which we retained from the nominal control (17) and (23).

The instantaneous difference between the state derivative of the system and the reference becomes:

\[
\begin{align*}
    \dot{\tilde{x}} &= \begin{bmatrix} \tilde{x}_1' \\ \tilde{x}_2' \\ \tilde{x}_3' \\ \tilde{x}_4' \\ \tilde{x}_5' \end{bmatrix} = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ x_5' \end{bmatrix} - \begin{bmatrix} -k_4 \tilde{x}_1 + b_1 u_{lad} - \Gamma_1 z_1 \\ -k_3 \tilde{x}_2 + b_2 u_{2ad} - \Gamma_2 z_2 \\ a_{10} \tilde{x}_1 - k_2 \tilde{x}_3 \\ 0 \\ a_{14} \tilde{x}_2 \tilde{x}_3 + -k_1 \tilde{x}_5 \end{bmatrix}
\end{align*}
\]  

(27)

Let us notice that the first two equations do not depend on the variables \( \tilde{x}_3, \tilde{x}_4 \) and \( \tilde{x}_5 \).
in the third equation if $\bar{x}_1 \rightarrow 0 \Rightarrow \bar{x}_3 \rightarrow 0$

in the fourth equation we have $x_4 = 0 \Rightarrow \dot{x}_4 = 0$

in the fifth equation if $\bar{x}_2 \rightarrow 0 \Rightarrow \bar{x}_5 \rightarrow 0$

In the continuation, for the determination of $u_{ad}$ let us consider the subsystem:

$$\tilde{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$  \hspace{1cm} (28)

Whose dynamics results from the system (27).

$$\dot{\tilde{x}} = S \cdot z$$

$$\dot{\tilde{x}} = \begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} = \begin{bmatrix} -k_4\bar{x}_1 + b_1u_{lad} - \Gamma_1z_1 \\ -k_3\bar{x}_2 + b_2u_{lad} - \Gamma_2z_2 \end{bmatrix}$$  \hspace{1cm} (29)

From system (29) we can write it in a matrix form:

$$\dot{\tilde{x}} = H(\tilde{x}) + \tilde{B}u_{ad} - \Gamma z$$  \hspace{1cm} (30)

$$H(\tilde{x}) = \tilde{A}\tilde{x} \text{ et } \tilde{A} = \begin{bmatrix} -k_4 & 0 \\ 0 & -k_3 \end{bmatrix}, \tilde{B} = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}$$  \hspace{1cm} (31)

In this case for the determination of internal model we introduce a resent implicit fault tolerant control approach which does not rest on the resolution of the Sylvester equation. The internal model takes then this form:

$$\begin{cases} \dot{\xi} = S\xi + N(\tilde{x}) \\ \dim(\xi) = \dim(z) = 2n_f \end{cases}$$  \hspace{1cm} (32)

For the calculation of the additive control we introduce a new variable.

$$\chi = M(\xi - z) - G\tilde{x}$$

$$\dot{\chi} = M(\dot{\xi} - \dot{z}) - G\dot{\tilde{x}}$$  \hspace{1cm} (33)

After replacement we find:

$$\dot{\chi} = MS(\xi - z) + M N(\tilde{x}) - G(H(\tilde{x}) - u_{ad} + \Gamma z)$$  \hspace{1cm} (34)

Then $u_{ad}$ is chosen like: $u_{ad} = \tilde{B}^{-1}\Gamma\xi$  \hspace{1cm} (35)

Consider the systems (30) and the additive term given by (35) in this case we have:

$$\dot{\tilde{x}} = H(\tilde{x}) + \Gamma \cdot (\xi - z)$$  \hspace{1cm} (36)

The new variable of error is considered:

$$e = (\xi - z)$$  \hspace{1cm} (37)

Its derivative compared to time:

$$\dot{e} = \dot{\xi} - \dot{z} = S \cdot \xi + N(\tilde{x}) + S \cdot \dot{z}$$

$$\dot{e} = S \cdot e + N(\tilde{x})$$  \hspace{1cm} (38)
The equations describing the dynamics of the errors in closed loop are thus:

\[
\begin{aligned}
\dot{\tilde{x}} &= \tilde{A} \cdot \tilde{x} + \Gamma \cdot e \\
\dot{e} &= S \cdot e + N(\tilde{x})
\end{aligned}
\]  

(39)

It is necessary to find the expression of \( N(\tilde{x}) \) which cancels the error of observation of the faults \( e \) and makes it possible at the same time to reject their effect for it cancels also \( \tilde{x} \).

That is to say the Lyapunov function of the system (39):

\[
V = \frac{1}{2} \tilde{x}^T \cdot \tilde{x} + \frac{1}{2} e^T \cdot e
\]  

(40)

After develop of calculates \( \dot{V} \) becomes:

\[
\dot{V} = \tilde{x}^T \cdot \tilde{A} \cdot \tilde{x} + e^T \cdot \Gamma^T \cdot \tilde{x} + e^T \cdot N(\tilde{x})
\]  

(41)

In this case the choice of \( N(\tilde{x}) \) is given by:

\[
N(\tilde{x}) = -\Gamma^T \cdot \tilde{x}
\]  

(42)

Finally \( \dot{V} \) is written:

\[
\dot{V} = \tilde{x}^T \cdot \tilde{A} \cdot \tilde{x} \leq 0
\]  

(43)

The system (39) becomes:

\[
\begin{aligned}
\Gamma \cdot e &= 0 \\
\dot{e} &= S \cdot e
\end{aligned}
\]  

(44)

The objective of the control is achieved by adopting the procedure suggested and we able to compensate the faults effect on the system \( (x \rightarrow 0) \) and to reproduce \( (e \rightarrow 0) \) thanks to the internal model.

5. SIMULATION RESULTS

In the Fig.2 we introduce at \( t=0.2 \) sec a load torque equal to the nominal torque then at \( t=0.4 \) sec a variation of 80% in the electrical and mechanical parameters of machine, after that the effect of only one fault in the stator generating one harmonic of frequency 50Hz, amplitude 8 and null phase at \( t=0.6 \) sec.

For Fig.3 we consider the same situation (Fig.2) but in this case we introduce at \( t=0.6 \) sec the effect of two faults one in the stator and another one in the rotor generating three harmonics of null phases, frequency 10, 15, 20 Hz and of amplitude 8, 10, 5 respectively.

In Fig.4 and Fig.5, we simulate the closed loop system with the recent implicit FTC approach. The FTC approach which we synthesized rejects the effect of the load torque, the parametric disturbances and also the effect of one fault or two faults (in the stator and rotor).
Fig. 2 Simulations of backstepping control in the presence of one fault in the stator.
Fig. 3 Simulations of backstepping control in the presence of two faults (stator and rotor).
Fig. 4 Simulations of the recent FTC approach in the presence of one fault.
Fig. 5  Simulations of the recent FTC approach in the presence of two faults.
6. CONCLUSION

In this paper, a recent implicit FTC technique is studied and applied to induction motor. The backstepping control (nominal control) which we have synthesizes present robustness compared to the parametric disturbances and the load torque. But, present an insufficiency as for the rejection of the effect of one fault (in the stator) or two faults (in the stator and rotor). We have shown how a resent implicit FTC approach can be designed in order to compensate the faults effect starting with generating from the internal model state, an additive term wish we add to the nominal control. The approach that we drew from the literature rests on the resolution of Sylvester equation which is at the origin of its disadvantages. These disadvantages were eliminated by the introducing of a resent implicit fault tolerant control approach for the calculation of the internal model. Simulation results have been presented in order to show the effectiveness of this recent approach.

REFERENCES


| Rated Values | Power | 1.08 | KW |
| Voltage | 220 / 380 | V |
| Frequency | 50 | Hz |
| \( n_p \) | 2 |

| Rated Parameters | | |
| \( R_s \) | 10 | \( \Omega \) |
| \( R_r \) | 6.3 | \( \Omega \) |
| \( L_s \) | 0.4642 | H |
| \( L_r \) | 0.4612 | H |
| \( M \) | 0.4212 | H |
| \( J \) | 0.02 | Kg.m² |
| \( f \) | 0.0005 | IS |