Permanent magnet synchronous motors (PMSMs) are exceptionally promising thanks to their many advantages compared with other types of electrical machines. Indeed, PMSMs are characterized by their important torque density, light weight, high air gap flux density, high acceleration, high efficiency and strong power-to-weight ratio. A surface-mounted PMSM (SPMSM) is used in this work. The SPMSM is built using a 2D finite element method (FEM). Cogging torque, torque ripples and back-EMF are examined during the design process in order to obtain sinusoidal back-EMF and to minimise torque ripples which are one of the major problems with PMSMs. Two procedures are used to reduce the cogging torque of SPMSM: the effect of slot opening and the influence of skewing the stator laminations. Cogging torque factor $t_c$ and the torque ripples factor $t_r$ have been calculated to compare the two configurations (open slots and closed slots). Then, the configuration with closed slots is utilised with skewing the stator laminations for different angle 0˚, 10˚ and 15˚.

Keywords: Torque ripples; permanent magnet motor; finite element method; slot opening; skewing the stator.

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1. Introduction

Of late, electric vehicles (EVs) have hogged the limelight in attempts to come up with solutions to cut down on fuel consumption, for both economic and environmental reasons. The electric motor is one of the most important parts of the transmission chain of any EV. In this respect, PMSMs are exceptionally promising thanks to their many advantages compared with other types of electrical machines. Indeed, PMSMs are characterized by their important torque density, light weight, high air gap flux density, high acceleration, high efficiency and strong power-to-weight ratio [1],[2],[3]. One major problem with PMSMs is cogging torque which causes torque ripples and non-sinusoidal back-EMF [4],[5],[6],[7]. Several design methods have been suggested in the literature with the aim of reducing torque ripples and cogging torque like magnet pole width modification [8], slot opening[9][10], skewing stator laminations[10][11], magnet shifting [12], magnet skewing [1],[13],[14], rotor shifting [1],[15]. This work focuses only on two methods occupied by stator slots design of the PMSM: slot opening and skewing stator lamination [11],[16],[17]. The first part of the paper comes in the form of a description of the initial model design of SPMSM used in this work. The second part is dedicated to a FEM validation and simulation of the studied motor. The finite element program Opera-2D is used to simulate motor in this study. Cogging torque, torque ripples and back-EMF are examined during the design process of SPMSM. Finally, the third part concludes the paper by summarising the major findings of the study.
2. Initial prototype of SPMSM

The studied motor is a SPMSM suitable for EV applications. The motor used in this study is designed with 12 open slots and 4 pole pairs. The 12 teeth are actually 6 main teeth between which 6 teeth are inserted to minimise leakage flux and to improve the electromotive wave form. The motor structure and dimensions are well described and studied in [5][18]. A half-cross section of the initial SPMSM is shown in Figure 1.

![Figure 1: Half-cross section of the initial SPMSM](image)

2.1. Analytical modeling of back-EMF

Electric parameters and motor dimensions are developed by the analytical method developed in previously published work [5],[18]. Three sizing ratios define the motor structure. These ratios help to determine the form of the desired electromotive force which is sine wave form. The first is $r_1$, ratio of the magnet angular width by the pole pitch. The second ratio is $r_2$, ratio of the main tooth angular width by the permanent magnet angular width. The last one is $r_3$, ratio of the inserted tooth angular width by the main tooth angular width [19].

\[
    r_1 = \frac{A_{FM}}{A_p} \tag{1}
\]

\[
    r_2 = \frac{A_{tooth}}{A_{FM}} \tag{2}
\]

\[
    r_3 = \frac{A_{tooth}}{A_{tooth}} \tag{3}
\]
With

\[ A_p = \frac{\pi}{p} \]  

(4)

Where, \( A_{PM} \) the permanent magnet angular width, \( A_p \) the pole pitch, \( A_{tooth} \) the principal tooth mean angular width, \( A_{tooth} \) the inserted tooth mean angular width and \( p \) the number of pole pairs. The \( r_1, r_2 \) and \( r_3 \) ratios are fixed in [18] and they take the values of 2/3, 3/2 and 1/5, respectively.

The rotor turns from an initial position with angular speed \( \Omega \), Equation 5. The back-EMF caused by the coil depends on the derivative of flux. Then, it is important to express the flux as a function of the magnets position. The induction in the air gap is depicted in Figure 2 [20].

\[ \Omega = \frac{d\theta}{dt} \]

(5)

Where \( \theta \) is the magnets position.

![Figure 2 Induction in the air gap.](image)

The magnetic induction is assumed to be perfectly rectangular in the air gap. Leakage between the air gap and the stator is neglected. With these assumptions, the flux captured by one coil \( \phi_{coil} \) can take the following form.

\[ \phi_{coil} = \int_{S_{coil}} B_a(\theta) \, dS = \int_{S_{coil}} \frac{B_a}{2} \, dS = \frac{B_a}{2} \int_{S_{coil}} \, dS. \]

(6)

Where \( B_a \) is the induction in the air gap and \( S_{coil} \) the coil surface.

The overall back-EMF of one phase is proportional to the spire number per phase \( N_{spil} \), the angular speed \( \Omega \) and the flux variation. The back-EMF value is expressed in Equation (7):

\[ E = -N_{spil} \frac{d\phi_{coil}}{dt} \Omega \]

(7)
According to geometrical parameters values and permanent magnets position, four different intervals are defined as shown in Table 1 [18].

<table>
<thead>
<tr>
<th>Position (Rad)</th>
<th>Flux $\Phi_{\text{bin}}$ (Wb)</th>
<th>E (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{\pi r_2}{2p}$ $(1-r_3) \leq \theta \leq \frac{\pi r_2}{2p}$ $(1-r_3)$</td>
<td>$N_{e_p} N_{p_m} B_a$</td>
<td>0</td>
</tr>
<tr>
<td>$-\frac{\pi r_2}{2p}$ $(1-r_3) \leq \theta \leq \frac{\pi}{2p}(1 + r_2)$</td>
<td>$2N_{e_p} N_{p_m} B_a$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2p}(1-r_3) \leq \theta \leq \frac{\pi}{2p}(1 + r_2)$</td>
<td>$2N_{e_p} N_{p_m} B_a$</td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{2p}(1 - r_3) \leq \theta \leq \frac{\pi}{2p}(1 + r_2)$</td>
<td>$2N_{e_p} N_{p_m} B_a$</td>
<td></td>
</tr>
</tbody>
</table>

The Back-EMF is a periodic function. Its form resembles the sinusoidal function of period T. The back-EMF over an electric period is provided in Equation (8).

$$\text{emf}(t) = \sum_{n=1}^{\infty} b_n \sin \left( \frac{\pi n \theta}{T} \right)$$  \hspace{1cm} (8)

With

$$b_n = \frac{1}{T} \int_{0}^{T} B_{\text{emf}} \sin \left( \frac{\pi n \theta}{T} \right) d\theta$$ \hspace{1cm} (9)

The first harmonic of back-EMF at no load can be attained from the preceding equations.

$$\text{emf}_1(t) = \frac{8}{\pi} N_{e_p} L_{m} \frac{D_A}{2} B_a \sin \left( \frac{\pi}{2} r_2 \right) \sin \left( \frac{\pi}{2} r_1 \right) \cos \left( \frac{\pi}{2} r_1 \right) \sin (p\Omega t)$$ \hspace{1cm} (10)

Where $L_m$ is the motor length and $D_A$ the average diameter of the motor.

2.2. Electromagnetic torque

The instantaneous electromagnetic torque of the motor $T_{\text{em}}$ (t) presented in Equation (11).

$$T_{\text{em}}(t) = \frac{1}{\Omega} \sum_{i=1}^{n} \text{emf}_i(t) i_i(t)$$ \hspace{1cm} (11)

Where $\text{emf}_i$ (t) and $i_i$ (t) are the electromotive force and the current of the i phase, respectively.

Table 2 contain motor dimensions and specifications which are fixed in [5],[18],[21].
Table 2: Motor dimensions and specifications

<table>
<thead>
<tr>
<th>Motor dimensions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor length</td>
<td>L</td>
</tr>
<tr>
<td>Stator outer diameter</td>
<td>D_{So}</td>
</tr>
<tr>
<td>Stator inner diameter</td>
<td>D_{Si}</td>
</tr>
<tr>
<td>Slots height</td>
<td>H_S</td>
</tr>
<tr>
<td>Slots mean angular width</td>
<td>A_{slot}</td>
</tr>
<tr>
<td>Principal tooth mean angular width</td>
<td>A_{tooth}</td>
</tr>
<tr>
<td>Inserted tooth mean angular width</td>
<td>A_{toothi}</td>
</tr>
<tr>
<td>Rotor outer diameter</td>
<td>D_{Ro}</td>
</tr>
<tr>
<td>Rotor inner diameter</td>
<td>D_{Ri}</td>
</tr>
<tr>
<td>Permanent Magnet height</td>
<td>H_{PM}</td>
</tr>
<tr>
<td>Permanent Magnet width</td>
<td>A_{PM}</td>
</tr>
<tr>
<td>Air gap</td>
<td>e</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Materials</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator material</td>
<td>M_S</td>
</tr>
<tr>
<td>Rotor material</td>
<td>M_R</td>
</tr>
<tr>
<td>Permanent magnets material</td>
<td>M_{PM}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pair</td>
<td>p</td>
</tr>
<tr>
<td>Number of phases</td>
<td>N</td>
</tr>
<tr>
<td>Number of slots</td>
<td>N_{slot}</td>
</tr>
<tr>
<td>Stator flux amplitude</td>
<td>f</td>
</tr>
</tbody>
</table>

Average torque and Back-EMF amplitude are calculated by referring to the data in Table 2 which are respectively 111 Nm and 226 V.

3. Finite element Analysis

The motor is examined using the rotating machines (RM) analysis software Opera-2D. The FEM is used in order to examine the motor's performance dynamically. A time stepping solution to transient electromagnetic equation is obtained allowing the rotor to rotate by appropriate angle at each time step [4]. In order to obtain accurate results from a finite element model, it is essential to ensure that the finite elements are small enough to model the field gradients as portrayed in Figure 3. Cogging torque, electromagnetic torque and Back-EMF can easily be obtained from FEA.
3.1. Winding configurations

Using concentrated winding offers some important advantages as minimisation in the copper volume and losses in the end region, higher filling factor and ease of construction [22]. When the stator windings were created, each region was also given a conductor number to make it easy to define the circuits. Opera-2D refers to current directions; GO (positive conductor numbers) and RETURN (negative conductor numbers); for compatibility with other application areas as demonstrated in Table 3. Figures 4 and 5 shown the winding layout and the linear representation of the winding of the SPMSM, respectively.

Table 3 Current directions on the conductors

<table>
<thead>
<tr>
<th>windings</th>
<th>Conductor set</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>-1, 2, 8, -7</td>
</tr>
<tr>
<td>W2</td>
<td>-5, 6, 12, -11</td>
</tr>
<tr>
<td>W3</td>
<td>-3, 4, 10, -9</td>
</tr>
</tbody>
</table>

Figure 4 The winding layout of the studied SPMSM

Figure 5 Linear representation of the winding layout in SPMSM
In the RM model of Opera-2D, the armature windings will be connected to a 3-phase power supply using external circuits. Three circuits, representing the three phases, are to be defined for this model, which is drawn by using the circuit editor of Opera-2D [4]. Figure 6 presents the circuit layout. The vector map and the field lines distribution for a rotor position are presented in Figure 7 and Figure 8, respectively. Besides, while Figure 9 depicts the module of induction in the air gap for a rotor position, Figure 10 shows its curve as a function of the mechanical angle. A decrease in the value of the induction is observed in front of the slots which are related to the reluctance of stator teeth effects. The maximum value of the induction is approximately equal to 0.9 T.

![Drive circuit for the RM analysis](image1)

![Vector map potential of SPMSM for a rotor position.](image2)

![Field lines distribution of SPMSM for a rotor position.](image3)
3.2. Solving for back-EMF and cogging torque

Due to the interaction between the stator teeth and the rotor permanent magnet flux in PMSM, an electromagnetic torque exists even without exciting the stator winding. This torque is the cogging torque. Then, at no-load condition, the FEA Opera-2D can be used to easily obtain the cogging torque as well as the open circuit back-EMF by removing the alternative current source and setting the resistance of the resistors to 100 MΩ, as detailed in Figure 11 [4]. Back-EMF and cogging torque are represented in Figure 12 and Figure 13, respectively. As is clearly shown in from Figure 12, Back-EMF amplitude is equal to 226 V which is almost the same to analytical value.
4. Reducing torque ripples

Reducing cogging torques and torque ripples is one of the most interesting advantages of multi-phase machines, in order to obtain sinusoidal back-EMF [6],[14],[23].

4.1. Influence of the stator slot opening

The stator slot opening has an important effect on cogging torque and torque ripples [5],[21]. To show this, two finite element simulations with open slots and closed slots were performed. The same SPMSM is used in this part with the exception of the slot opening dimensions modification in order to compare slot opening effect on torque ripples, Back-EMF and cogging torque.

Figure 14 presents the SPMSM with open slots and closed slots. The stator winding is connected to three phase balanced sinusoidal currents. Figure 15, Figure 16 and Figure 17 represent respectively the effect of the slot opening on the cogging torque, on the back-EMF and on the electromagnetic torque, respectively.
Figure 14 Motor with open slots and closed slots

Figure 15 Effect of the slot opening on cogging torque.

Figure 16 Effect of slot opening on Back-EMF.

Figure 17 Effect of the slot opening on electromagnetic torque.
Cogging torque factor $t_c$ can be defined as follows [25]:

$$t_c = \frac{T_{cpp}}{T_{av}} \times 100$$

(12)

Where $T_{cpp}$ is the peak-to-peak value of cogging torque and $T_{av}$ is the average torque value.

The torque ripples factor $t_r$ has been defined as follows [25]:

$$t_r = \frac{T_{max} - T_{min}}{T_{av}} \times 100$$

(13)

Where $T_{max}$ and $T_{min}$ are respectively the maximum and the minimum values of torque.

In order to compare cogging torque and torque ripples of the two configurations, the $t_c$ and $t_r$ factors are calculated. Table 4 presents both factors according to slot opening. What can be noted is that the structure with closed slots has the lowest cogging torque and torque ripples. It is clear that the slot form has an important influence on cogging torque and torque ripples. The $t_c$ factor of motor with closed slot decrease 18.11% while the $t_r$ falls by 3.5%. To conclude, despite the fact that back-EMF of open slots is closer to the pure sine form, the motor generates less torque ripples with closed slots. Thusly, it is better to opt for the configuration with closed slots.

<table>
<thead>
<tr>
<th>Slot form</th>
<th>$T_{max}$ (Nm)</th>
<th>$T_{min}$ (Nm)</th>
<th>$T_{av}$ (Nm)</th>
<th>$t_r%$</th>
<th>$T_{cpp}$ (Nm)</th>
<th>$t_c%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
<td>109.76</td>
<td>85.55</td>
<td>99</td>
<td>24.45</td>
<td>21.32</td>
<td>21.53</td>
</tr>
<tr>
<td>Closed</td>
<td>109.94</td>
<td>94.92</td>
<td>102.5</td>
<td>16.74</td>
<td>3.50</td>
<td>3.42</td>
</tr>
</tbody>
</table>

4.2. Influence of skewing the stator laminations

Skewing of the stator laminations is an effective method to reduce torque ripple [10]. The same dimensions and specifications of SPMSM with closed slots are utilised in this part with including the skew technique of the stator laminations. It is common practice to choose a skew angle comparable to the stator slot pitch. The best skew angle to eliminate as much cogging torque as possible is expressed in Equation 14 [23],[24].

$$\theta_{sk} = \frac{360}{M}$$

(14)

Where $M$ is the smallest common multiple of slot number $N_{slot}$ and pole number $2p$. For the case of the studied motor, $M$ is equal to 24, so the optimal skew angle $\theta_{sk}=15^\circ$.

Back-EMF for different values of skew angle and the EMF harmonic spectrum are presented in Figure 18 and Figure 19, respectively. In turn, Figures 20 and 21 portray cogging torque and electromagnetic torque for different values of skew angle, respectively.
Figure 18 Back-EMF for different values of skew angle.

Figure 19 EMF Spectrum for different values of skew angle.

Figure 20 Cogging torque for different values of skew angle.
For 15°, EMF harmonic spectrum yields better balance, which validates the calculated skew angle value. The total harmonic distortion (THD) content of each skew angle is reported in Table 5. In addition, Table 6 shows torque-ripple factor and cogging torque factor for different values of skew angle.

Table 5 Total harmonics distortion

<table>
<thead>
<tr>
<th>Skew angle (°)</th>
<th>0</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD (%)</td>
<td>12.6</td>
<td>8.04</td>
<td>4.46</td>
</tr>
</tbody>
</table>

Table 6 Torque ripples factor and cogging torque amplitude factor

<table>
<thead>
<tr>
<th>Skew angle (°)</th>
<th>( T_{\text{max}} ) (Nm)</th>
<th>( T_{\text{min}} ) (Nm)</th>
<th>( T_{\text{av}} ) (Nm)</th>
<th>( t_r ) %</th>
<th>( T_{\text{cpp}} ) (Nm)</th>
<th>( t_c ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>109.94</td>
<td>94.92</td>
<td>102.5</td>
<td>16.74</td>
<td>3.50</td>
<td>3.42</td>
</tr>
<tr>
<td>10</td>
<td>106.84</td>
<td>95.50</td>
<td>100.72</td>
<td>11.26</td>
<td>1.16</td>
<td>1.15</td>
</tr>
<tr>
<td>15</td>
<td>101.87</td>
<td>95.54</td>
<td>98.49</td>
<td>6.42</td>
<td>0.40</td>
<td>0.41</td>
</tr>
</tbody>
</table>

According to the aforementioned results, it can be remarked that average torque is practically the same for 0°, 10° and 15° skew angle. Nevertheless, peak-to-peak cogging torque value and torque ripples decrease noticeably when the skew angle increases. For 15° skew angle: \( t_r \) and \( t_c \) have the lowest values, back-EMF is closest to the sine form. Hence, 15° is the optimal skew angle which confirms analytical calculation of \( \theta_{sk} \).

5. Conclusion

In this paper, an analytical study of SPMSM is effected to present the back-EMF and the electromagnetic torque. A 2D finite element simulation is performed to examine the effects of stator slot opening and stator skewing on cogging torque, back-EMF and electromagnetic torque. Cogging torque factor \( t_c \) and the torque ripples factor \( t_r \) were calculated to compare the two configurations with open slots and closed slots. The \( t_c \) factor of the motor with closed slot decrease 18.11% while the \( t_r \) falls by 3.5%. The motor generates less torque ripples with closed slots configuration. Thus, it is better to opt for the configuration with
closed slots. Then, the motor with closed slots is utilised with skewing the stator laminations for different angle 0°, 10° and 15°. For 15° skew angle: t₁ and t₂ have the lowest values. Hence, the use of a skew angle comparable to the stator slot pitch allows: eliminating as much cogging torque as possible, minimising torque ripples and better balancing EMF harmonic spectrum yields. To conclude, choose the combination design of the motor with closed stator slots and skewed stator laminations seem to be a good solution to reduce torque ripples.

References


