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J. Electrical Systems 13-4 (2017): 790-805

Regular paper

Power flow analysis for islanded micro-grid in hierarchical structure of control system using optimal control theory

Journal of Electrical Systems

In environmental uncertainties, the power flow problem in islanded microgrid (MG) becomes complex and non-trivial. The optimal power flow (OPL) problem is described in this paper by using the energy balance between the power generation and load demand. The paper also presents the hierarchical control structure which consists of primary, secondary, tertiary, and emergency controls. Clearly, optimal power flow (OPL) which implements a distributed tertiary control in hierarchical control. MG consists of diesel engine generator (DEG), wind turbine generator (WTG), and photovoltaic (PV) power. In the control system considered, operation planning is realized based on profiles such that the MG, load, wind and photovoltaic power must be forecasted in short-period, meanwhile the dispatch source (i.e., DEG) needs to be scheduled. The aim of the control problem is to find the dispatch output power by minimizing the total cost of energy that leads to the Hamilton-Jacobi-Bellman equation. Experimental results are presented, showing the effectiveness of optimal control such that the generation allows demand profile.

Keywords: Microgrid, hierarchical control, optimal control, photovoltaic, wind power, diesel power, optimal power flow, Hamilton-Jacobi-Bellman equation, distributed generation.

Article history: Received 10 August 2017, Accepted 16 November 2017

1. Introduction

In recent years, the climate change all over the world gives birth to the renewable energy opportunity such that reduction of the impart of climate change. Particularly, renewable energy plays a key role to deal with the climate change. In reality, power systems are continuously subject to various disturbances such as changes in the loads and the availability of components. Moreover, the conventional electric power systems are rapid changes to alleviate environmental conditions, bring intelligent grids and solutions, and respond to the customer demands. In this work, our focus is on the dealing with changes in loads, and uncertainty of wind power, photovoltaic (PV) arrays that separate the system into islanded microgrid. The concept of microgrid was proposed and has recently emerged, developed in many institutions to introduce a smart grid. Microgrid is a local grid in which distributed generations (DGs) and electric loads are placed together and controlled efficiently in an integrated manner [1]. In fact, microgrid plays an important role to provide the electrical energy to islands, the installation of DG with small-scale power generation technologies has rapidly increased in many countries at a reduced cost and a higher efficiently. Microgrids can operate in grid-connected, or grid-isolated modes [2], [3], [4], [5]. The principle roles of the microgrid control structure are as voltage, frequency, and phase angle regulation for operating modes [2], [3], [6], [7], [8], [9]; proper load sharing and distributed energy resources; microgrid synchronization with the main grid (national

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grid); power flow control between the microgrid and the main grid, or within microgrid; and optimizing the microgrid operating cost.

In grid-isolated mode, the local loads should be supplied by the DG units that the control is carried out as controlled voltage sources (CVS) as in Ref [10]. In order to avoid circulating currents between the DGs and make sure that the operation is stable and efficient, the MGs need some form of control [7]. Moreover, in this mode, the microgrid operation can be subject to several problems such as [11]: voltage oscillations, frequency oscillations, and power quality issues. Therefore, the hierarchical control structure needs considering which consists of four levels, namely, *the primary*, *secondary*, *tertiary*, and *emergency control* [2], [12] (see also in [13] for the case of wind power plant). It is reported in [2] that the primary control is in normal operation, and droop control methods involve an inherent trade-off between power sharing and voltage and frequency regulation, and include fundamental control hardware. The secondary control is in off-normal operation, and compensate for the voltage and frequency deviations after the primary control achieves the power sharing. Particularly, the tertiary control is used to restore the secondary reserve, and manages the power flow between the microgid and the grid, and within the microgrid for the case of islanded mode that lead to the economic issues.

Concerning the problem of optimal power flow in the tertiary control level, basically we must consider the uncontrollable nature of wind, solar power as well as load change raises uncertainty for power system operation on the one part, the application and integration of DGs into the system is complex on the other part. Among these aspects are, the investigation of impact of distributed generation on the power fluctuations from penetration of wind, photovoltaic power is presented in [14], [15], and [16]. According to [17], [18], [19], the authors have considered the hybrid power system whose the energy storage/thermal unit has a high potential for providing regulation power to meet the reverse requirements. Energy management of microgrid was considered by Basaran et al. [20] and Han et al [21] with simulation model. The authors [20] proposed a novel method for design of a wind-PV hybrid system in order to operate both on-grid and off-grid. On the other paper as in [21], the authors considered on PV DC power system with batteries, supercapacitors, and hydrogen fuel.

One of important works as in Ref [22], by using the interval number that is easier than probability distribution and fuzzy numbers. The main contribution that the work focus on examining the optimal problem as the investigation of optimal power flow by adopting the interval optimization in which the wind power is defined in range of values; the interval power flow problem for DC power flow can be formulated as a non-convex and nonlinear programming. In consequence, to ensure system reliability, the forecasting uncertainty must be considered into generation scheduling, and interval power flow provides promising approach to achieve the boundary information of system statuses under uncertainties. More importantly, the contribution in [22] is a landmark for class of optimal power flow problem. For the load flow problem which is typically formulated as a set of non-linear equations as functions of bus voltages has taken some advantage. Such stochastic demand has been developed and presented in [23]–[25], those authors used stochastic, fuzzy, and probability programming techniques to model the uncertainties. Although the works in [23]–[25] have been specified to load flow problem with some algorithms, the aforementioned methods are typically dependent on predefined probability distribution function or membership function of uncertainty. Furthermore, it is hard to identify accurate probability distribution functions due to data availability and stochastic nature of the uncertainty. The research in all these directions [14]–[22] for the case of power flow was based on non-convex, and non-linear programming and the DG units have known locations and running all the time. To add to the complexity of the problem, in the real life systems, the operation of these DG units, undergoes different scenarios according to the strategies of the electricity producers and the needs of the consumers. Therefore, some uncertainties are introduced in the operation of such units and thus, stochastic modeling of systems involving DG units becomes of great interest. The sources of the uncertainties in the operation of the DG systems at a certain hour of the day include. These uncertainties affect the modeling and evaluation of the system capacity, power losses.

In this paper, firstly we review hierarchical control strategy, then consider the tertiary control level of hierarchical control in order to investigate the power flow under uncertainties by minimizing the cost of electricity production. Therefore utilizes the energy balance equation in real time to formulate the optimal power flow problem as optimal control problem of a linear system by using dynamic programming. To do this it, will be expedient to derive an algorithm similar to the Bellman principle where the optimality conditions satisfy Hamilton-Jacobi-Bellman (HJB) equations, and the value function is convex. In addition, the HJB equation leads optimal feedback control for power flow problem, and is to make the resulting system relatively insensitive to fluctuations that can deal with uncertainties of power system considered.

2. Notation

The notation used throughout the paper is stated below.

Indexes:

k	Distributed generation unit index
t	time interval (hour) index considered in the time horizon

Constants:

$X\left(t\right)$	energy (cumulative electricity production) at time *t* [kWh]
$D\left(t\right)$	load demand at time *t* [kW]
$U(t)$	control variable at time *t* [kW]
$G\left(.\right)$	running cost function [$/Time Unit]
$J(t;.)$	overall Cost of system [$]
$v\left(t,.\right)$	value function [$]
C^{+}	unit surplus cost at time *t* [$/kWh]
C^{-}	unit backlog cost at time *t* [$/kWh]
P_{DEG}	diesel engine generator output power [kW]
P_{WTG}	wind turbine generator output power [kW]

P_{PV}	photovoltaic output power [kW]
$P^{DEG}_{\min/\max}$	upper/lower bounds of P_{DEG} [kW]
$P^{WTG}_{\min/\max}$	upper/lower bounds of P_{WTG} [kW]
$P^{PV}_{\min/\max}$	upper/lower bounds of P_{PV} [kW]

3. Microgrid structure and control

This section presents the typical structure of microgrid (MG) with loads, nDGs such that photovoltaic (PV), wind turbine generator (WTG), and diesel engine generator (DEG). Figure 1 shows the typical structure of MG, and only the islanded operation mode will be considered as study objective in this study.

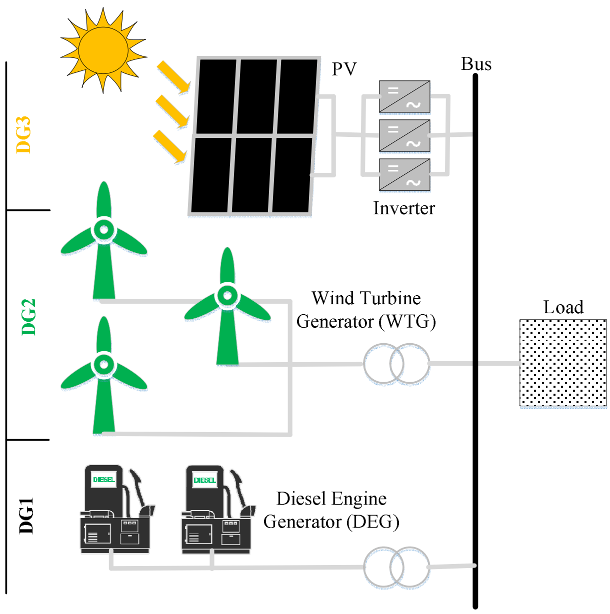


Figure 1. Simplified schematic of islanded microgrid

In reality, the MG can be as the flexible hybrid power system that is able to import/export from the grid through the intelligent bypass switch (IBS) and the point of common coupling (PCC), control the active and reactive-power flows, as well as manage the energy storage. The system always uses electronic devices or the interface between DGs and the microgid which are called *inverters*. During operation time, the microgrid can be subject to a physical disturbance or more critically when there is a fault occurred and make the inverter be disconnected from the grid that leads to the instability on the system. Regarding the aspect of stability and control [26], theoretically, the advantage of the electrical form of energy is that it can be *transported* and *controlled* with relative ease and with a high degree of efficiency and reliability. Hence, MG operation shall meet the fundamental requirements such the quality of power supply with regards to the constancy of frequency, voltage, and level of reliability. In order to adapt the stability and control of the system, the microgrid could operate as an autonomous with the adopting of the following four control levels [27], [2], [3] and [12]:

- *Level 0* (*Inner control loop*): Identification of the control type is considered from the physical system of microgrid. Current and voltage, feedback and feedforward, stochastic and deterministic, and linear and nonlinear control loops can take place of dynamics system to regulate the output voltage and to control current, the output power and to control power generation of fuel DGs, and make sure the stability of the system considered.
- *Level 1* (*Primary control*): Under the normal operation, the droop control method is used for modeling of the physical behavior, which makes the system stable and more damped. In this level, the control loops of all generating units respond within a few seconds.
- *Level 2* (*Secondary control*): In Off-normal operation, after the primary control archives the power sharing, the secondary control takes over the remaining frequency and power deviation after tens to few minutes, and can re-establish the microgrid voltage to nominal values. In addition, the supervisor sends proper signals by using bandwidth communications. A synchronization of microgrid to the main grid needs carrying out in this level.
- *Level 3* (*Tertiary control*): Similarly to Level 2, the tertiary control is used to restore the secondary reserve. The balance of power is significant to consider in this level; the energy production level controls the power flow between the microgid and the grid, and within the microgrid. The power flow problem is, normally, economic issues and may be considered as optimal control problem in order to make decision in the microgrid and associated to the implementation in a SCADA system.
- *Level 4* (*Emergency control*): In the case of off-normal operation, if there is a serious load-generation imbalance that occurs a significant fault, the secondary control can unable to re-establish frequency, then the emergency control must be used to decrease the risk of cascade faults.

4. Strategy of control hierarchy

In this section, we review briefly the hierarchical control strategy of MG and present the power flow and optimal operation on the tertiary control level. However, we focus on the optimal control of power flow in islanded mode of operation as the essential idea of this paper. The hierarchical control strategies of MG were investigated in the several works [2],[7], [3], [28], [6] of which we now are in the cutting edge of the state of the art to present in what following.

4.1. Islanded Mode of Operation

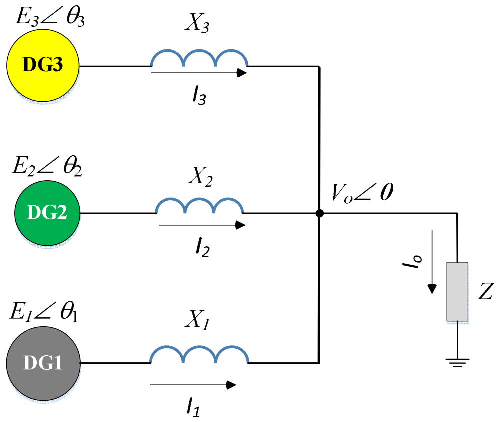


Figure 2. Equivalent circuit of three parallel-connected DGs

As in Figure 1, a microgrid is define as a cluster of loads and DG units serviced by a distribution system. In the aspect of operation in different modes (*grid-connected mode*, *islanded mode*, and *transition mode*), control, protection, operational issues, and energy management strategies of a microgrid are significant. The islanded mode of operation occurs in the following scenarios [3]:

- *Planned islanded operation*: islanded operation shall be started if any event in the main grid are presented, such long-time voltage dips or general faults.
- *Nonplanned islanded operation*: the microgrid shall be able to detect the critical faults and failures as blackout in the main grid if the blackout occurs and disconnect. The detection should use the proper algorithms.

In islanded mode, the DG units play main role to regulate frequency and amplitude voltage of the microgrid. By meaning of active power balance, DG units will support all active power either inject or absorb active power proportionally to the frequency deviation. The DG units share the power while feeding the system for nominal voltage and frequency stability. Therefore, the microgrid acts like a voltage source and current source, the power system stability must be considered as an important issue [26]. Equally, in autonomous mode, the microgrid must satisfy the three issues: voltage and frequency management, supply and demand balancing, and power quality.

For efficient operation, the control strategies must be considered as optimal control problem of power flow for the power balance condition. From the microgrid structure, the control strategies are responsible for the following [4]:

- If the total generation of DG_1 + …+ DG_n prior to islanding is less (greater) than the MG load demands, formation of the autonomous MG results in reduction (rise) in the frequency. Also, frequency restoration is applied to adjust the frequency to standard, through balancing the power generation set-points of all DGs, during steady-state operation.
- The total real power demand is provided by DG_1,.., and DG_n. All DGs are coupled to the MG through power electronics, they can respond to the load dynamics faster than DEG, and thus it can effectively maintain frequency and enhance angle stability.
- Reactive power demand of the total load and voltage profile is provided by DG_1,.., and DG_n and reactive power sources.

4.2. Primary Control

This control level is concerning about change in real power demand at any point of a network by a change in frequency. The frequency of a power system is dependent on real power balance when connecting DGs in parallel, circulating active and reactive power can appear as presented in Figure 2. The objective of the control is to adjust the frequency and amplitude of voltage reference provided to the inner current and voltage control loops. The droop control is utilized to avoid communication wires while obtaining good power sharing, ensuring *P* and *Q* [6], [29] flow control that is the well-known *P/Q* droop method:

$$\omega = \omega^{ref} - D_p\left(P - P^{ref}\right), E = E^{ref} - D_q\left(Q - Q^{ref}\right) \tag{1}$$

where ω and E address the frequency and amplitude of the output voltage, ω^{ref} and E^{ref} are the reference frequency and amplitude, P^{ref} and Q^{ref} are the active and reactive power references normally set to zero in islanded MG [30], [31], D_p and D_q are the droop coefficients, and V_o and I_o are output voltage and output current, respectively.

4.3. Secondary Control

This control level is required because primary control action is not usually sufficient to restore the system frequency, especially in an interconnected power system. Therefore, the objective of the secondary control is to restore the voltage, frequency, and compensate for the frequency deviation.

In case of AC microgrid, the error signals of frequency and voltage are processed by individual controllers as in equations (2) [27]; the resulting signals ($\delta\omega$ and δE) are send to the primary controller of the DER (distributed energy resource) to compensate for frequency and voltage deviations [2], [27], [6]:

$$\begin{cases} \delta\omega = K_{p\omega}\left(\omega - \omega^{ref}\right) + K_{I\omega}\int\left(\omega - \omega^{ref}\right)dt \\ \delta E = K_{pE}\left(E - E^{ref}\right) + K_{IE}\int\left(E - E^{ref}\right)dt \end{cases} \tag{2}$$

where $K_{p\omega}$, $K_{I\omega}$, K_{pE}, and K_{IE} are the control parameters of the secondary compensator. In this case, $\delta\omega$ and δE must be limited in order to do not exceed the maximum allowed frequency and amplitude deviations [2]. The angular frequency level ω are measured and compared to the reference ω^{ref} and the errors processed by the PI compensator are sent to all the DGs in order to restore frequency of MG.

4.4. Tertiary Control

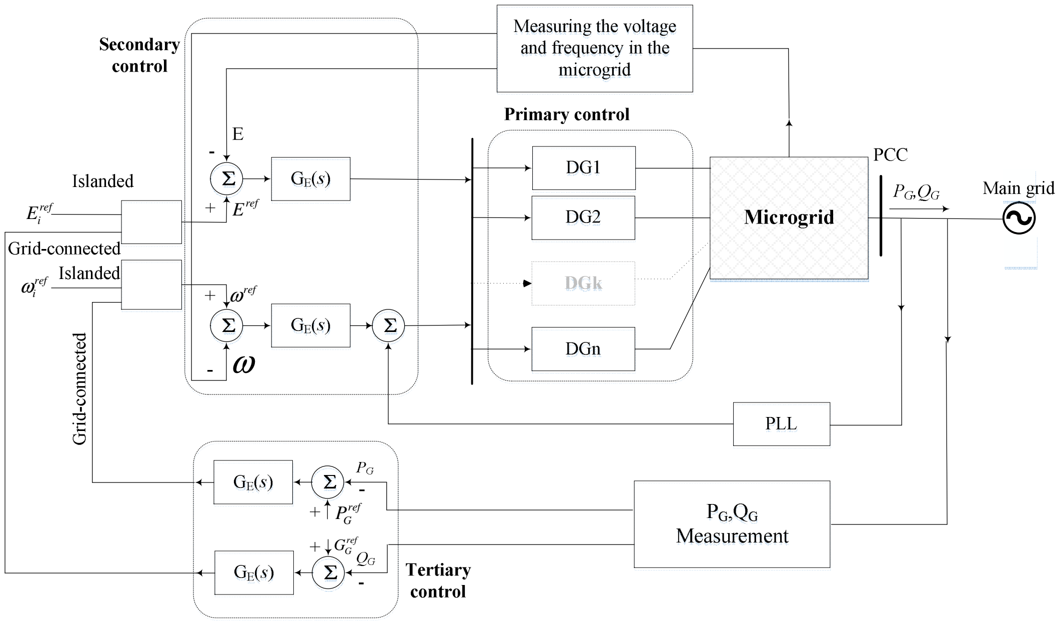


Figure 3. Block diagram of secondary and tertiary controls [6]

This control level concerns the economic issues in the optimal operation of the microgrid, and controls the power flow between microgrid and main grid by adjusting the frequency and amplitude of the voltage inside the microgrid [2],[6]. Figure 3 represents the block diagram of the primary, secondary, and tertiary controls. The primary and secondary controls carry out in the islanded mode of operation, and the former one does in the connected-grid mode operation by measuring the power flow P_G and Q_G. As can be seen in Figure 3, active and reactive output power of the microgrid, P_G, Q_G are measured through the static bypass. Then these quantities are compared with references P_G^{ref}, Q_G^{ref}. The control laws can be addresses:

$$\begin{cases} \omega^{ref} = K_{pP}\left(P_G^{ref} - P_G\right) + K_{iP}\int\left(P_G^{ref} - P_G\right)dt \\ E^{ref} = K_{pQ}\left(Q_G^{ref} - Q_G\right) + K_{iQ}\int\left(Q_G^{ref} - Q_G\right)dt \end{cases} \tag{3}$$

where K_{pP}, K_{iP}, K_{pQ}, and K_{iQ} are the control parameters of the tertiary control compensator [2]. The tertiary control also provides an economically optimal operation [32]. In order to lead the optimal operation of the MG, every DG is equipped with a droop control. This ensures grid stability in islanded operation and enable the use of public communication network. The optimal operation, therefore, is optimal power flow [32], [33].

4.5. Emergency Control

This level is about emergency condition, i.e., *off-normal condition*, an immediate change in the output power control of the MG must take place of situation. As a result, it can change from a dispatched power mode to one controlling frequency and/or voltage of the islanded section of the network. The islanding plan can be considered as the most important emergency control scheme in the MG systems. In islanded mode, the voltage/frequency might go beyond the power quality limits. That is necessary exploit controllable micro sources, storage devices, local load, and load shedding schemes and special protection plan in a cooperative way. In reality, there are few reports on the role of distributed renewable energy systems (RESs) in emergency conditions [12]. The impact of distributed utilities on transmission stability is represented in [34]. A review on emergency control and protection plans in microgrids is addressed in [35].

4.6. Optimal control of power flow in the tertiary control level

As mentioned above, the tertiary control level is to control the power flow in the global terms (microgrid import/export energy) or local terms (hierarchy of spending energy), and provides economic issues [2]. The power flow data is used to prevent line overload, the objective is to minimize the energy stock exchange. According to [32], the optimal operation is solved by using a grossing algorithm. Generally, the optimal operation is satisfied when all operate at equal marginal costs [32], [33], [36]. In the next section, we present the novel control approach for optimal power flow by using the dynamic programming principle that is different from a population dynamics approach or a grossing algorithm as in [33], [36].

5. Optimal Feedback Control of Power Flow

In this section, we consider the hybrid power system on the islanded AC microgrid including AC loads, two photovoltaics (PVs), two wind turbine generators (WTGs), and two diesel engine generators (DEGs), and assume that the DEGs are always *available* in continuous time. Figure 4 shows the power flow of MG.

The optimal power flow problem is considered in finite time (i.e., $0 < T < \infty$) in order to find the DEGs output power. Clearly, to formulate a new model, first of all we shall assume that the power generation in the real time and the total power generation of DGs satisfies the demand such as:

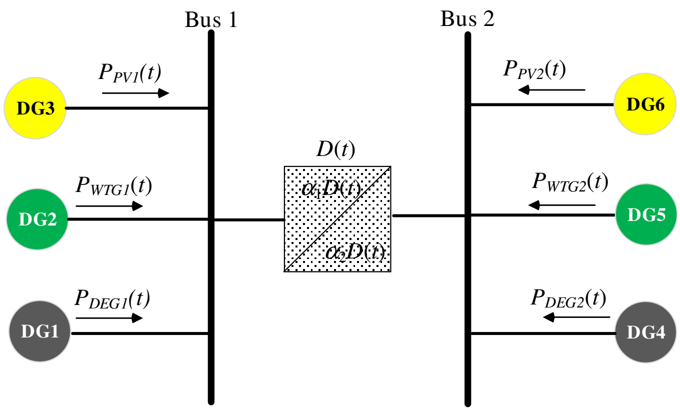


Figure 4. Power flow of islanded microgrid

$$P_{DEG}(t) + P_{WTG}(t) + P_{PV}(t) = D(t) \tag{4}$$

The state equation is, then, represented as follows:

$$\frac{dX(t)}{dt} = U(t) - \left(D(t) - \left(P_{WTG}(t) + P_{PV}(t)\right)\right) \tag{5}$$

or

$$\frac{dX(t)}{dt} = \sum_{k=1}^{2}\left[U_k(t) - \left(\alpha_k D(t) - \beta_k\left(P_{WTGk}(t) + P_{PVk}(t)\right)\right)\right]$$

$$\frac{dX(t)}{dt} = f\left(t, X(t), U(t)\right) \tag{6}$$

Constraints:

$$P_{\min}^{WTG1,2}(t) \le P_{WTG1,2}(t) \le P_{\max}^{WTG1,2}(t) \tag{7}$$

$$P_{\min}^{PV1,2}(t) \le P_{PV1,2}(t) \le P_{\max}^{PV1,2}(t) \tag{8}$$

$$P_{\min}^{DEG1,2}(t) \le P_{DEG1,2}(t) \le P_{\max}^{DEG1,2}(t) \tag{9}$$

where $U(t) = U_1(t) + U_2(t)$ with $U_1(t) = P_{DEG1}(t)$ and $U_2(t) = P_{DEG2}(t)$ is the *control variables* in equation (5) and $U(t) \in \Re^+ = [0, +\infty)$ in [kW], $D(t)$, $P_{WTG1}(t)$, $P_{WTG2}(t)$, and $P_{PV1}(t)$, $P_{PV2}(t)$ are within their forecasted upper bounds $P_{\max}^{WTG1,2}(t), P_{\max}^{PV1,2}(t), P_{\max}^{DEG1,2}(t)$ and lower bounds $P_{\min}^{WTG}(t)$, $P_{\min}^{PV}(t)$, $P_{\min}^{DEG}(t)$ (in [kW]), $X(t)$ is the *energy* in [kWh] and $X(t) \in \Re = (-\infty, +\infty)$, $f(t, X, U)$ is the *state function* and satisfies the Lipschitz condition:

$$\left|f(t, X_1, U) - f(t, X_2, U)\right| \le K_\rho \left|X_1 - X_2\right| \tag{10}$$

where K_ρ is constant, and the participation factors

$$\begin{cases} \beta_k = \dfrac{P_{\max}^{DEGk}}{P_{\max}^{DEG1} + P_{\max}^{DEG2}} \\ \alpha_k = \dfrac{P_{\max}^{PVk} + P_{\max}^{WTGk} + P_{\max}^{DEGk}}{\sum_{k=1}^{2}\left(P_{\max}^{PVk} + P_{\max}^{WTGk} + P_{\max}^{DEGk}\right)} \end{cases} \tag{11}$$

The behavior of the state variable $X(t)$ will be specified shortly in the subsection 5.2. Now let us define cost function (*cost-to-go*) which is given by

$$J(t, X; U) = \int_t^T G(s, X(s), U(s))ds \tag{12}$$

where $G(t, X(t), U(t))$ is the *running cost function*: $G(.) = C^+X^+ + C^-X^-$ with C^+ representing a unit surplus cost at time t, C^- the unit backlog cost at time t, $X^+ = \max(X, 0)$, and $X^- = \min(0, -X)$. Thus, the function $J(t, X, U)$ is called *an overall cost* of the system. The aim of this section is to evaluate the minimum cost over all possible control policies, which we denote as

$$v(t,X)=\inf_{U(.)\in\Omega(t,.)} E\left[J(t,X;U)\right] \tag{13}$$

where $\Omega(t,X)$ is the set of admissible control. Our motivation is to obtain *admissible control* $U(t,X)\in\Omega(t,X)$ that optimizes the cost function (12).

5.1 Bellman Optimality Principle

Taking $t<t+\Delta t<T$, we have that

$$\begin{aligned} v(t,X) &= \inf_{U(.)\in\Omega(t,.)} E\left\{\int_t^{t+\Delta t} G(s,.)ds+\int_{t+\Delta t}^{T} G(s,.)ds\right\} \\ &= \inf_{U(.)\in\Omega(t,.)} E\left\{\int_t^{t+\Delta t} G(.)ds\right\}+v\big(t+\Delta t, X\left(t+\Delta t\right)\big) \end{aligned} \tag{14}$$

The Bellman principle of optimality in Ref [37] (see also [38] and [39] for more details), states that if $U^*(t,.)$ is an optimal control policy exercised over the time interval [t, T] for a given start state at time t, then if we operated this policy up to time $t+\Delta t$ then the remaining component of the policy will be optimal for the control problem over [$t + \Delta t$, T] with start state being the current state at time $t + \Delta t$. If we assume the existence of such an optimal policy, then, as $\Delta t\to 0^+$, we are lead to the partial differential equation (*Hamilton-Jacobi-Bellman* equation) for $v(t, X)$.

$$0=\inf_{U(.)\in\Omega(t,.)}\Big\{G(t,.)+v_t\ (t,X)+\Big(U\left(t,.\right)-\big(D\left(t\right)-P_{WTG}\left(t\right)-P_{PV}\left(t\right)\big)\Big)v_X\left(t,X\right)\Big\} \tag{15}$$

where the terms $v_t\,(t,X)$, and $v_X\,(t,X)$ denote the gradient of value function respect to time t and state variable X, respectively.

In principle, once a minimizing solution $U^*(t,X)$ is known, it may be used as closed loop feedback: that is, the control policies are taken as these functions of the current state parameters. The optimal policies are therefore given by

$$\begin{cases} U^*\left(t,X\right)=U_1^*\left(t,X\right)+U_2^*\left(t,X\right) \\ U^*\left(t,X\right)\in \arg\min\limits_{U(.)\in\Omega(.)}\Big\{G\left(t,.\right)+f\left(t,.\right)v_X\left(t,X\right)\Big\} \end{cases} \tag{16}$$

The Proof of this optimal policies is developed from the results in page 8 of Ref [40].

5.2. Optimal control strategies

The production target is specified for energy $X(t)$ having to make by time T, the production period. The cumulative electricity production is the total energy produced by time t. The cumulative electricity production must equal to total demand at time T; that is one of the objectives is to ensure that

$$\int_{t_0}^{t} \left[U(s) + P_{WTG}(s) + P_{PV}(s) \right] ds = \int_{t_0}^{t} D(s)\, ds \tag{17}$$

By definition, the energy in equation (6) whose value is described as follows:

$$X(t) = X_o + \int_{t_0}^{t} \left[U(.) - \left(D(.) - \left(P_{WTG}(.) + P_{PV}(.) \right) \right) \right] ds \tag{18}$$

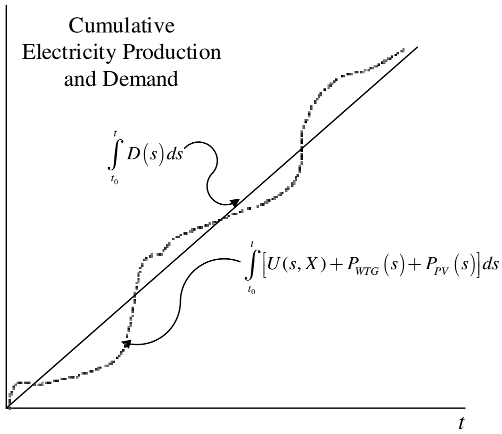


Figure 5. Electricity Production to track demand load

In the Figure 5 which is modified from the framework in [41], the dotted line is cumulative demand by time *t*, and the straight line is cumulative electricity production by time *t*. Our objective is to meet production as closely as possible at every time *t* for period *T*, or, equivalent to keep *X*(*t*) close to zero.

5.3 Supply and demand control (SDC) system [1]

In order to minimize energy stock exchange (*X* in *kWh*), the control is carried out on operation plan, using short-time load forecast. In reality, the SDC system forecasts a next day's load pattern and makes a next day's operation plan taking into account economics and efficient operation of the DGs. Based on weather forecast data, wind data, and PV data, next day generation is forecasted for wind power and PV power generation. The forecasting methods are presented in [42]–[45], [46]. The load forecast can be done at next fifteen minutes to three hours based on the demand at present, the accuracy of the forecast for the system considered so that' the operation plan must be modified for improving.

6. Numerical Example

In this example, the forecast of load demand, total of PV power, and total of wind power generations are represented in Figure 6. The constraints of system parameters in equations (7-11) for MG presents in the Table 1.

Table 1: Parameters of optimal power flow model

No	Description	Lower	Upper
1	Time interval [h]	0	24
2	Total of WTG power [kW]	0	1200
3	Total of PV power [kW]	0	1000
4	DEG1 power [kW]	300	3000
5	DEG2 power [kW]	400	4000
6	Load demand [kW]	0	7000
7	C^{-} Backlog cost [$/kWh]	10	-
8	C^{+} Surplus cost [$/kWh]	1	-

By using Kushner's approach [47], we solve the HJB equation (15) includes the gradient of value function of $v(t, X)$. Let $\Delta X > 0$ and $\Delta t > 0$ denote the length of the finite difference interval of the variable X and t respectively. The first-orders partial derivative of the value functions $v_t(.)$ and $v_x()$ in equation (15) are replaced by the following expressions:

$$v_t\left(t, X\right) = \frac{v\left(t+\Delta t, X\right) - v\left(t, X\right)}{\Delta t} \tag{19}$$

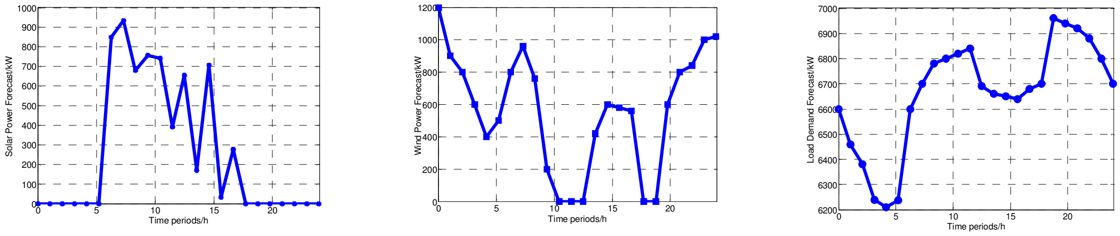
$$v_X\left(t, X\right) = \begin{cases} \dfrac{v\left(t, X+\Delta X\right) - v\left(t, X\right)}{\Delta X} & \text{if } f\left(t,.\right) \geq 0 \\ \dfrac{v\left(t, X\right) - v\left(t, X-\Delta X\right)}{\Delta X} & \text{otherwise} \end{cases} \tag{20}$$

Using ΔX and Δt, and after manipulations, the HJB equations can rewritten as follows:

$$0 = \min_{U^{\Delta}(.)} \left\{ G(.) + \frac{v^{\Delta}(t+\Delta t, X) - v^{\Delta}(t, X)}{\Delta t} + \left(U^{\Delta} - D + \left(P_{WTG} + P_{PV}\right)\right) \frac{\left(\mp\right) v^{\Delta}(., X \pm \Delta X) \mp v^{\Delta}(.)}{\Delta X} \right\} \tag{21}$$

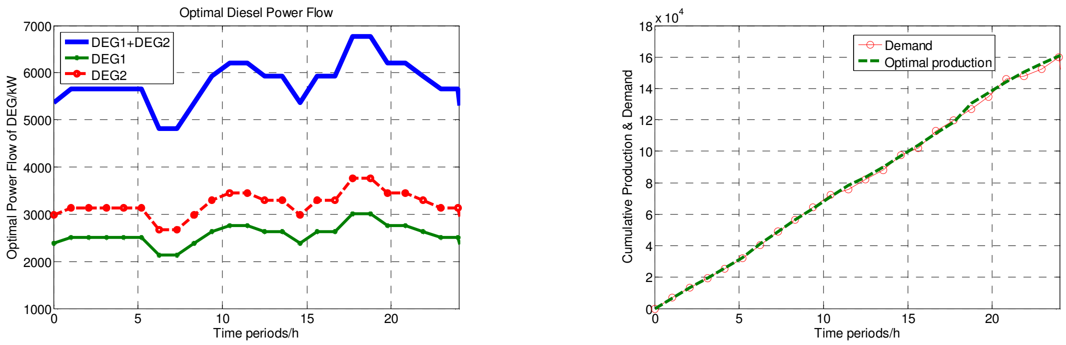
The results are illustrated in Figure 7. Figure 7a represents the optimal production of DEG output power versus time t in the interval [0,24h]. For the chosen parameters, the DEG1 and DEG2 follow demand profile (demand forecast). At the peak hour, both two dispatches generate at their nominal power according to the demand factor chosen in this case. Moreover, the demand is adequately covered: $D = \sum_{k=1}^{2} \left(P_{DEGk} + P_{WTGk} + P_{PVk}\right)$ for all t.

Figure 7b represents the cumulative electricity production of MG and the load demand versus time t. This Figure shows that, the effectiveness of optimal control gives the birth to optimal power flow of MG, thus the characteristic is linear instead of being non-linear as in Figure 5, showing that the optimal control approach can be implemented with appropriate results.



a) PV power forecast [kW] b) Wind power forecast [kW] c) Demand forecast [kW]

Figure 6. Data Forecast



a) Optimal output power of DEGs [kW] b) Cumulative production [kWh]

Figure 7. Optimal output power of DEGs [kW]

7. Conclusions

An approach based on dynamic programming (Bellman's principle) applied to the dispatch of DGs in a microgrid have been formulated. The main objective of study is optimal power flow control with uncertain power injection to implement a distributed *tertiary control level* in hierarchical control strategy of the microgrid, which allows the dispatches to operate at an economic optimum. As a result, the new model is as a stochastic control problem whose optimality conditions have been established as Hamilton-Jacobi-Bellman equation. Moreover, the proposed model makes considered system joint between the optimal power flow and optimal control problem. Also, this paper reviews existing methodologies to enhance the control performance of the microgrid, which lie within the context of hierarchical control structure consisting of primary control, secondary control, tertiary control, as well as emergency control.

We applied our proposed model to a real life system of an islanded MG at small-scale with demand, PV power, and WTG power forecast, and the DEG output power is control variable. The results of test system have demonstrated the effectiveness of the proposed method.

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