

Combination of mathematical indices and probabilistic neural network to detect the type of winding fault in transformers

This paper presents a new method for determination the type of transformer winding fault through transfer function (TF) analysis. For this purpose, probabilistic neural network (PNN) is used. Outset of all, the required measurements are carried out on two groups of transformers, under both intact and faulted conditions of different degrees in axial displacement, radial deformation, disc space variation and short circuit on the winding. Then, using algorithms based on mathematical methods, appropriate indices from frequency responses are extracted with the required accuracy. The extracted features are finally used as the inputs to PNN classifier in order to perform the multi-category fault classification. The obtained results reveal the ability of proposed method in comparison with two other distinguished methods.

Keywords: Transformer; fault type; measurement; transfer function; probabilistic neural network.

1. Introduction

As it is found in the related published literatures [1]-[5], the transfer function (TF) evaluation method is the most feasible method for detection of winding faults in transformers. TF method is a comparative method; in this method the results of a new measurement is compared with the referential measurement. If the deviations were remarkable, the direction and magnitude of deviations should be studied and analyzed to see it is possible a fault to be happen if a fault happened, can we estimate the fault type, its level and its location?

The important winding faults, which are most likely to be detected using the TF analysis, can be classified as follows [6]-[9]:

- a- Axial Displacement (AD),
- b- Radial Deformation (RD),
- c- Disc Space Variation (DSV) and
- d- Short circuit (SC)

These faults have been studied analytically and experimentally in [1]-[9]. These researches have studied the sensitivity of TFs parameters against those faults individually; however the methods for detection of a specific winding fault and their classification into one of the AD, RD, DSV or SC types are not discussed. In practice, it is very important for most of the applications to specify the type of fault.

In [10] a method for comparison of TFs based on application of mathematical indices is proposed to detect the fault type of transformer winding. This method is proposed to distinguish only the AD, RD and DSV fault types (SC is not included). Therefore, if changes occur in TFs due to SC this method fails. The investigation carried out in [11] introduces a new index to specify the fault type. However the proposed method was not verified. Although these studies gave important results, such results are not efficient to

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detect the type of winding fault in transformers without performing additional investigations. In order to contribute to such additional investigations, a general method for transformer winding's fault detection is proposed in this work with the help of an intelligent method based on TF analysis.

2. Notation

The notation used throughout the paper is stated below.

$f_{k,ti}$	the i -th frequency in trough points in intact case
$f_{o,ti}$	the i -th frequency in trough points in faulted case
$f_{k,pi}$	the i -th frequency in peak points in intact case
$f_{o,pi}$	the i -th frequency in peak points in faulted case
$A_{k,ti}$	the i -th amplitude in trough points in intact case
$A_{o,ti}$	the i -th amplitude in trough points in faulted case
$A_{k,pi}$	the i -th amplitude in peak points in intact case
$A_{o,pi}$	the i -th amplitude in peak points in faulted case
j	the level of AD
s	the level of RD
w	the location (state) of DSV
z	the level of DSV
l	the location of SC
x	each of the matrix rows (in equation 3)
μ	the mean value of x
sd	the standard deviation of x
X	the normalized vector of x

3. Problem definition

In this analysis, the TF method is analyzed by a comparative method. However, in general the comparative algorithms introduced in the literature, can be classified into four major categories as follows:

3.1. Algorithms based on mathematical and statistical methods

These algorithms employ a number of mathematical and statistical indices for comparison of TFs. Indices such as; the index of frequency ratio (IFR), index of amplitude ratio (IAR), correlation coefficient (CC) and so on belongs to this category. Some researchers [7], [10], [12] have been used these algorithms to evaluate fault level and fault location.

3.2. Algorithms based on electric circuit models

Some researches have been done to get information about the faults from the measured TFs by describing the TFs using an equivalent electric circuit model [13]. The parameters

of the model representing the TFs can be calculated by the means of the details of the transformer design [7] or can be estimated using terminal measurements [13].

3.3. Algorithms based on estimation methods

TFs of transformers can be approximated by rational functions consisting of two polynomials with real coefficients. To estimate the poles and zeros of the TFs (in frequency domain), efficient methods are introduced. One of these is the vector fitting (VF) method [14]. It is exact and effective method for estimating the TFs of Transformers. In [6], [8] based on detected poles using VF method, a new index is introduced for comparison of TFs. The obtained results showed that the proposed index can evaluate the fault level. Another work that belongs to this group, [11] describes an algorithm based on estimated coefficients of rational function, which is able to detect the type, level and location of faults.

3.4. Algorithms based on artificial intelligence methods

The smart methods such as artificial neural networks (ANN), genetic algorithm (GA), fuzzy logic, and probabilistic neural network (PNN) and so on can be included in this group. Reference [15] has used some of these methods to locate the winding faults.

Among these intelligent methods, PNN is known as one of the best methods for solving the classification problems. Its ability for detection of electrical faults in transformer [16] is proved.

As a result, in this paper PNN is used for classification of winding faults. The most important factor which can result in successful fault recognition through PNN application is the selection of proper features as input. Since the capabilities of mathematical methods in evaluation of the fault level are proved [7], [10] to extract the suitable features for training and testing of PNN, mathematical methods are used.

At first, the required measurements are carried out on two groups of transformers, under both intact and faulted conditions of different degrees in AD, RD, DSV and SC on the winding. Then, using algorithms based on mathematical methods (IFR, IAR), appropriate indices are extracted with the required accuracy. The features extracted with IFR, IAR are then used as the inputs to PNN classifier in order to classify the winding faults in transformer.

The group 1 transformers contain four model transformers of equal size; on each of them one of the four fault types is applied. The measured data obtained from this group of transformers is used in PNN training process. The group 2 transformers include four equal size windings of a real transformer; on each of them one of the four fault types are applied and their measured data is employed for testing and accuracy evaluation of the PNN.

4. Test objects and measurements

Two groups of test objects are employed in this study. Their test results are used to evaluate the accuracy and validity of the proposed method in identification of the transformer winding fault type. All measurements were executed in the time domain then

using Fast Fourier Transform (FFT) acquired the frequency response of TFs which is illustrated in [7].

4.1. The group 1 of test objects

Four model transformers are employed in this study and several tests were performed on them. These tests are discussed in the following.

4.1.1 Study of AD

The test object for study of AD is a high voltage winding which has 31 double inverted discs (there are 6 turns in each disc), beside a low voltage winding in form of a four layer concentric winding (there are 99 turns in each layer). These particular windings were manufactured for the special experimental purposes, and they correspond to windings of a transformer with a rated voltage of 10 kV and a rated output of 1.3 MVA. Its specific construction permits a gradual axial movement of the internal layer winding with respect to the outer winding. Since, the test object is 82.7 cm high therefore; a 1cm axial displacement in this winding is equivalent to 1.2% displacement.

4.1.2. Study of RD

As a test object for the study of radial deformation a high voltage winding with 30 double inverted discs, where 11 turns exists in each disc, and a one layer low voltage winding, having 23 turns are used. The double disc winding has a rated voltage of 10 kV and a rated output of 1.2 MVA. The deformation has occurred on the double disc winding in four degrees, as follows:

1. Degree 1: The sixth up to the 54-th discs were all radially deformed on one side. Deformation was around 7% of the disc radius
2. Degree 2: The sixth up to the 54-th discs were all radially deformed on two opposite sides. Deformation was around 7% of the disc radius
3. Degree 3: The sixth up to the 54-th discs were all radially deformed on three sides with 90° with respect to each other. Deformation was around 7% of the disc radius
4. Degree 4: The sixth up to the 54-th discs were all radially deformed on four sides with 90° with respect to each other. Deformation was around 7% of the disc radius

4.1.3. Study of DSV

For studying DSV the same winding that is used in previous section (4.1.2) experimented. For this purpose, another intact winding that space between its discs is 5mm, is selected. To study the effect of DSV on TFs, the space between adjacent discs were changed to 7.5, 10, 15, 20 and 25 mm, afterwards the TF is measured in each step. For a better study the following states are experimented:

- State 1 of DSV: space variation between disc 2 and disc 3
- State 2 of DSV: space variation between disc 4 and disc 5
- State 3 of DSV: space variation between disc 8 and disc 9

State 4 of DSV: space variation between disc 12 and disc 13

State 5 of DSV: space variation between disc 16 and disc 17

Although some of the DSV faults given in the above states seem to be an exaggeration and those are impossible to occur on real transformers, they have been executed in this work in order to accumulate more data about DSV and to obtain precise information on TF variations. Additionally, since the higher distances implemented between discs are compensable with smaller winding circumferences covered with the DSV, consequently the DSVs carried out in this work could be more similar to real faults in transformers.

4.1.4. Study of SC

To study the winding’s SC, a high voltage winding with 30 double inverted discs, where 9 turns exists in each disc is used. The double disc winding has a rated voltage of 10 kV and a rated output of 1.2 MVA. All discs of the test winding are provided with a tap, whereby the recording of the impulse voltage distribution is enabled along the winding. The input terminal of the winding is subjected to the impulse voltage, the earth current at a selected tap along the winding recorded as the response signals. To experimentally determine the effect of short circuits and their spatial arrangement on the TF, a SC is applied between two discs (in different places along the winding). For each individual position of the SC the TF is measured.

To investigate the sensitivity of TF measurements, for AD, RD, DSV and SC, different terminal conditions have been studied, as shown in Figure 1.

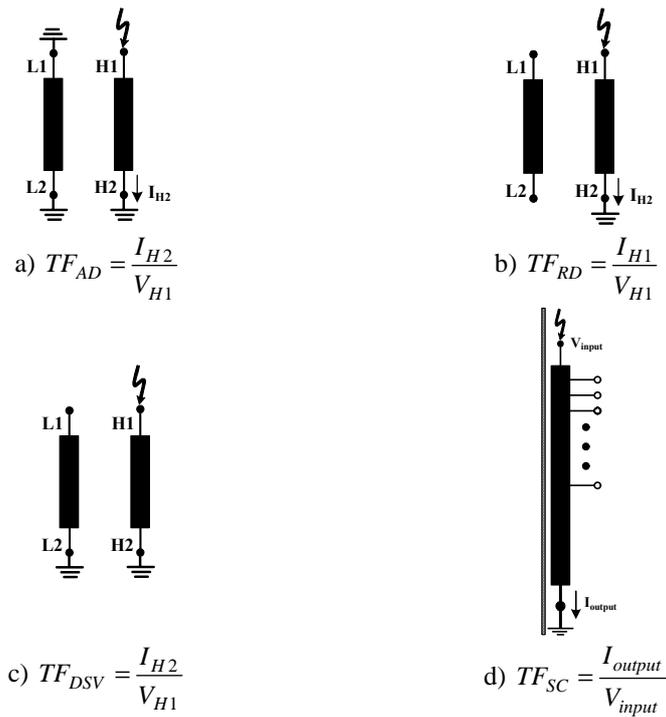


Figure 1. Different terminals connections, for analysis of winding faults: a) AD, b) RD, c) DSV, and d) SC

The measured results show that the winding faults (AD, RD, DSV and SC) affect the TFs and change them, differently. For example, the measured TF_{DSV} is showed in Figure 2.

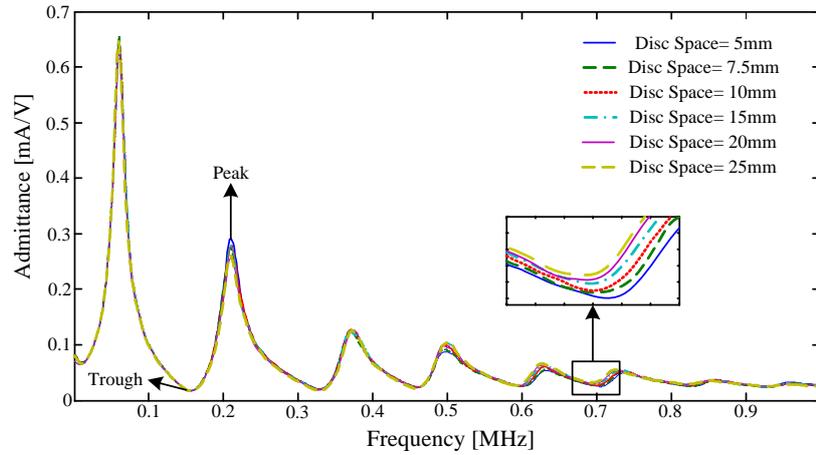


Figure 2. The measured TF_{DSV} of group 1 of transformer winding

4.2. The group 2 of test objects

In order to evaluate and validate the proposed method, another test object of similar type (layer-disc winding) with a different rating (6.5 MVA and 20 kV) is examined. Specifications of this test object are:

- 1- High voltage winding with 60 inverted discs, where 13 turns are present in each disc
- 2- Low voltage winding with four layer concentric, where 60 turns are present in each layer

To study the effect of AD, RD, DSV and SC similar terminal connections (Figure 1) have been experimented. Experiments carried out to study different degrees of AD and RD are closely similar to group 1, but to study the effect of DSV and SC, total condition of group 1 hasn't been experimented and these experiments have been done for DSV only in one location (between discs 6 and 7) and for SC in two locations (between discs 1, 2 and 2, 3). Similar to group 1, the measured results in this group show that the winding faults (AD, RD, DSV and SC) affect the TFs and modify them. For example, the measured TF of TF_{RD} is showed in Figures 3.

5. A description of the methods used

Since in this study a combination of mathematical indices (IFR, IAR) and PNN are employed to detect the fault type in transformer winding, therefore these methods will be described briefly in here.

5.1. Mathematical Indices

When faults occur in the winding, the most important changes observed in TF characteristics are in peak and trough points (This is shown in Figures 2, 3). Hence, the frequency and amplitude variations in these points can be used as reliable indices to train the PNN. The variation of *i*-th frequency, in peak and trough points; also referred to as the *i*-th index of frequency ratio (IFR) is defined as follows:

$$IFR_{ti} = \frac{f_{k,ti}}{f_{o,ti}} \quad , \quad IFR_{pi} = \frac{f_{k,pi}}{f_{o,pi}} \tag{1}$$

Where $f_{k,ti}$ and $f_{o,ti}$ represent the *i*-th frequency in trough points and $f_{k,pi}$ and $f_{o,pi}$ are the *i*-th frequency in peak points (*k* indicates fault condition and *o* is indication of intact condition).

Similarly, the variation of amplitude in the peak and trough points, are represented through the index of amplitude ratio (IAR) as follows:

$$IAR_{ti} = \frac{A_{k,ti}}{A_{o,ti}} \quad , \quad IAR_{pi} = \frac{A_{k,pi}}{A_{o,pi}} \tag{2}$$

Where $A_{k,ti}$ and $A_{o,ti}$ represent the amplitude of TF at the *i*-th trough point, and $A_{k,pi}$ and $A_{o,pi}$ are the amplitude of TF at the *i*-th peak point, respectively.

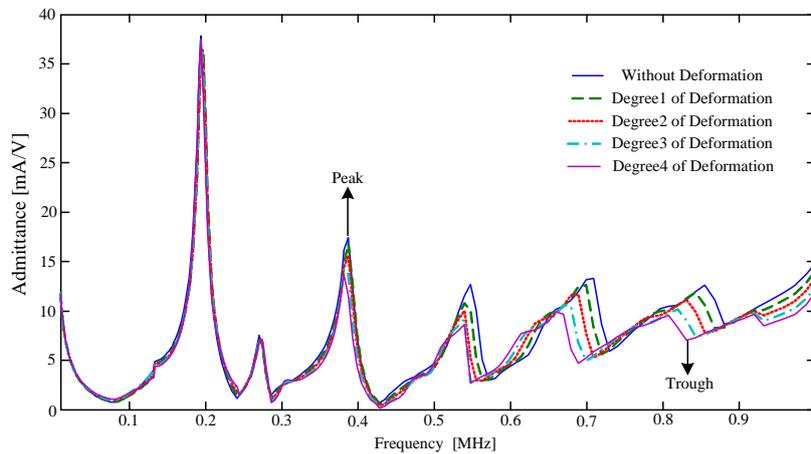


Figure 3. The measured TF_{RD} of group 2 of transformer winding

5.2. Probabilistic Neural Network (PNN)

The PNN introduced by D. Specht [17] is a special type of neural network that is widely used in the classification applications [16]. A PNN has a fast training process to provide a strong adaptability, needing only a single-pass, without any iteration, to adjust weights for network training. A PNN can function as a classifier to learn to place test examples into one of two or more categories. The architecture of the PNN is shown in Figure 4 [18].

As it is showed in Figure 4, the PNN contains three layers: the input layer, radial basis layer and competitive layer. When an input is presented, the radial basis layer computes distances from the input vector to the training input vectors, and produces a vector whose elements indicate how close the input is to a training input. The competitive layer sums these contributions for each class of inputs to produce as its net output a vector of probabilities. Finally, a “compet” function on the output layer picks the maximum of these probabilities, and produces a 1 for that class and a 0 for the other classes.

It is assumed that there are Q input vector/target vector pairs. Each target vector has K elements. One of these elements is 1 and the rest is 0. Thus, each input vector is associated with one of K classes.

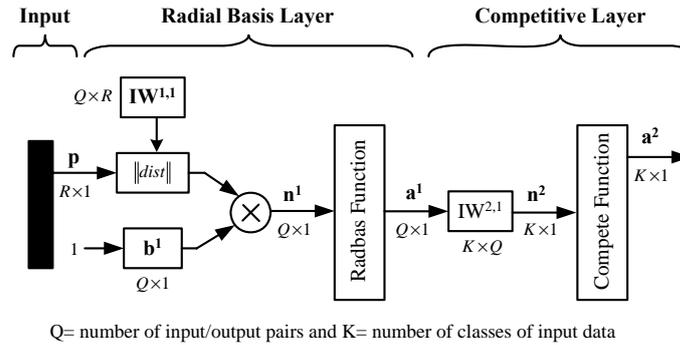


Figure 4. The structure of PNN

The first-layer input weights ($IW^{1,1}$), are set to the transpose of the matrix formed from the Q training pairs. When an input is presented the $\|dist\|$ box produces a vector whose elements indicate how close the input is to the vectors of the training set. These elements are multiplied, element by element, by the bias and sent the “radbas” function. An input vector close to a training vector is represented by a number close to 1 in the output vector a^1 . If an input is close to several training vectors of a single class, it is represented by several elements of a^1 that are close to 1.

The second-layer weights ($IW^{1,2}$), are set to the matrix T of target vectors. Each vector has a 1 only in the row associated with that particular class of input, and 0's elsewhere. The multiplication Ta^1 sums the elements of a^1 due to each of the K input classes. Finally, the second-layer function, “compet”, produces a 1 corresponding to the largest element of n^2 , and 0's elsewhere. Thus, the network has classified the input vector into a specific one of K classes because that class had the maximum probability of being correct.

6. Fault type detection

In this work, the PNN is used to classify the winding faults. Fault identification using PNN is formed from three steps: in the first step measurements should be carried out to acquire TFs needed. The second step is related to feature extraction, in which the most proper features found for classification. In the third step, using features found in the prior step, the classification will be done. Extracted features in previous items are used for training PNN. Then, using trained PNN decision will be made on new data.

6.1. Feature Extraction

One of the important stages in any pattern recognition process is the feature extraction. If proper features are selected for a problem, pattern recognition will be done more successfully.

In detection of transformer winding fault, features extraction is based on using the information of TFs. At the intact conditions, the measured TFs are considered as a reference TF and the other TFs (in faulted conditions) are compared against the reference one.

One of the possible methods for comparing TFs with reference TF is using statistical indices such as CC. Because of similarity of TFs, the outcome will be same at many cases, so it doesn't has necessary efficiency for PNN training. Beside, PNN work best when many training data are available, while using parameters such as CC, the number of input data of PNN will be restricted. So, in PNN training it is important to use an index which has both a lot of data and high reliability.

Realizing that the frequency and amplitude in peak and trough points can be used as reliable indices to train the PNN when faults occur in the winding, indirectly TFs are compared through examination of these points which can be detected easily by simple calculations. In this regard, the defined indices in (1), (2) can be applied as an input to PNN.

6.2. Training Procedure

To train PNN first of all, its structure (input/output data) should be determined. For this purpose, the defined indices in equations (1), and (2) are used for training of PNN. Therefore, input matrix can be defined as follows:

$$I = \begin{bmatrix} IFR_{t1,AD_j} & IFR_{t1,RD_s} & IFR_{t1,DSV_{w,z}} & IFR_{t1,SC_l} \\ \vdots & \vdots & \vdots & \vdots \\ IFR_{tk,AD_j} & IFR_{tk,RD_s} & IFR_{tk,DSV_{w,z}} & IFR_{tk,SC_l} \\ IFR_{p1,AD_j} & IFR_{p1,RD_s} & IFR_{p1,DSV_{w,z}} & IFR_{p1,SC_l} \\ \vdots & \vdots & \vdots & \vdots \\ IFR_{pi,AD_j} & IFR_{pi,RD_s} & IFR_{pi,DSV_{w,z}} & IFR_{pi,SC_l} \\ IAR_{t1,AD_j} & IAR_{t1,RD_s} & IAR_{t1,DSV_{w,z}} & IAR_{t1,SC_l} \\ \vdots & \vdots & \vdots & \vdots \\ IAR_{tk,AD_j} & IAR_{tk,RD_s} & IAR_{tk,DSV_{w,z}} & IAR_{tk,SC_l} \\ IAR_{p1,AD_j} & IAR_{p1,RD_s} & IAR_{p1,DSV_{w,z}} & IAR_{p1,SC_l} \\ \vdots & \vdots & \vdots & \vdots \\ IAR_{pi,AD_j} & IAR_{pi,RD_s} & IAR_{pi,DSV_{w,z}} & IAR_{pi,SC_l} \end{bmatrix} \quad (3)$$

Where:

k, i: represent the number of trough and peak points in TFs, respectively,
 j, s: show the level of AD and RD, respectively,
 w: illustrates the location (state) of DSV, and z: shows level of DSV,
 l: demonstrates the location of SC

Output of PNN can have four different classes (according to four faults) which are:

Class number 1: AD

Class number 2: RD

Class number 3: DSV

Class number 4: SC

As a result, the output of PNN is single dimension vector that shows the type of fault.

After finding relation (3) from the first group of transformers, results of these calculations are applied to PNN as an input.

It should be noted that in order to obtain a suitable measure for comparison, the feature extracted (equations 3) should be normalized using the following equation:

$$X = \frac{x - \mu}{sd} \quad (4)$$

Where x stands for each of the matrix rows (in equation 3), μ and sd denote the mean value and standard deviation of x, and X is the normalized vector of x.

6.3. Classification Results and Discussion

After training, the obtained data from group 2 transformers are applied to the PNN as a testing input for prediction of fault classes. The PNN response to these test data are shown in Tables 1, 2. A close observation of these tables shows that the PNN is able to identify the fault type correctly in most of the cases.

In order to prove the capabilities of the proposed method, a comparison between PNN technique and past methods [10]-[11] is carried out here. As displayed in Tables 1, 2 the accuracy of PNN in the detection of fault type is much better than previously proposed methods.

With a careful examination of the output of PNN, as shown in tables 1, 2 the network is 100% correct in recognition of the faults of classes 1, 2 and 4 (AD, RD and SC). It can be concluded that in different cases of AD, RD and SC the amount of variations in the measured TFs has a regular trend, if compared with its prior state which is different from other faults. For example, variation of TF characteristics due to AD in the winding occurs in the high frequency range (approximately above 100 KHz); whereas the variations due to occurrence of SC have been distributed in the entire frequency ranges [11]. These differences result in specific changes in the frequency and magnitude of peak and trough points under various faults conditions and consequently PNN recognizes the fault type correctly. However, interference was identified in two cases in class number 3 with class numbers 1 and 2. This shows that the patterns of DSV under a small variation (7.5 mm) are close to AD patterns. Similarly, patterns of DSV under large variation (25 mm) are close to RD patterns. Therefore, network can't distinguish between them and classification has been wrongly done in these cases. Consequently, PNN was able to correctly (in most of the cases) recognize the type of winding fault for transformers of different sizes, while its data

is not employed in training of network. So, it can be used for detection of the type of fault in transformer winding.

Table 1: Results of fault type detection using different methods

Fault type (actual)	Prediction		
	PNN	α [10]	FI [11]
AD ₁	AD	AD	AD
AD ₂	AD	AD	AD
AD ₃	AD	AD	AD
AD ₄	AD	AD	AD
AD ₅	AD	AD	AD
AD ₆	AD	AD	RD
AD ₇	AD	AD	RD
AD ₈	RD	DSV	RD
RD ₁	RD	AD	AD
RD ₂	RD	RD	RD
RD ₃	RD	RD	RD
RD ₄	RD	RD	RD
DSV ₁	AD	DSV	AD
DSV ₂	DSV	DSV	AD
DSV ₃	DSV	DSV	DSV
DSV ₄	DSV	DSV	DSV
DSV ₅	RD	DSV	DSV
SC ₁	SC	DSV	SC
SC ₂	SC	DSV	SC

Table 2: Accuracy (in percent) of different methods in winding fault detection

Method	PNN	α [10]	FI [11]
Fault class			
Class 1 (AD)	100	87.5	62.5
Class 2 (RD)	100	75	75
Class 3 (DSV)	60	100	60
Class 4 (SC)	100	0	100
Total	89.4	78.9	68.4

7. Conclusion

Due to importance of fault type detection in transformer winding and lack of reliable method in this field, a new method for winding fault type determination is proposed by application of mathematical indices and PNN technique. The proposed method is able to distinguish different fault types of AD, RD, DSV and SC. For training and testing purposes of the PNN algorithm, the measured data related to two groups of transformers is employed. After finding the peak and trough points of the measured TFs, the frequency and magnitude of these points in intact condition with respect to the faulted condition, of group 1 transformers are applied to PNN algorithm, for its training. And the similar measured parameters from group 2 transformers are used for validation of the method. The testing process reveals that the proposed method, based on PNN, has a high accuracy of about 89.4%. Comparing with the available methods in the literatures, this can be recognized as a reliable method for detection of transformers winding fault type.

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