Study of approximate methods accuracy for analyzing dynamic performance of stressed power systems

Heavily loaded, stressed power systems exhibit complex nonlinear dynamic behaviors which cannot be analyzed and described accurately by conventional linear methods such as eigen-analysis. Normal form of vector fields, a well established mathematical method, and Modal Series technique, a relatively newly established approach have been used as tools to analyze, characterize and quantify some of these sophisticated behaviors such as low frequency inter-area oscillations in stressed power systems. Normal Form method has been used extensively in recent years for analysis of nonlinear modal interaction and this interaction's role in causing inter-area oscillations after occurrence of the large disturbances. However, Normal Form has some shortcomings which must be further highlighted. In this paper some of these shortcomings are addressed by use of simple examples. The IEEE 50-generator test system is simulated and based on the obtained results from linear modal, Normal Form, and Modal Series methods, performance and accuracy of these methods are investigated. It is shown that normal form technique cannot simulate stressed power system well in some regions of its operating space. Also superiority of Modal Series method and its larger region of validity in the state space of the systems is illustrated.

Keywords: Nonlinear modal interaction, Modal Analysis, Normal Form, Modal Series, Power system dynamics

1. INTRODUCTION
Modern interconnected power systems are operating in stressed condition, especially with the advent of restructuring and deregulation. Heavily loaded and stressed power systems exhibit complex dynamic behaviors when subjected to disturbances whether large or small. When a stressed power system is subjected to a large disturbance, it exhibits complex behavior, not detectable from linear system analysis. For example the inter-area mode phenomenon in stressed power systems and auto- and hetero-parametric resonances can be addressed as some of these complex behaviors [1-3]. In recent years Normal Form (NF) analysis has been used to investigate and quantify nonlinear interactions between power system modes [4-9]. Applications include control system design [4], [10-12], approximation of stability boundary [13-15], and predicting of inter-area separation [6-7], [16-17]. [18] is a relatively complete survey on the applications of normal form method in small signal stability of power systems. The method of normal form is a well-established mathematical procedure for simplifying nonlinear differential equations, [19-23]. Using this method, provided certain conditions are met, a set of nonlinear differential equations can be transformed, up to a desired order, into a set of linear differential equations by performing a sequence of nonlinear coordinate transformations. The transformed equations are in their simplest form, i.e., in their normal form, and allow for the study of essential modal characteristics. An important characteristic of this approach is that it provides an approximate closed form solution for the system state variables. These last two features are key elements in the analysis of power system stability. Normal Forms are used to identify the nonlinear interaction among the power system natural modes of oscillation. These interactions are then quantified in terms of the modal solutions in the original system states. The interaction coefficients obtained via the NF analysis identify the interacting modes. However, as normal form technique is an approximate method, solutions it provides do not
coincide exactly with the actual responses of the system. In [24] this feature of the NF method has been considered and several indices for its better utilization have been represented, but the drawbacks to this method have not yet been discussed. The method of Modal Series (MS) is another approximate technique for the analysis and study of nonlinear dynamical systems. It is looked up as an efficient complement or alternative to NF, [25, 26].

In this paper, it will be shown that in some region around stable equilibrium point, even near it, NF technique fails to simulate nonlinear dynamic behaviors of power systems. Also, it will be shown that the region of validity of NF technique shrinks near resonance condition, and that at some regions of state space, linear analysis (LA) works better than NF in simulating nonlinear systems. The obtained results show that MS method provides results which simulate the behavior of stressed power system more accurately than either linear analysis or NF method.

The paper has been organized as follows: In sections 2 a summary of approximate methods are presented. Shortcomings of the normal form method have been addressed in Section 3. The power system modeling and simulation and comparison of the accuracy of different methods are carried out in Section 4. Finally conclusion is presented in section 5.

2. APPROXIMATE METHODS

At first, fundamental and common concepts for both normal form and Modal Series methods are presented and then each method is summarily introduced.

2.1 Taylor Series Expansion and Jordan Form Transformation

A wide class of nonlinear dynamical systems, including power systems, can be modeled by differential equations of the form:

\[ \dot{X} = F(X) \]  

(1)

where, \( X \) is the \( N \) dimensional state vector and \( F : R^N \rightarrow R^N \) is a smooth vector field (when only sinusoidal nonlinearity is considered, it would be analytic as well). Often, behavior of the system in a neighboring of an equilibrium point is desired and studied. Expanding (1) in a Taylor series around a stable equilibrium point \( X_{SEP} \) and using again \( X \) and \( x_i \) as the new state vector and state variables, to refer to \( X - X_{SEP} \) and \( x_i - x_{SEP} \), yields the following representation:

\[ \dot{x}_i = A_i X + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} H_{k}H_{j} x_k x_j + \frac{1}{6} \sum_{P=1}^{N} \sum_{Q=1}^{N} \sum_{R=1}^{N} P_{PQR} x_P x_Q x_R + \cdots \]  

(2)

where, \( X \) belongs to the convergence domain of the Taylor series \( \nu \subseteq R^N \), and \( i = 1, 2, .., N \) \( A_i \) is the \( i \)-th row of Jacobian matrix \( A = \left( \frac{\partial F}{\partial X} \right)_{X_{SEP}}, H_i = \left( \frac{\partial^2 F}{\partial X^2} \right)_{X_{SEP}} \) is Hessian matrix, \( P_{PQR} = \left( \frac{\partial^3 F}{\partial X_P \partial X_Q \partial X_R} \right)_{X_{SEP}} \), and so on. Assuming the system has \( N \) distinct eigenvalues, \( \lambda_j, j = 1, 2, ..., N \) and denoting the matrices of the right and left eigenvectors of \( A \) by \( U \) and \( V \), respectively, the similarity transformation \( X = U Y \) yields the following equivalent set of differential equations for (2).
\[ \dot{y}_j = \lambda_j y_j + \sum_{k=1}^{N} \sum_{l=1}^{N} C_{jl}^{k} y_k y_l + \sum_{p=1}^{N} \sum_{q=1}^{N} \sum_{r=1}^{N} D_{pq}^{r} y_p y_q y_r + \cdots \]  

(3)

where, \( Y \) belongs to the linear mapping of \( \Theta \) denoted by \( \Theta \subset C^N \) under defined linear transformation. Here and from now on \( j=1,2,\ldots,N \),

\[ C^j = \frac{1}{2} \sum_{p=1}^{N} \sum_{q=p}^{N} \sum_{r=p}^{N} \sum_{l=1}^{N} \sum_{k=1}^{N} C_{pqr}^{kl} y_p y_q y_r y_{rl} \]

(4)

\[ D^j = \frac{1}{6} \sum_{p=1}^{N} \sum_{q=1}^{N} \sum_{r=1}^{N} \sum_{l=1}^{N} \sum_{k=1}^{N} \sum_{m=1}^{N} \sum_{n=1}^{N} D_{pqr}^{lm} y_p y_q y_r y_{lm} \]

(5)

\( Y_p \) is the \( p \)th element of the \( P \)th eigenvector and so forth.

### 2.2 Linear Modal Analysis

Starting with the set of (3), in linear modal method of analysis, only the linear first term is retained to obtain:

\[ \dot{y}_j = \lambda_j y_j \]

(6)

\[ y_j(t) = y_{j0} e^{\lambda_j t} \]

(7)

Inverse transformation of (6) yields:

\[ x_i(t) = \sum_{j=1}^{N} u_{ij} y_{j0} e^{\lambda_j t} = \sum_{j=1}^{N} L_{ij} e^{\lambda_j t} i = 1,2,\ldots,N \]

(8)

where, \( y_{j0} \) is the \( j \)th element of \( Y_0 = U^{-1} X_0 \), \( X_0 \) is the initial condition in physical state-space, and \( L_{ij} = u_{ij} y_{j0} \). In linear systems, \( e^{\lambda_j t} \)s are called modes of system. Although a linear modal analysis gives a physical insight into system behavior, it is only valid in a small region containing the equilibrium point. This method of analysis is not capable of describing stressed power system behavior accurately when it is subjected to an even moderately larger disturbance. Neglecting higher order terms in Taylor series expansion is the origin of this weakness.

### 2.3 Normal Form Analysis

The version of normal form of a vector field employed in this work, (Poincare’s normal form), is the simplest member of an equivalence class of vector fields, all exhibiting the same qualitative behavior [19]. To get this normal form, successive polynomial transformations are used. By these transformations, in the absence of resonance condition, which is defined later, the smallest order of nonlinear terms in the new coordinates is increased. To have the closed form approximate solution, high order nonlinear terms in the new coordinate are ignored and a linear system is obtained [21]. This linear system has the form \( \dot{Z} = \Lambda Z \) where \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n) \) and \( \lambda_i \) is an eigenvalue of the original system for
each $i$. The response of the linear system with initial condition $z_0$ is calculated and by applying successive transformation, approximate closed form solution in the original coordinates is obtained. In normal form theory a set of system modes is said to be resonant of order $r$ if $\lambda_i = \sum_{j=1}^{N} m_j \lambda_j$ and $r = \sum_{j=1}^{N} m_j$ for $i \in \{1, \ldots, N\}$, where $r$ and $m_j$ are integer and $\lambda_j$’s are linear modes of the system. By neglecting the third and higher order terms in (3), and considering the case without second order resonance, normal form technique offers the transformation [4]:

$$Y = Z + h2(Z) \quad \text{or} \quad y_j = z_j + h2^j(Z)$$  \hspace{1cm} (9)

where,

$$h2^j(Z) = \sum_{k=1}^{N} \sum_{l=1}^{n} h_{kl}^j \bar{z}_k \bar{z}_l \quad j = 1, \ldots, N$$  \hspace{1cm} (10)

$$h_{kl}^j = \frac{C_{kl}^j}{\lambda_k + \lambda_l - \lambda_j}$$  \hspace{1cm} (11)

In $Z$-coordinates, the system (3) takes the form:

$$\dot{z} = \lambda_j z_j + \text{Order}(|z|^3)$$  \hspace{1cm} (12)

By neglecting higher order terms in (12), an explicit second order approximate solution can be found as:

$$z_j(t) = z_{j0} e^{\lambda_j t}$$  \hspace{1cm} (13)

$$y_j(t) = z_{j0} e^{\lambda_j t} + \sum_{k=1}^{N} \sum_{l=1}^{n} h_{kl}^j z_k z_l e^{(\lambda_k + \lambda_l) t}$$  \hspace{1cm} (14)

$$x_i(t) = \sum_{j=1}^{N} u_{ij} z_0 e^{\lambda_j t} + \sum_{j=1}^{N} u_{ij} \left[ \sum_{k=1}^{N} \sum_{l=1}^{n} h_{kl}^j z_k z_l e^{(\lambda_k + \lambda_l) t} \right]$$  \hspace{1cm} (15)

where,

$$Y_0 = VX_0 = U^{-1} X_0$$  \hspace{1cm} (16)

$$Y_0 = Z_0 + h2(Z_0)$$  \hspace{1cm} (17)

### 3.3 Modal Series method

By using Modal Series method, it is possible to represent nonlinear dynamic systems, as well as stressed power systems, in a manner which yields a good deal of physical insight into the problem under consideration. This method of solution also has the great conceptual
advantage of presenting a nonlinear system as a rather straightforward generalization of the linear case. Moreover, this method provides a solution to the differential equations even in the case of resonance condition. As with normal form technique, this method is restricted to polynomial nonlinearity; therefore, the Taylor series expansions of other nonlinearity types are needed.

It has been shown that the solution of (3) for the initial condition \( Y_0 = [y_{10}, y_{20}, \ldots, y_{N0}]^T \) can be written as [25, 26]:

\[
y_j(t) = y_{j0}(t) + y_{2j}(t) = \left( Y_0 - \sum_{k=1}^{N} \sum_{l=1}^{N} h_{kl}^j y_{k0} y_{l0} \right) e^{\lambda_j t} + \sum_{k=1}^{N} \sum_{l=1}^{N} h_{kl}^j y_{k0} y_{l0} e^{(\lambda_k + \lambda_l) t} + \sum_{k=1}^{N} \sum_{l=1}^{N} C_{kl}^j y_{k0} y_{l0} t e^{\lambda_j t} \right) e^{(\lambda_k + \lambda_l) t} \right) e^{(\lambda_k + \lambda_l) t}
\]

The coefficients of \( h_{kl}^j \)'s are defined in (11) and the set \( R_2 \) contains all three tuples \( (k, l, j) \), which cause the second order resonance condition, i.e., satisfy \( \lambda_k + \lambda_l = \lambda_j \). Similar procedure may be carried out to calculate higher order terms. The condition \( |\lambda_k + \lambda_l - \lambda_j| \leq 0.001|\lambda_j| \) is so-called second order quasi-resonance and denotes by \( R'_2 \) the set of all three tuples \( (k, l, j) \), which cause the second order quasi-resonance. Rearranging second order modal effects i.e., the second term in (19), and defining new constants \( L_{i,j}^{MS}, S_{i,kl}^{MS} \) and \( M_{i,j}^{MS} \) as:

\[
L_{i,j}^{MS} = u_j \left( y_{j0} - \sum_{k=1}^{N} \sum_{l=1}^{N} h_{kl}^j y_{k0} y_{l0} \right) \right) e^{(\lambda_k + \lambda_l) t}
\]

\[
S_{i,kl}^{MS} = u_j h_{kl}^j y_{k0} y_{l0} \right) e^{(\lambda_k + \lambda_l) t} \right) e^{(\lambda_k + \lambda_l) t}
\]

\[
M_{i,j}^{MS} = \sum_{k=1}^{N} \sum_{l=1}^{N} u_j C_{kl}^j y_{k0} y_{l0} \right) e^{(\lambda_k + \lambda_l) t} \right) e^{(\lambda_k + \lambda_l) t}
\]

The second order approximate response becomes:

\[
x(t) = \sum_{j=1}^{N} L_{i,j}^{MS} e^{\lambda_j t} + \sum_{j=1}^{N} M_{i,j}^{MS} t e^{\lambda_j t} + \sum_{k=1}^{N} \sum_{l=1}^{N} S_{i,kl}^{MS} e^{(\lambda_k + \lambda_l) t}
\]

\[
= \sum_{j=1}^{N} \left( L_{i,j}^{MS} + M_{i,j}^{MS} t \right) e^{\lambda_j t} + \sum_{k=1}^{N} \sum_{l=1}^{N} S_{i,kl}^{MS} e^{(\lambda_k + \lambda_l) t}
\]
When \( R^2 \) is empty, i.e. there is no second-order resonance condition, the approximate solution is given by:

\[
x(t) = \sum_{j=1}^{N} L_{n,j} e^{\lambda_j t} + \sum_{k=1}^{N} \sum_{l=1}^{N} S_{n,m} e^{(\lambda_k + \lambda_l) t}
\]  

(21)

3. SHORTCOMINGS OF NORMAL FORM TECHNIQUE

There are a few issues that limit the use of NF method in studying stressed power system. These issues (one is tempted to call them problems) are as follows:

(I) The nonlinear transformation needed for NF method is neither onto nor one to one. This may result in multiple solutions in some cases or non-convergence of the algorithm used for calculation of initial conditions, \( z_0 \).

(II) The nonlinear transformation introduces variables, and obtains the solution in terms of these variables, that do not directly correspond to physical state or modal phenomena.

(III) Solving nonlinear algebraic equation to obtain \( z_0 \) for a practical large size power system is not an easy task;

(IV) In general normal form technique fails to obtain a closed form solution, when there is second, or higher, order resonance condition. However, an approximation solution can be found when some conditions hold [8].

The shortcomings of the normal form technique to simulate nonlinear system behavior are illustrated using two examples. In the first example, a one-dimensional system is used to show the problems that are caused by the nonlinear transformation in the normal form technique. The aim of the second example is to demonstrate the effect of the near resonance condition on the accuracy of normal form results.

**Example1:** by using normal form, we want to eliminate the second order term of the system:

\[
\dot{y} = \lambda y + \frac{1}{2} Hy^2
\]

From (9)-(11) one can obtain the following transformation:

\[
y = z + \frac{1}{2\lambda} Hz^2
\]

(23)

Applying this transformation to (22) yields:

\[
\dot{z} = \lambda z + \left( 1 - \frac{2 + \frac{1}{2\lambda} Hz}{1 + \frac{1}{2\lambda} Hz} \right) Hz
\]

(24)

Expanding the fractional term of (24) by introducing limitative assumption, \( |(\lambda\dot{z})Hz| < 1 \) yields:

\[
\dot{z} = \lambda z + \left( \frac{1}{2\lambda} Hz^3 - \frac{1}{8\lambda^2} Hz^4 + \frac{1}{8\lambda^3} Hz^5 + \cdots \right)
\]

(25)

From now on, any effort to eliminate higher order terms is subject to that limitative assumption and introduces other limitations. These sequential limitations cause the region of validity of the closed form solution obtained from normal form to shrink. Neglecting the nonlinear term in (24) gives an approximation closed form solution of (22), called second order modal, as:
\[ y(t) = z_0 e^{\lambda t} + \frac{1}{2\lambda} H z_0^2 e^{2\lambda t} \]  \hspace{1cm} (26)

Linear approximate solution of (22), called linear modal, is given as:
\[ y(t) = y_0 e^{\lambda t} = \left( z_0 + \frac{1}{2\lambda} H z_0^2 \right) e^{\lambda t} \]  \hspace{1cm} (27)

Is the accuracy of the (26) always better than (27)? To answer this question, let us define two nonlinearity measures as the ratio of the absolute values of nonlinear terms to linear term in (22) and (24) by \( R_y \) and \( R_z \) respectively:
\[
R_y = \left| \frac{H}{2\lambda} \right| = \left| \frac{H}{2\lambda} \left( z + \frac{1}{2\lambda} H z_0^2 \right) \right| = \left| Z^2 - Z \right| \]  \hspace{1cm} (28)
\[
R_z = \left| \frac{Z^2(2-Z)}{1-2Z} \right| \]  \hspace{1cm} (29)

where \( Z = -\frac{H}{2\lambda} z \). In Fig.1, \( R_y \) and \( R_z \) are plotted. As shown in the figure \( R_z > R_y \) for \( Z > 0.2324 \), i.e., linear modal is more accurate than second order modal for \( Z > 0.2324 \).

Simulation result in Fig.2 proves this conclusion for \( Z = 0.5 \), equivalent to \( y_0 = 0.05 \), \( H = 1 \) and \( \lambda = -0.1 \).
Another problem originates from the nonlinear transformation. Polynomial transformations that are used by normal form technique are neither one to one nor onto. Therefore, some parts of state space may not be covered by those transformations. For example, assuming \( \lambda \) to be real and negative, Fig.3, plot of (23), shows that for the region specified by \( y > - \frac{\lambda}{2H} \) there is no solution for \( z \). Therefore if initial condition is such that \( y \) belongs to that region, second order modal approximate solution based on normal form fails, and (17) does not have any actual solution.

\[
\begin{align*}
\text{Fig. 3: Plot of Normal Forms Technique Transformation} \\
\end{align*}
\]

Example2: This second example shows that near resonance conditions may exacerbate the aforementioned problem.

Let, \( x_{ILA}(t), x_{INF}(t), x_{MS}(t) \) and \( x_{NL}(t) \) denote linear modal, second order normal form, Modal Series and nonlinear time domain step by step simulation results for state \( i \), respectively, and also \( X_{ILA}(\omega), X_{INF}(\omega), X_{MS}(\omega) \) and \( X_{NL}(\omega) \) denote their Fourier transformation magnitudes. A likeness or similarity index between nonlinear and other approximated simulations is defined as:

\[
L_m = \sum_{i=1}^{N} \int_{0}^{\infty} |X_{IL}(\omega) - X_{other}(\omega)| d\omega 
\] (30)

For the following numerical system, the domain within \( L_m \leq 2.5 \) is obtained for each approximate solution’s simulation and for two values of \( \alpha \).

\[
\dot{X} = AX + \begin{bmatrix} X^TH^1X & X^TH^2X \end{bmatrix} 
\] (31)

where:
Also, to be able to compare Modal Series and normal form solutions under similar conditions, only two terms of Taylor series expansion are used in both cases. This system has two eigenvalues, $\lambda_1 = -0.2$ and $\lambda_2 = \alpha$. If $\alpha$ is chosen such that $2\lambda_1 = \lambda_2$, then normal form fails to give closed form solution, because a resonance condition results. The region of stability of the system has been shown in Figs. 4.a and 4.b. The filled regions in these figures are the regions where normal form cannot span. These figures show that the domain of attraction is not changed considerably but, for $\alpha = -0.41$, i.e., near resonance condition (Fig. 4.a); the region around origin that normal form is applicable becomes smaller than that for $\alpha = -0.45$ (Fig. 4.b). For more details, we focus our attention on the region that is surrounded by rectangle in each figure.

In Fig. 5, circular, ellipsoidal and lunulate show the regions of validity of linear, Modal Series and normal form approximation for $L_m < 2.5$. It can be seen in the Fig. 5 that the accuracy region of the modal series technique is wider than those of normal form and linear modal methods. On the other hand there are some regions around the stable equilibrium point where the accuracy of linear approximation is better than that of the second order normal form.
4. STUDY ON THE STRESSED POWER SYSTEMS
In this section, the 50-bus IEEE power system is analyzed using the LA, NF, and MS methods.

4.1 Power system model
The dynamical behavior of the system is governed by the dynamic characteristics of the individual elements such as synchronous machines, excitation systems, loads, and the power network. It is assumed that in an $n$-generator system, $m$ generators are represented by two-axis model and equipped with exciter and the remaining $n-m$ generators are presented by classical model. The block diagram of the exciter model used is shown in Fig.6.

![Excitation system model (IEEE Type AC4A)](image)

All the loads are represented by constant impedances, and the network is reduced to the generator internal nodes [27]. With these assumptions, the state vector of the system will be:

$$X = \left[ E_{d1}', E_{q1}', \omega_1, \delta_1, E_{FD1}, X_{E11}, X_{E12}, ..., E_{qm}', E_{dm}', \omega_m, \delta_m, E_{FDm}, X_{E1n}, X_{E2n}, ..., \omega_n, \delta_n \right]^T$$

$E_{d1}', E_{q1}'$: transient direct and quadrature axes EMFs of stator;
$\omega$: rotor speed with respect to a synchronous reference frame;
$\delta$: rotor angle;
$E_{FD}$: stator EMF corresponding to the field voltage;
$X_{E1}, X_{E2}$: exciter state variables as shown in Fig. 1.

$X$ is the $N$ dimensional state vector ($N = 7 \times m + 2 \times (n - m) - 1$). In general, the dynamics of the system can be described by (1) where $F: R^N \rightarrow R^N$ is a smooth vector field.
4.2 Case studies (Numerical simulation)

The IEEE 50-generator test case system is shown in Fig. 7. This system demonstrates a wide range of dynamic characteristics at different loading levels and fault scenarios. The system is known to exhibit the inter-area mode behavior in response to certain disturbance locations. The inter-area mode is seen as the instability in the positive direction of a large number of generators. In some cases, one machine is perturbed in the negative direction. The Generators at buses 93, 104, 105, 106, 110, and 111 are represented by a fourth-order d-q axis model and a fast exciter. The other generators are represented by classical models. The system was subjected to a number of three phase faults in different operating conditions. Fault locations were chosen so that the system response exhibit both plant and inter-area modes. We push the system to higher stressed conditions by increasing of generations of units 93 and 110 each from 700 MW to 1300 MW.

As fault clearing times are increased, the normal form will reach a boundary for each operating condition beyond which it fails to converge. We determined values of these clearing times for different fault locations and for each operating condition heuristically. The results are shown in Table 1. Although different initial guesses for solving nonlinear algebraic equation $\frac{Z_0 - h_2(Z_0) - Y_0}{Z_0} = 0$ results in convergence to different solutions for $Z_0$, the main conclusion that with increase of stress, normal form domain of validity shrinks stays valid. This confirms the result obtained in example 2 of section 3. This problem is more prominent in cases which result in inter-area mode type behavior. In the plant mode
type behavior, normal form method often responds up to the stability boundary of the system.

Table 1: Maximum fault clearing times beyond which Normal Form transformation fails to converge, [ms]

<table>
<thead>
<tr>
<th>Fault Location at Bus</th>
<th>Operating Condition of Gen.93 &amp; Gen.110</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2*700 MW</td>
</tr>
<tr>
<td>100</td>
<td>263</td>
</tr>
<tr>
<td>112</td>
<td>257</td>
</tr>
<tr>
<td>1</td>
<td>132</td>
</tr>
<tr>
<td>7</td>
<td>120</td>
</tr>
<tr>
<td>25</td>
<td>110</td>
</tr>
<tr>
<td>124</td>
<td>143</td>
</tr>
</tbody>
</table>

The following error index or distance measure is defined and used to measure accuracy of the results obtained from different approximate methods. By using this index, the similarity between time responses of linear modal, normal form and Modal Series methods with nonlinear time domain response are compared.

\[
P_{m,pe} = \frac{\int_0^\infty X_{NLi}(\omega) - X_{otheri}(\omega) d\omega}{\int_0^\infty X_{NLi}(\omega) d\omega}
\]

where \( X_{NLi}(\omega) \) and \( X_{otheri}(\omega) \) are as in (30).

Due to space constraints and since the emphasis in this paper is on dynamic behaviors of stressed power systems, only a highly stressed condition has been selected for reporting. The load level of each generator at plant A at buses 93 and 110 are set to 1100 MW. The faults are applied and cleared with no line removed. The data for the system are taken from [28]. The system is subjected to three phase stub fault at buses 1, 7, 112, 100 and 25. Fig. 8 shows the time evolution of four state variables of the system obtained from nonlinear time domain simulation (the exact solution) and the three approximate simulations. This figure shows the much better accuracy of the Modal Series simulation compared to either liner or normal form simulation.

Table 2: Comparison of distance measure (error index) obtained from different approximate methods

<table>
<thead>
<tr>
<th>Fault at Bus 1, cleared in 90 [msec]</th>
<th>Fault at Bus 25, cleared in 60 [msec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>State # ( \omega_{43} ) Linear Modal</td>
<td>State # ( \delta_4 ) Linear Modal</td>
</tr>
<tr>
<td>38.93%</td>
<td>25.13%</td>
</tr>
<tr>
<td>35.98%</td>
<td>43.85%</td>
</tr>
<tr>
<td>24.71%</td>
<td>15.44%</td>
</tr>
<tr>
<td>( E_{q6} ) Linear Modal</td>
<td>( E_{d2} ) Linear Modal</td>
</tr>
<tr>
<td>21.48%</td>
<td>10.33%</td>
</tr>
<tr>
<td>6.41%</td>
<td>57.02%</td>
</tr>
<tr>
<td>2.52%</td>
<td>19.53%</td>
</tr>
<tr>
<td>( E_{d4} ) Linear Modal</td>
<td>( E_{q2} ) Linear Modal</td>
</tr>
<tr>
<td>14.65%</td>
<td>10.01%</td>
</tr>
<tr>
<td>7.53%</td>
<td>4.58%</td>
</tr>
<tr>
<td>2.51%</td>
<td>1.54%</td>
</tr>
<tr>
<td>( \omega_1 ) Linear Modal</td>
<td>( \omega_5 ) Linear Modal</td>
</tr>
<tr>
<td>46.23%</td>
<td>35.74%</td>
</tr>
<tr>
<td>29.31%</td>
<td>22.25%</td>
</tr>
<tr>
<td>12.96%</td>
<td>10.29%</td>
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Also, numerical values for distance measures for a few states for two fault scenarios have been calculated and shown in Table 2. The results validate our judgments about more accurate performance of the Modal Series method. The selected fault clearing times in Table 2, and in simulations of Fig.8, are less than the critical clearing times of the system. This is because of divergence problem of normal form method in determination of initial conditions, $z_0$'s, which does not allow selecting longer fault clearing times.
The time evolutions shown in Fig.8 are due to three phase fault at bus 1 which causes inter-area mode oscillations as do faults at buses 7 and 25. Three phase faults in buses 1, 7, and 25, however, exhibit plant mode oscillations. In cases in which inter-area mode behavior emerges, nonlinearity effects are more significant and differences between linear and nonlinear responses are more pronounced.

5. CONCLUSION
The method of normal form has been gainfully used as a tool for the study of dynamic performance of stressed power systems. In spite of its wide spread use, the method suffers from some shortcomings. These shortcomings are pointed out as: (a) under resonance conditions, it fails; (b) its nonlinear transformation is neither onto nor one to one resulting in multiple solutions; (c) determination of initial conditions in normal form coordinates, $z_0$’s, specifically for highly stressed cases may not be possible. Under these situations, the numerical solution algorithms have difficulty converging. Modal Series method, on the other hand, does not require or use nonlinear transformation and therefore does not suffer from the above shortcomings. Using a simple example, it was shown that the normal form may return more than a single solution even for a low dimension dynamical system. The same example also showed that for a range of initial conditions
normal form failed to provide solutions. The other example illustrated that changing the operating point of the system and moving closer to resonance condition, shrinks the validity region of the normal form method. Also validity regions of linear modal, Modal Series and normal form methods were compared. The results show that Modal Series has a much larger validity region. Dynamic behavior of the IEEE 50-generator test case system under stressed condition was also simulated under severe three phase fault condition. Time domain nonlinear step by step numerical simulations was used as exact system behavior and the results of simulations using the three approximate methods were compared with it. It was shown that the performance of Modal Series method is more accurate and obtainable under wider sets of conditions.

References


