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An efficient digital signal processing method for RRNS-based DS-CDMA systems

This paper deals with an efficient method for achieving low power and high speed in advanced Direct-Sequence Code Division Multiple-Access (DS-CDMA) wireless communication systems based on the Residue Number System (RNS). A modified algorithm for multiuser DS-CDMA signal generation in MATLAB is proposed and investigated. The most important characteristics of the generated PN code are also presented. Subsequently, a DS-CDMA system based on the combination of the RNS or the so-called Redundant Residue Number System (RRNS) is proposed. The enhanced method using a spectrally efficient 8-PSK data modulation scheme to improve the bandwidth efficiency for RRNS-based DS-CDMA systems is presented. By using the C-measure (complexity measure) of the error detection function, it is possible to estimate the size of the circuit. Error detection function in RRNSs can be efficiently implemented by Look-Up Table (LUT) cascades.

Keywords: RRNS-based DS-CDMA; 8-PSK modulation; error detection circuit; complexity measure; MATLAB.

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1. Introduction

CDMA technology was first used in military systems, and it was later adapted for commercial applications (e.g., UMTS, WIFI, GPS). Applications of CDMA systems are introduced in Table 1 [1].

Table 1: Applications of CDMA systems.

Applications	Military	Commercial
Anti-jamming	X	X
Multiple access	X	X
Low detectability	X	
Message privacy	X	X
Selective calling	X	X
Identification	X	X
Navigation	X	X
Multipath protection	X	X
Low radiated flux density	X	X

The current CDMA system is considered as an interference-limited system mainly due to the existence of Multiple-Access Interference (MAI) and Multipath Interference (MI). Many problems of communication systems based on CDMA technology stem from the unitary spreading codes. In multiuser DS-CDMA systems, the spreading codes fall into two categories, one for channelization (orthogonal Walsh-Hadamard codes) and another for

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scrambling (PN codes) [2-6]. There are two primary advantages to using Residue Number System (RNS) in DS-CDMA systems. First, RNS-based DS-CDMA can improve the bandwidth efficiency of a communication system by incorporating more bits per symbol for orthogonal modulation. Second, by adding a redundant digit to an RNS, we have a Redundant Residue Number System (RRNS) that detects a single-digit error. In the past, to detect an error in RRNS, only a method using Mixed Radix Number System (MRNS) or a method using Chinese Reminder Theorem (CRT) were known. Thus, the estimation of the size of the error detection circuit was difficult. This paper presents a design method without using MRNS or CRT. By using the C-measure of the error detection function, can be estimated the size of the circuit. This paper is organized as follows: Section 2 discusses design, simulation and analysis details of the proposed DS-CDMA system in MATLAB. Section 3 discusses the proposed DS-CDMA system based on the RNS. Section 4 derives upper and lower bounds on the C-measure of error detection function for a given RRNS, and with these bounds, it is possible to estimate the size of the error detection network implemented by a Look-Up Table (LUT) cascade.

2. Analysis of the proposed DS-CDMA system in MATLAB

We have proposed the modified algorithm for multiuser DS-CDMA signal generation in MATLAB. First, the algorithm for DS-CDMA signal generation of each user, where each user has I and Q data streams has been proposed. The universal algorithm for an $N \times N$ (in our case $N = 64$) Hadamard matrix for Walsh-Hadamard code generation and a linear feedback shift register for PN code generation has been proposed, respectively. For filtering of the user-coded data after scrambling, the Raised Cosine (RC) pulse shaping FIR filter with the roll-off parameter set to 0.3, oversampling = 4 and a group delay = 10 has been used. The resulting algorithm for multiuser DS-CDMA signal generation has been proposed by summation of each user signal. Finally, the algorithm for calculating the autocorrelation function and the power spectral density of the multiuser DS-CDMA signal has been proposed, respectively.

Implementation of the modified algorithm for multiuser DS-CDMA signal generation is illustrated in the flowchart in Figure 1. The simulation procedure of multiuser DS-CDMA signal generation is described as follows: Generated baseband user data (each user has I and Q data streams) is first oversampled in order to perform spreading. The oversampling rate is equal to the spreading factor since each bit is represented by N chips that represent a Walsh-Hadamard code. Then each user data is spread by a Walsh-Hadamard code providing orthogonality with other users, and the resulting user-coded data is further scrambled by a PN code of the same length. Consequently, each scrambled user-coded data stream is passed through an RC pulse shaping FIR filter and then summed to yield a multiuser DS-CDMA signal.

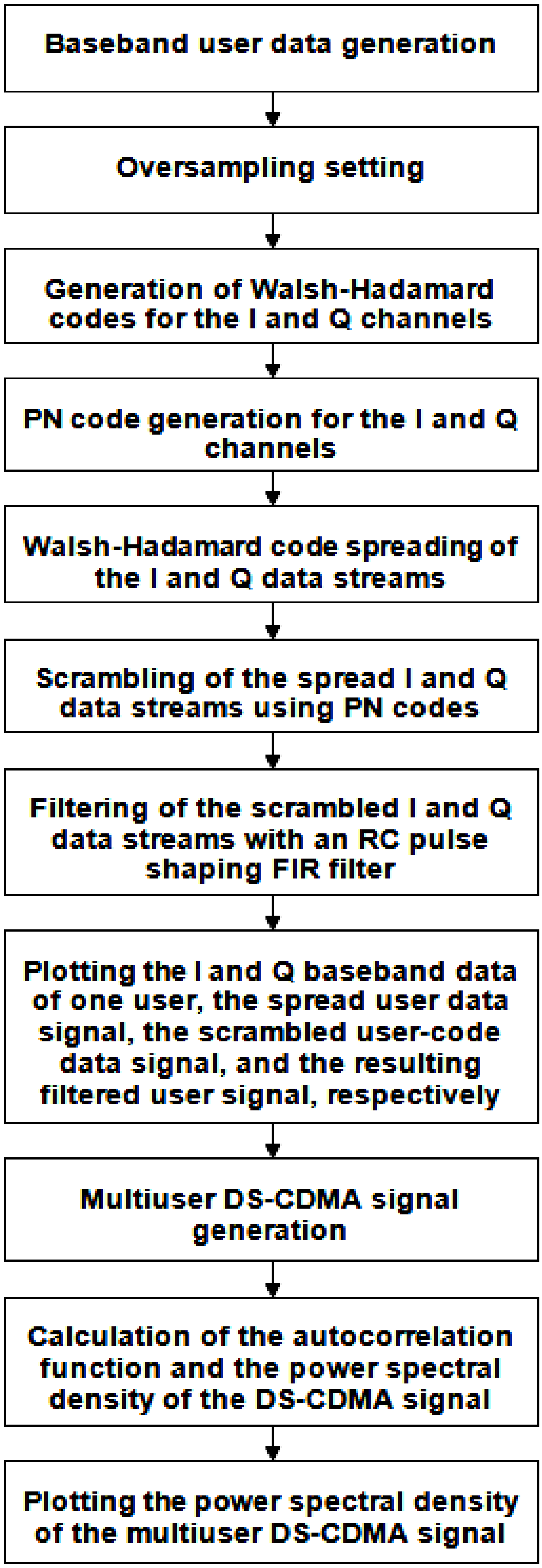

Figure 1: Flowchart of the simulation procedure of multiuser DS-CDMA signal generation in MATLAB.

Proposed MATLAB realization of the PN generator model based on the 12-stage linear feedback shift register for PN code generation is illustrated in Figure 2. The PN code sequence used for scrambling user-coded data is characterized by the characteristic polynomial $p(x) = x^{12} + x^6 + x^4 + x + 1$. The most important characteristics of the generated PN code, i.e., the autocorrelation function and its corresponding power spectral density are depicted in Figure 3 and 4, respectively.

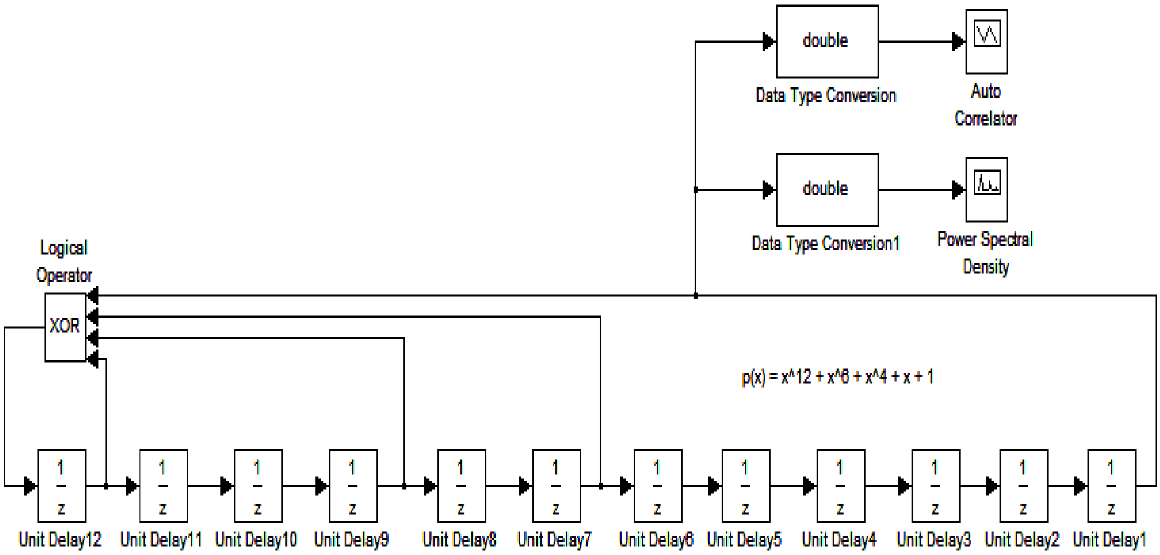

Figure 2: Proposed and simulated PN generator based on the 12-stage linear feedback shift register for PN code generation in MATLAB.

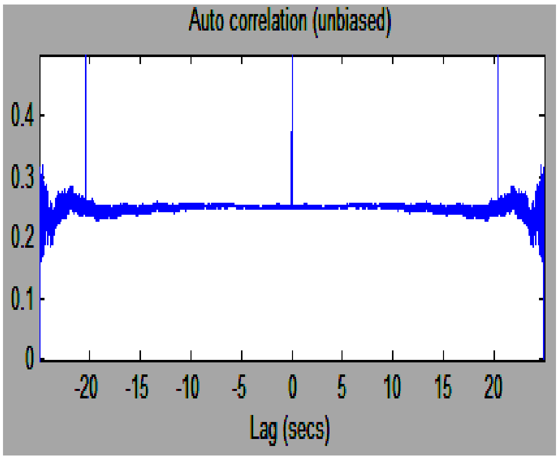

Figure 3: Autocorrelation function of the generated PN code.

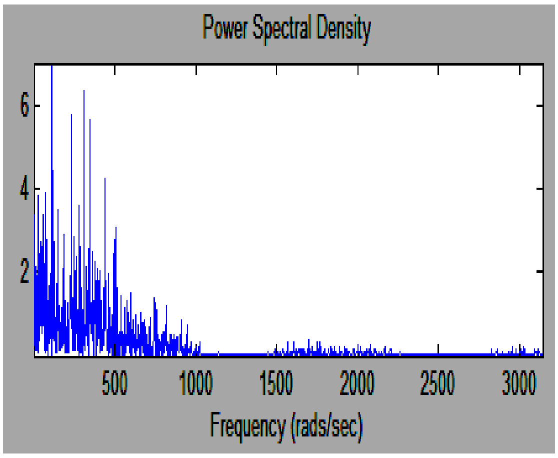

Figure 4: Power spectral density of the generated PN code.

Finally, the simulation results of DS-CDMA signal generation are depicted in Figure 5.

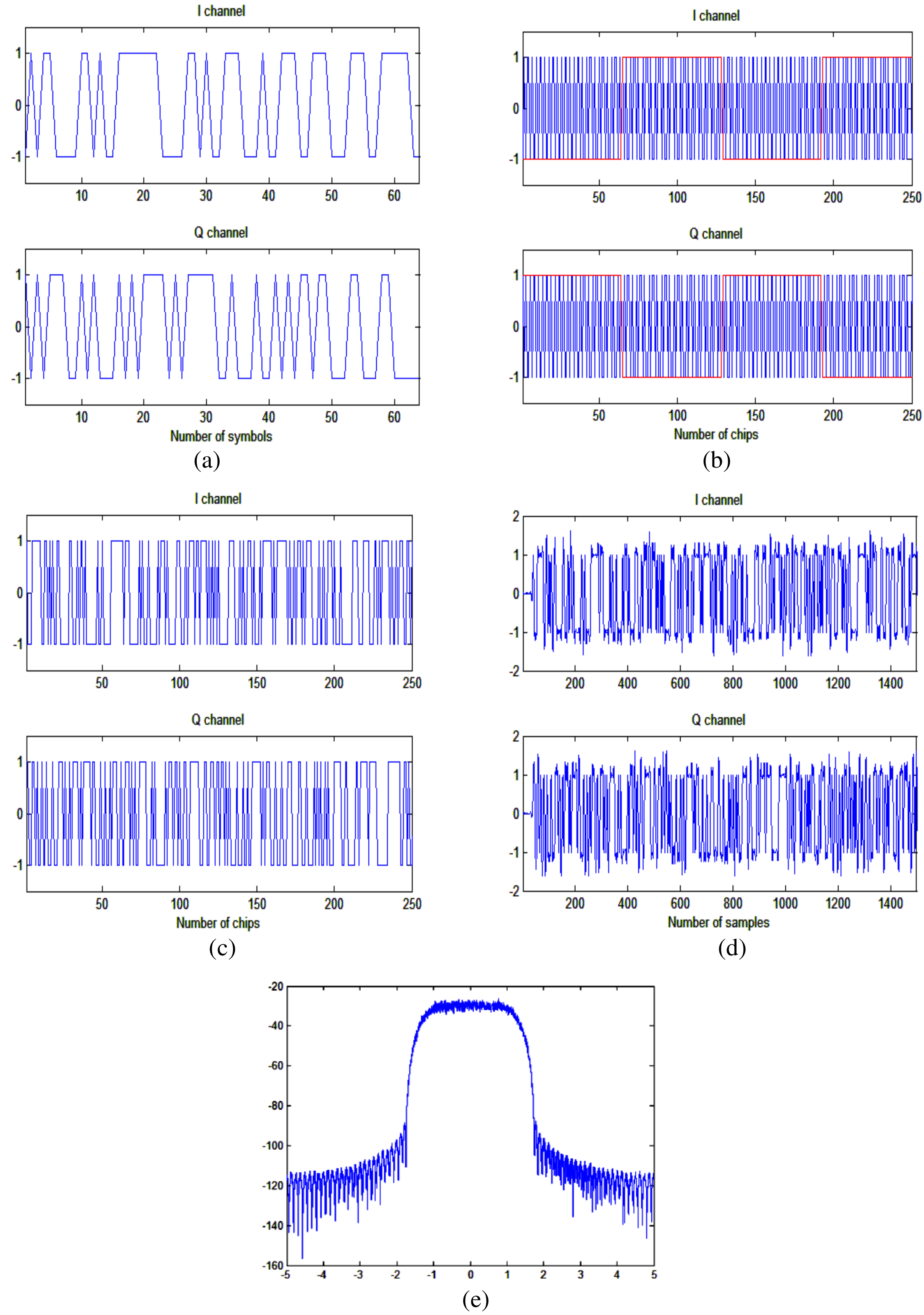

Figure 5: Simulation results of DS-CDMA signal generation. (a) I and Q baseband data of one user, (b) User data after spreading by Walsh-Hadamard code, (c) User-coded data after scrambling by a PN code, (d) Resulting filtered user signal by a baseband RC filter, and (e) Power spectral density of a multiuser DS-CDMA signal.

3. RNS-based DS-CDMA systems

The orthogonality of spreading codes is imperfect in general, and some of them are not orthogonal at all when they are used in multipath transmission channels. This is due to high cross-correlation values between the spreading sequences with arbitrary time shifts. This is the main source of interference in a DS-CDMA system where orthogonal codes are employed. A signaling scheme, where a set of orthogonal signals is transmitted in parallel and these signals are selected according to the so-called Residue Number System (RNS) [7-11] is presented. The first important property of RNS is absence of carry-propagation in addition and multiplication, carry-propagation being the most significant speed-limiting factor in these operations. The second is that because the residue representations carry no weight-information, an error in any digit-position in a given representation does not affect other digit-positions. And the third there is no significance-ordering of digits in an RNS representation, which means that faulty digit-positions may be discarded with no effect other than a reduction in dynamic range. With these inherent properties of the RNS, residue arithmetic offers a variety of new approaches to the realization of digital signal processing algorithms, such as digital modulation and demodulation, and the fault-tolerant design of arithmetic units. It also offers new approaches to the design of error-detection and error-correction codes. This section discusses the design of the DS-CDMA system based on residue arithmetic.

3.1. Practical utilization of RNS-based DS-CDMA

Residue Number System (RNS) is defined by a set of relatively prime integers called the moduli. The moduli set is denoted as $\{m_1, m_2, \ldots, m_v\}$. If all the moduli are pairwise relative primes, any integer X, describing a message (binary symbol), can be uniquely and unambiguously represented by the so-called residue sequence $\{r_1, r_2, \ldots, r_v\}$ in the range $0 \leq X < M$, where $r_i = X\left(\bmod m_i\right)$ represents the residue of X upon division by m_i, and $M = \prod_{i=1}^{v} m_i$ is the dynamic range. According to the so-called Chinese Reminder Theorem (CRT) [12] for any given v-tuple $\{r_1, r_2, \ldots, r_v\}$, where $0 \leq r_i < m_i$, $i = 1, 2, \ldots, v$ there exists one and only one integer X such that $0 \leq X < M$ and $X\left(\bmod m_i\right) = r_i$. The same congruence can be written in an alternative notation as follows:

$$\left|X\right|_{m_i} = r_i. \tag{1}$$

However, the real-time implementation of the CRT is not practical, as it requires modular operations with respect to a large M. In order to avoid processing large valued integers, fast algorithms for the computation of X have been proposed in [13].

If an RNS is proposed not only for the representation of data, but also for the protection of data, usually we design the RNS using so-called redundant moduli, so that the system has the capability of self-checking, error detection and error correction [7]. In this case X is limited to the so-called information dynamic range M_I of $\left[0, M = \prod_{i=1}^{v} m_i\right)$, where $v \leq u$, and $m_1, m_2, \ldots, m_v$ are referred to as the information moduli, while

$m_{v+1}, m_{v+2}, \ldots, m_u$ are the so-called redundant moduli. We express the product of the redundant moduli as $M_R = \prod_{j=1}^{u-v} m_{v+j}$. The error detection and correction properties of RNS are well established. In general $u-v$ redundant moduli can detect $u-v$ errors and can correct up to $u-v/2$ errors. If the SNR per bit given by E_b/N_0 is sufficiently high, the Redundant Residue Number System (RRNS)-based parallel DS-CDMA system using one or more redundant moduli can achieve a lower BER than the RNS-based parallel DS-CDMA system without redundant moduli.

Based on [7], we have proposed the modified moduli set as a tradeoff between BER performance and the computational complexity of RNS converters. Let $\{3,5,8,11,13,17\}$ is the proposed RNS moduli set. The moduli set assumes five residue channels in which five moduli $\{m_1, m_2, m_3, m_4$ and $m_5\}$ are non-redundant and the last, $\{m_6\}$ is used for error detection and correction. The product of the information moduli is called the information dynamic range $\left(\text{i.e., } M_I = \prod_{i=1}^{5} m_i = 17160\right)$ and determines the maximum number of bits possible in a symbol $\left(\text{i.e., } k = \left\lfloor \log_2 \prod_{i=1}^{5} m_i = 14 \right\rfloor\right.$, where $\lfloor x \rfloor$ represents the largest integer less than equal to $\left. x\right)$, provided that $M_I = \prod_{i=1}^{5} m_i \geq 2^k$. So, any symbol of 14 bits can be chosen within the given dynamic range. For any number within the given dynamic range, each of the moduli, m_i, $1 \leq i \leq 6$ will at most produce m_i residues. These residues are then mapped into an orthogonal code from Walsh-Hadamard transform of selected processing gain (spreading factor) and multiplexed for transmission. The total number of orthogonal spreading codes required is the sum of all moduli values $\left(\text{i.e., } \sum_{i=1}^{6} m_i = 57\right)$ which is equivalent to the minimum number of orthogonal codes required for the mapping of residue digits for parallel transmission. Adjacent channel interference is minimal due to the orthogonality of the individual residue channels. The multiplexed spread code corresponding to each residue symbol is scrambled using a scrambling code (PN code) and transmitted after pulse shaping.

On the other hand, in the conventional M-ary orthogonal DS-CDMA system, the number orthogonal signals required will then be $2^{14} = 16384$, several orders of magnitude higher than that of the RNS-based DS-CDMA system.

3.2. Proposed method for high-speed data transmission using RRNS in DS-CDMA systems

Even though the residue representation helps to reduce the computational complexity in M-ary DS-CDMA signaling systems, the data transmission rate and thereby bandwidth efficiency can be significantly increased further by modifying the modulation method in residue channels.

Consider a bandwidth-efficient modulation scheme for the case of 14 bits/symbol. The symbol is represented by our proposed moduli set $\{3,5,8,11,13,17\}$, in which the last element is redundant moduli. The residues can vary from $\{0-2, 0-4, 0-7, 0-10, 0-12, 0-16\}$, respectively. The maximum possible residue values for each modulus and its binary representation are given in the first and the second column of Table 2. Assuming 8-PSK modulation format for the data symbols, these binary residue values are split into spread code index and data symbol as shown in third and fifth columns of the Table 2. One extra orthogonal code is required for all moduli to accommodate spread code index 0. As shown in Table 2, for 8-PSK data modulation, the proposed method reduces the required number of orthogonal codes from 57 to 10.

Table 2: Example of the proposed method using an efficient modulation scheme for the RRNS-based DS-CDMA $\left(\text{number of bits per symbol} = 14, \text{ moduli set is} \{3,5,8,11,13,17\}\right)$ where 8-PSK data modulation is assumed, and u is the number of residues.

Maximum residue values		**Proposed method using a spectrally efficient 8-PSK data modulation scheme**		
		Index to orthogonal code		
Decimal (r_i)	Binary	Binary	Decimal (x_i)	Data
2	00010	00	0	010
4	00100	00	0	100
7	00111	00	0	111
10	01010	01	1	010
12	01100	01	1	100
16	10000	10	2	000
Total: $\sum r_i + u = 51 + 6 = 57$		Total: $\sum x_i + u = 4 + 6 = 10$		

Table 3: Comparison of computational complexities for the M-ary DS-CDMA, proposed RRNS-based DS-CDMA, and enhanced RRNS-based DS-CDMA systems.

	M-ary DS-CDMA system	**Proposed RRNS-based DS-CDMA system**	**Enhanced RRNS-based DS-CDMA system**
Number of orthogonal codes	16384	57	10
Minimum processing gain (spreading factor)	16384	64	16
Number of parallel channels	1	6	6
Data bits/symbol	Nil	Nil	3
Error correction	Nil	Can correct one residue channel	Can correct one residue channel
Correlator banks/symbol	16384	57	10
Number of complex multiplication per symbol	16384^2	$57 \times 64 = 3648$	$10 \times 16 = 160$

The corresponding performance of the proposed RRNS-based DS-CDMA system is numerically evaluated for the required system parameters. Table 3 compares the computational complexity of the M-ary DS-CDMA, proposed RRNS-based DS-CDMA and enhanced RRNS-based DS-CDMA systems. The number of bits per symbol for all three systems is 14. For the enhanced RRNS-based DS-CDMA system, the moduli set $\{3,5,8,11,13,17\}$, and 8-PSK data modulation are assumed. The complexity of RRNS-based systems is minimal when considering minimum number of complex multiplication required per symbol.

4. C-measure of error detection function in RRNS-based DS-CDMA

The C-measure (complexity measure) of a logic function $f(X)$ is the maximum value of $\mu(f : X_1 | X_2)$, where $X = (x_1, x_2, \ldots, x_N)$, $X_1 = (x_1, x_2, \ldots, x_k)$, and $X_2 = (x_{k+1}, \ldots, x_N)$. By repeatedly applying functional decompositions to a given function, we have a Look-Up Table (LUT) cascade shown in Figure 6. An LUT cascade has a regular structure, and is easier to design than a random logic network [14,15].

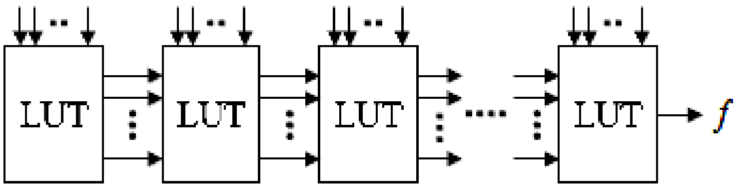

Figure 6: LUT cascade.

An LUT cascade consists of cells, and the signal lines connecting adjacent cell are rails. A logic function with a small C-measure can be realized by a compact LUT cascade. The C-measure is equal to the maximum width of the Binary Decision Diagram (BDD), where the variable ordering is $(x_1, x_2, \ldots, x_N)$. Let μ be the C-measure of a function f. Then, f can be implemented by an LUT cascade, whose cells have at most $\lceil \log_2 \mu \rceil + 1$ inputs, and $\lceil \log_2 \mu \rceil$ outputs.

4.1. Lower bound on the C-measure of the error detection function

Let μ be the C-measure of the error detection function in the RRNS $(m_1, m_2, \ldots, m_{n+1})$. Then, the following relations hold:

$$M_U + 1 \le \mu \tag{2}$$

and

$$M_L + 1 \le \mu, \tag{3}$$

where $M_U = \prod_{i=1}^{k} m_i$ and $M_L = \prod_{i=k+2}^{n+1} m_i$; k is the largest integer satisfy the relation $M_U \le M_L$.

4.2. Upper bound on the C-measure of the error detection function

Let μ be the C-measure of the error detection function in the RRNS $\left(m_1, m_2, \ldots, m_{n+1}\right)$. Then, the following relation holds:

$$\mu \leq \max_{t=0}^{s}\left\{\min\left[2^t M_U, \left(M_L\right)^{2^{s-t}}\right]\right\} + 1, \tag{4}$$

where $M_U = \prod_{i=1}^{k} m_i$ and $M_L = \prod_{i=k+2}^{n+1} m_i$; k is the largest integer satisfy the relation $M_U \leq M_L$, and s denotes the number of bits to represent $\hat{x}_{k+1}$.

4.3. Practical design of LUT cascade for error detection function for a given RRNS

For RRNS $\{3,5,8,11,13,17\}$, it can be obtained C-measure and designed the cascade for the error detection function. In this case, $\hat{x}_1$ is represented by 2 bits, $\hat{x}_2$ and $\hat{x}_3$ is represented by 3 bits, $\hat{x}_4$ and $\hat{x}_5$ are represented by 4 bits, and $\hat{x}_6$ is represented by 5 bits. Thus, the total number of primary inputs is $N = 21$. A single-digit error detection in a RRNS is considered. From Section 3.1, we found that the dynamic range is $M = 17160$. First, obtain the lower bound on μ. In the RRNS $\left(m_1, m_2, \ldots, m_{n+1}\right)$, if $\prod_{i=1}^{n+1} m_i$, then for each tuple $\left(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_k\right)$, there exist a unique tuple $\left(\hat{x}_{k+1}, \ldots, \hat{x}_{n+1}\right)$, that represents the same integer, where $\hat{x}_i \in \left[0, 1, \ldots, m_i - 1\right]$. Since, $3 \times 5 \times 8 < 11 \times 13 \times 17$, we have $k = 3$. From this, we have $1 + 3 \times 5 \times 8 = 121 \leq \mu$ and $1 + 13 \times 17 = 222 \leq \mu$. Second, obtain the upper bound on μ. From Section 4.2, we have $M_U = 3 \times 5 \times 8 = 120$, and $M_L = 13 \times 17 = 221$. Note that $s = \log_2\left\lceil \hat{x}_{k+1} \right\rceil = \log_2\left\lceil 11 \right\rceil = 4$. Consider the function $g\left(t\right) = \min\left\{2^t M_U + 1, \left(M_L\right)^{2^{s-t}} + 1\right\}$. This function takes its maximum when $t = 3$. Thus, we have $\mu \leq 2^t M_U + 1 = 2^3 \times 120 + 1 = 961$. By constructing the BDD, we found that the C-measure is $\mu = 397$ [15] which is much smaller than the dynamic range. Since $w = \left\lceil \log_2 \mu \right\rceil = \left\lceil \log_2 397 \right\rceil = 9$, the number of rails in the LUT cascade is at most 9. In an LUT cascade that realizes the function f, let N be the number of input variables; s be the number of cells; w be the maximum number of rails (i.e., the maximum number of signal lines between cells); u be the number of inputs for a cell; μ be the C-measure of the function f; and $u \geq \left\lceil \log_2 \mu \right\rceil + 1$. Then, an LUT cascade satisfying the following condition exists:

$$s \leq \left\lceil \frac{N - w}{u - w} \right\rceil. \tag{5}$$

When cells with $u = 12$ inputs are used, we have $s = \left\lceil \frac{21-9}{12-9} \right\rceil = \left\lceil \frac{12}{3} \right\rceil = 4$. This means that the function can be implemented by the LUT cascade with 4 cells. However, detailed analysis of lower and upper bounds on the C-measure of the error detection function shows that this function requires only 3 cells as shown in Figure 7.

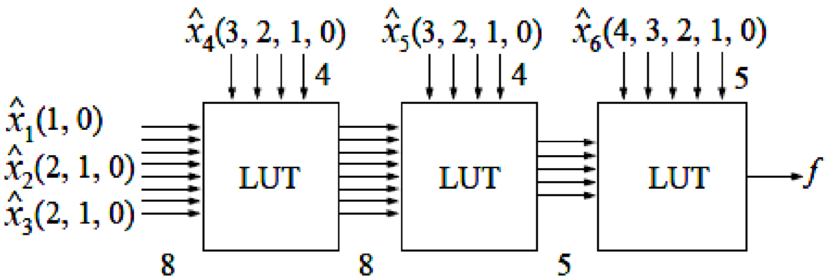

Figure 7: LUT cascade for error detection function.

The first cell has 12 external inputs: $\hat{x}_1(1,0)$, $\hat{x}_2(2,1,0)$, $\hat{x}_3(2,1,0)$ and $\hat{x}_4(3,2,1,0)$. Let $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4 | \hat{x}_5, \hat{x}_6)$ be the partition of the input variables. Since $1 + M_L$ is 222, the first cell has 8 $(\lceil \log_2 222 \rceil)$ outputs. The second cell has 8 rail inputs and 4 external inputs $\hat{x}_5(3,2,1,0)$. Let $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5 | \hat{x}_6)$ be the partition of the input variables. Since $1 + M_L$ is 18, the second cell has 5 $(\lceil \log_2 18 \rceil)$ outputs. The last cell has 5 rail inputs and 5 external inputs $\hat{x}_6(4,3,2,1,0)$.

5. Conclusion

The modified algorithm for multiuser DS-CDMA signal generation in MATLAB has been discussed in this paper. Subsequently, an enhanced method using a spectrally efficient 8-PSK data modulation scheme has been proposed for RRNS-based DS-CDMA systems to improve the bandwidth efficiency. The computational complexity of the RRNS-based DS-CDMA system is significantly reduced by using the proposed method. It is primarily because the proposed system uses a lower processing gain (spreading factor) to carry more number of data bits in a symbol. By using higher modulation for data, one can improve data transmission rate, but detection accuracy of the proposed system is decreasing. It has been obtained C-measure of the error detection function for a given RRNS, and demonstrated that C-measure is much smaller than the dynamic range. Thus, error detection circuit can be implemented by compact LUT cascades. In our practical design, M, the dynamic range of RRNS is chosen so that it matches 14-bit precision. Considering the bandwidth efficiency, and robustness against channel impairments, the proposed system can be considered as an alternative to high-speed data transmission mobile communication systems.

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