Regular paper

Linearizing control of a photovoltaic structure and stability by Lyapunov directly on bond graph

The photovoltaic powered water pumping system investigated in this paper consists mainly of a photovoltaic generator, a boost converter, a three phase inverter, and an induction motor-pump. Then, we present a method which resolved directly on the bond graph the problem of input/output linearization of the nonlinear system while basing itself on input/output causal inversion of its BG model. Analysis of stability by Lyapunov directly on BG model of the structure is presented. Results of simulation are given for various variables of the structure.

Keywords: Bond graphs, induction motor-pump, converter, photovoltaic, input/output linearization, causal inversion, control, stability by Lyapunov

1. INTRODUCTION

The Bond Graph methodology provides a particularly formalism to represent the energy transformations in the heterogeneous systems because of its unifying character with respect to the various fields of physics [1] based on the energy exchange between under systems; it is a power transfer. It is very simple to specify on a model BG the causal relations between the signals associated with the physical magnitudes with the system. Once causality is assigned, we can derive directly from the BG of the very current causal models in the automatic (state equation, transfer function).

We present in this communication, in the first time, an average model BG of photovoltaic structure, then, since our system is nonlinear, we propose to use a nonlinear technique of control which was well developed during the last decade, this technique of command allows to obtain a linear ordering of the system by holding account of all non the linearity by exploiting of advantage the approach bond graph in the determination of the expression of the linearizing control. The method bases on the input-output causal inversion of the bond graph of the system following the concept of bicausality [2]. It consists in deducing the laws from input-output control directly on bond graph.

Then we study the dynamic of the zeros by the 2nd Lyapunov method directly on the bond graph of the installation, finally, we validate this study of stability by the digital simulation of the nonlinear model in closed loop.

2. BOND GRAPH MODELLING.

the diagram block of the photovoltaic system of pumping is given by figure 1.

While referring to the electric diagrams of the motor in a reference d-q dependent on the spinning field pattern and the equations mechanics and of the electromagnetic torque [6][13][16], and by directing rotor flow along the axis for a possible order, model BG corresponds is given by figure 2.
The figure below represents an association of centrifugal pump and a hydraulic network (canalizations, elbows...)

**Fig. 1.** Diagram block of installation statement of pumping

**Fig. 2.** BG model of the machine in a reference d-q turning according to the direction of rotor flow

**Fig. 3.** General diagram of the pump
The pump used is a surface pump; it is defined by a characteristic of the resistive torque:

\[
C_r(\Omega_m) = K_r \Omega_m^2 + f \Omega_m + C_{fc}
\]  

(1)

with \(K_r\) is the torque constant of pump, \(f\) is the viscous-friction coefficient, and \(C_{fc}\) is the load torque.

The centrifugal pump is also described by characteristic \(H(Q)\) given by [4]:

\[
H(Q) = a_0\Omega_m^2 + a_1\Omega_m Q + a_2 Q^2
\]  

(2)

where \(a_0\), \(a_1\) and \(a_2\) are constant parameters. \(H\) and \(Q\) are respectively the head and the flow of pumped water.

The hydraulic network is described by a load curve [4] given by:

\[
H(Q) = H_g + \Delta H
\]  

(3)

where, \(H_g\) is the geometrical head which is the difference between the free level of the water to pump and the highest point of the canalization, given by:

\[
H_g = Z_2 - Z_1
\]  

(4)

and \(\Delta H\) is the pressure losses in the whole canalization, their expression is given by:

\[
\Delta H = \rho g \Delta p
\]  

(5)

where:

\[
\Delta p = \left(\frac{\lambda}{d} + \xi\right) \frac{8 Q^2 \rho}{\pi^2 d^4}
\]  

(6)

With \(\rho\) density of water, \(g\) acceleration of gravity, \(\lambda\) is a coefficient of the regular pressure losses in the canalization., \(l\) is the length and \(d\) is the diameter of drain, \(\xi\) is a coefficient of the local or singular pressure losses in elbows, valves, and connections, of the canalization, thus, the preceding expression can be written:

\[
\Delta H = X Q^2
\]  

(7)

Thus, the bond graph model of the hydraulic structure is given by the following figure.
In the model of the figure 4, $\Omega_{\text{min}}$ being the minimal speed from which the pump starts to generate a pumped water flow. It is given by the following relation \[ 4 \]:

$$\Omega_{\text{min}} = \sqrt{\frac{-4(a_2 - X)Hg}{a_1^2 - 4(a_2 - X)a_0}}$$ \tag{8}

The circuits in electronics of power have a significant number of equations and the beach of the time-constants is relatively wide. This one can vary nanosecond with several milliseconds. The function switch "commutation" presents the disadvantage of introducing discontinuities. As for the chopper, the principal function of the inverter of tension is the energy transformation. The average model preserves this function but by removing the function switch. That causes to strongly decrease times of simulation. However, this approach is applicable only if the time-constants of the system external with the converter vary slowly by ratio the commutation period, and this case. Thus while being based on work of [8][15][11], the average bond graph model of association chopper booster and inverter of tension are given by the following figure:

**Fig.5.** model BG of the association booster and inverter of tension

With $\rho$ is the cyclic ratio of the chopper, $\eta$ is the logic of conduction, it checks the following expression:
\[ \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} = \eta \begin{pmatrix} V_c \end{pmatrix} \quad (9) \]

Where:

\[ \eta = \begin{pmatrix} \frac{2}{3} \alpha_1 - \frac{1}{2} \alpha_2 - \frac{1}{3} \alpha_3 \\ \frac{2}{3} \alpha_2 - \frac{1}{2} \alpha_1 - \frac{1}{3} \alpha_3 \\ \frac{2}{3} \alpha_3 - \frac{1}{2} \alpha_1 - \frac{1}{3} \alpha_2 \end{pmatrix} \quad (10) \]

However \( \alpha_1, \alpha_2, \alpha_3 \) are respectively cyclic ratios of the three arms of the inverter, when it acts of a symmetrical regular Pulse Width Modulation control [15], these ratios are defined by:

\[ \begin{align*}
\alpha_1 &= \frac{1}{2} + \frac{v_{\text{aref}}}{V_c} \\
\alpha_2 &= \frac{1}{2} + \frac{v_{\text{bref}}}{V_c} \\
\alpha_3 &= \frac{1}{2} + \frac{v_{\text{cref}}}{V_c}
\end{align*} \quad (11) \]

\( v_{\text{aref}}, v_{\text{bref}} \) and \( v_{\text{cref}} \) are modulating, they are generated by the adopted command to structure photovoltaic. While using (9) and (11), we obtain as follows:

\[ v_i = v_{\text{aref}} ; i = a, b, c \quad (12) \]

The passage of the three-phase representation to that diphasic is given by:

\[ P(\omega s) = \sqrt{2/3} \begin{pmatrix} \cos(\omega s t) & \cos(\omega s t - 2\pi/3) & \cos(\omega s t - 4\pi/3) \\ -\sin(\omega s t) & -\sin(\omega s t - 2\pi/3) & -\sin(\omega s t - 4\pi/3) \\ \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} \end{pmatrix} \quad (13) \]

This matrix preserves the powers, it's reversible, its opposite matrix is given by:

\[ P^{-1} = P^T \quad (14) \]

The bond graph model of the Park transformation is given by:
With $\omega_s$ is the stator pulsation in rd/s (estimated variable), it’s given by the relation of the following expression:

$$\omega_s = p_s \Omega_m + \frac{R_i L_m}{L_r} \frac{i_{qs}}{\Phi_r}$$  \hspace{1cm} (15)

Where $\Omega_m$ is the mechanical speed (measurable variable by a sensor) and $\Phi_r$ is rotor flow (estimated variable), it is given by the relation:

$$\Phi_r = \frac{L_m}{1 + s \tau_r} i_{ds}$$  \hspace{1cm} (16)

With $i_{ds}$ and $i_{qs}$ are the direct and quadratic components of the stator current (measured variable).

The general control diagram of the inverter is given by the figure 7:

Thus, we can say that:

$$v_{ds} = v_{dsref} \quad \text{and} \quad v_{qs} = v_{qref}$$  \hspace{1cm} (17)

$v_{ds}$ and $v_{qs}$ components of the stator voltage applied at the boundaries of the motor and $v_{dsref}$ and $v_{qref}$ the components d-q of modulating.
The pumping structure is fed by a photovoltaic generator, its bond graph model is given by [14][16].

\[
\text{Fig.8. Model BG of photovoltaic generator}
\]

3. INPUT-OUTPUT LINEARIZING CONTROL DIRECTLY ON BOND GRAPH

In this part, we propose to use a nonlinear approach of control which was developed during the last decade. This technique of order makes it possible to obtain a linear order by holding account of all non the linearity. The essential advantage of this approach is that for a broad part of the space of state, the controller does not need to bring back himself each time to the new point of operation to remake matrix algebra necessary. This approach is the linearizing order input-output which consists in applying to the system a change of reference frame and a return of nonlinear state in order to ensure a decoupling and the linearization of the relations between the entries and the exits [3]a found its applications in the machine to D.C. current [20] [8]et machines with AC current [10][9]. We propose of advantage of exploiting this approach in the determination of the expression of the linearizing order and the study of stability interns directly known the model jump graph of the photovoltaic structure considered. To determine the linearizing order, we follow the 4 following steps:

Steps 1 and 2: Model BG with standard and reverse causality

On BG model of figures 9 and 10, we affect standard causality, then we represent the minimal dynamic ways (mdw1), (mdw2) and (mdw3) or input-output ways (vds-Φr), (vqs-Ωm) and (ρ-lp) having a minimal number of integrators. The number of the latter on the (mdw) gives the relative degree of the input-output relation. In this case, the relative degrees are \( r_1 = 2, r_2 = 2 \) and \( r_3 = 1 \); \( r_T = r_1 + r_2 + r_3 = 5 \).

The causal inversion of the relations input-output were made while following the dynamic ways cdm1 and cdm2 of figures 9 and 10, which carries out us to have models BG given by figures 11 and 12.
Step 3: deduction of the input-output linearizing control

By supposing that the standard of flow is strictly positive in order to avoid the singularity of the machine during starting, we obtain the minimal dynamic equations edm1, edm2 and edm3 (return linearizing laws) following:

\[
\begin{align*}
    u_1 &= \frac{1}{R_\beta} v_1 + (R_\alpha + R_\alpha) p_{\alpha} - \left( R_\beta + \frac{R_\alpha^2}{R_\beta} \right) p_{\beta} - \omega p_{\alpha} \\
    u_2 &= \frac{J}{p_\beta} p_{\beta} - \left( R_\beta \frac{\omega}{p_{\alpha}} \right) p_{\alpha} - \left( f + R_\alpha + R_\alpha \right) p_{\alpha} \\
    &\quad - \left( \frac{f C_r}{J^2 p_\beta^2 p_{\alpha}} \right) p_{\beta} - \left( \frac{f C_r}{J p_\beta p_{\alpha}} \right) p_{\alpha} \\
    u_3 &= \frac{L_p}{Q_p/C_p} \left[ v_3 - V_p/L_0 \right] + 1
\end{align*}
\]
These controls are not defined for nulls flow and generator tension (not practical case) with $v_1 = P$, $v_2 = \dot{\Omega}$ et $v_3 = \dot{I}$. Given once the laws of linearizing control, it’s possible to apply one of the linear laws of control whose objective is to ensure a perfect continuation speed, flow and current of the panel to their respective values of reference $\Omega_{ref}$, $\Phi_{ref}$ et $I_{pref}$.

4. ANALYSIS STABILITY DIRECTLY ON BG

The second method of Lyapunov is used to analyze the stability of a model [17][18][20][19]. The interest to use this method is that it is valid as well for nonlinear models linear. In general, we use a square function as function of Lyapunov $V(x)$ candidate. In practice, the energy stored in the model is often selected to ensure an energy stability. However, the bond graph represents in a very explicit way this energy through elements I and C. This is why this method can be applied directly to the model bond graph. For the initial nonlinear model, the order is 6, the system is on the other hand linearized in input-output, and the order became 5, this linearizing control made then a state not observable $q_{cp}$ (in other words the terminal voltage of the capacitor $C_p$) starting from the new controls. According to the nonlinear control theory, this variety represents the dynamics of the zeros (placement with zero of the poles of the model in closed loop) of the system whose study of stability is strongly essential. The dynamics of the zeros is obtained by imposing the conditions $I_p=0$, $\Phi_r=0$ and $\Omega_m=0$. Compared to the bond graph of the figure above, this wants to say that the variables common to all the bonds attached to the junctions 1:$I_p$, 0:$\Phi_r$, et 1:$\Omega_m$ are cancel, consequently, the power in each bond as of the these junctions cancels itself. Thus model BG representing the dynamics of the zeros as follows:

![Fig. 13. The dynamic bond graph of the zeros](image)

Thus, the candidate function of Lyapunov $V(x)$ is:

$$V(q,p) = \frac{q_{cp}^2}{2 C_p}$$

The orbital derivative of this energy represents the difference between the electric output provided by the photovoltaic source $P_1$ and the consumption by the dissipative elements $P_2$, it’s given by:

$$\dot{V}(q,p) = P_1 - P_2 = 0$$

So, in the beginning, the model is asymptotically stable.

5. DIGITAL SIMULATION

We check this analysis of stability by simulation of the nonlinear model jump graph under 20 Sim. The diagram of simulation is given by the following figure:
We simulate the answers of the various variables of PV structure in order to study the performances of the selected control and to conclude on the stability of the system. Figures 15-a, 15-b, 16-a and 16-b respectively show the evolution of the Vp variables, Ip, rotor flow, mechanical speed and the varying flow for various insulations from 100 to 1000 W/m2, these simulations make it possible to validate the result relating to the stability of the model even for a weak illumination. For a nominal insulation (1000W/m2), we note a good continuation of the variables speed and flow compared to their references, a very short transient state and perfect decoupling between these variables. Following a fall of insulation, the PV generator variables change values while functioning at point of maximum capacity and that is due to the application of the linearizing control with a good continuation of the Ipopt reference. Figure 17 shows the temporal evolution of the flow for different insulation, in made, for a nice day, we record the values of the flow (fig 19) and we notice a peak max at semi-day corresponding an insulation of 1000 W/m2, even for this insulation, the pump starts to generate a water flow only starting from one minimal speed of about 213 rd/s (figure18), beyond this minimal speed, the flow to increase with speed to reach the end value of about 2 l/s for a speed of 300 rd/s (nominal value). Figure 19 illustrate the characteristic of the pump (hydraulic power) at constant speed of the pump corresponding to 300 rd/s. For a correct operation of the pump, we must observe the condition presented in figure 20 such as H(pump) and H(network) with null flow.

6. CONCLUSION

We presented the bond graph model of a photovoltaic chain, to then presenting us a method of nonlinear synthesis of control which develops entirely on the bond graph of our system by way of the input-output causal inversion using the bicausality concept, we derived an opposite model which allows the synthesis of the laws feedback achieving exact input-output linearization. The analysis of stability by Lyaponov directly on model BG of the structure is presented.
Fig. 15-a $V_p$ for various insulations $E_C$.

Fig. 15-b $I_p$ for various insulations $E_C$.

Fig. 16-a speed for various insulations $E_C$.

Fig. 16-b flow for various insulations $E_C$.

Fig. 17. flow for different insulations

Fig. 18 flow according to speed
Fig. 19 flow according to EC

Fig. 20 power hydraulic at constant speed

Fig. 21 head at constant speed

References

A. Fekih, « Approches de modélisation et de commande non linéaires appliquées à la machine asynchrone » thèse doctorat, ENIT, février 2002


Appendix:

**motor parameters**
- Power : P = 750W
- Voltage : 230 V
- Current : 1.6 A
- Power factor : \( \cos \varphi = 0.8 \)
- Rotoric resistance : \( R_r = 10.621 \Omega \)
- Statoric resistance : \( R_s = 7.001 \Omega \)
- Statoric cyclic inductance: \( L_s=1.0432 \text{ H} \)
- Rotoric cyclic inductance: \( L_r=1.0703 \text{ H} \)
- Mutual cyclic inductance : \( L_m=1.0154 \text{ H} \)
- Moment of inertia \( J = 0.015 \text{ Kg} \cdot \text{m}^2 \)
- Friction coefficient \( f=3.4399 \times 10^{-5} \text{ Kgm}^{-1} \cdot \text{s}^{-1} \)

**linear regulators parameters**
- \( K_1=99000 \)
- \( K_2=3500 \)
- \( K_3=19160 \)
K4=5000
K5=750

**Pump parameters**
pump torque constant  $K_r = 2.7729 \times 10^{-5}$
torque constant of static friction $C_{fc} = -0.01616 \, \text{N.m}$
geometric head $H_g = 7.4 \, \text{m}$
$a_0 = 1.61 \times 10^{-4} \, \text{m}^2 \, \text{s}^{-2} \, \text{rd}^{-2}$
$a_1 = 2.584 \times 10^{-3} \, \text{m}^2 \, \text{s}^{-1} \, \text{rd}^{-1}$
$a_2 = -0.49 \, \text{m} \, \text{l}^2 \, \text{s}^{-2}$
$X = 0.98388 \, \text{m} \, \text{s} \, \text{l}^{-1}$

**Boost parameters**
$L_p = 760 \, \text{mH}$
$C_p = 4700 \, \mu\text{F}$

**Panel parameters**
$I_{sc} = 8.143 \times 10^{-6} \, \text{A}$
$V_T = 6 \, \text{V}$
$I_{ph} = 1.4 \, \text{A}$