Optimal allocation of renewable distributed generation (RDG), i.e., solar and the wind in a distribution system becomes challenging due to intermittent generation and uncertainty of loads. This paper proposes an optimal allocation methodology for single and hybrid RDGs for energy loss minimization. The deterministic generation-load model integrated with optimal power flow provides optimal solutions for single and hybrid RDG. Considering the complexity of the proposed nonlinear, constrained optimization problem, it is solved by a robust and high performance meta-heuristic, Symbiotic Organisms Search (SOS) algorithm. Results obtained from SOS algorithm offer optimal solutions than Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and Firefly Algorithm (FFA). Economic analysis is carried out to quantify the economic benefits of energy loss minimization over the life span of RDGs.

Keywords: Optimal allocation, renewable distributed generation, loss minimization, symbiotic organisms search.

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1. Introduction

Electric power generation has entered into a new era of renewable energy generation. Solar and the wind are the most promising renewable energy technologies, with the potential to meet future energy demand in a sustainable way [1]. Hence, planning of renewable sources in distribution networks is the essential task. Optimally placed and sized solar DG (SDG) or wind DG (WDG) in distribution system reduces energy losses and improves voltage profile [2, 3]. Hybrid DG (HDG) that is a combined employment of energy sources enhance the utilization of energy sources and provide consistent energy generation [4]. Optimal allocation of such HDG offers significant loss reduction [5]. Optimal allocation of RDG in a distribution system to maximize their potential is a challenging issue due to their intermittency [6]. Also, RDG connection at non-optimal places or inappropriate sizing increases power losses. Hence, it is essential to identify optimal RDG allocation in a distribution network [7].

Various methodologies aim to optimally allocate RDG for minimizing losses in distribution systems. Analytical methods result in increased computational efforts with sub-optimal solutions [8]. Weighing factors method offers optimal solutions; however the quality of the solution is affected by the choice of weighing factors [9]. Probabilistic methods are formulated with large ‘generation-load’ states and these states formation involves clustering methods. Clustering obtained by the iterative process may converge to local minima producing sub-optimal solutions [10]. Intermittent renewable generation is also modeled probabilistically by convolution, but a large number of states involved in convolution and ‘generation-load state’ model affects the quality of solutions. This underscores the necessity of planning problem formulations for the power system to provide optimal solutions [11]. Thus, RDG allocation problems having large search space
needs an allocation methodology to explore the search space for obtaining optimal solutions.

The optimization algorithms used for optimal placement and sizing of DG mainly includes genetic algorithm (GA) [12], particle swarm optimization (PSO) [13, 14], Simulated Annealing (SA) [15], artificial bee colony (ABC) [16] and firefly algorithm [17]. The hybrid combination of algorithms such as ant colony optimization (ACO) and artificial bee colony (ABC) [18], (PSO) and gravitational search algorithm (GSA) [19] are also used for optimal allocation of DGs.

This paper develops an optimal solution methodology for allocation of RDG to minimize energy losses. A deterministic generation-load model is proposed from probabilistic generation model and IEEE-RTS load model. The proposed deterministic generation-load model, integrated with optimal power flow, provides optimal solutions. Then a deterministic generation-load model is proposed for HDG allocation using the expected generation of SDG and WDG. This nonlinear, constrained optimization problem is solved with a robust and high-performance symbiotic organisms search (SOS) algorithm. Comparative results from standard algorithms like GA, PSO, and FFA highlight the efficiency of SOS algorithm to offer better results for loss minimization. In the end, an economic model is included to get an insight of benefit of energy loss minimization by optimal allocation of RDG over the life period of renewable DG.

The paper is organized as follows. Section 2, presents the system modeling. The detail problem formulation is presented in section 3. The optimization algorithms, i.e., SOS is briefly discussed in section 4. Economic analysis to quantify the economic benefits of energy loss minimization is presented in Section 5. A case study and results are discussed in section 6.

2. System Modeling

2.1 Historical Data Processing

Historical data is used to obtain predicted RDG generation using probability distribution functions (pdfs). To obtain pdfs with accuracy, the whole year (Y) is segmented into three seasons (i.e. summer, monsoon and winter) considering average monthly temperatures. A day in the season is assumed to represent the whole season and each day is further divided into hourly time segments. Considering each month ‘M’ to have ‘D’ days, each time segment has Y* M* D solar irradiance and wind speed data points (i.e., ‘Y’ years * ‘M’ months per season * ‘D’ days per month).

2.2 Solar Power

The solar power modeling is done to obtain expected generation from it. Randomness of solar irradiance is expressed by beta probability distribution function (i.e., beta pdf $F_\beta$) [5]. The $F_\beta$ indicates the probability or fraction of time for which the solar irradiance is at a given irradiance ‘s’. The beta pdf $F_\beta(s)$ is as given below

$$F_\beta(s) = \begin{cases} 
\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} s^{(\alpha-1)}(1-s)^{\beta-1} & 0 \leq s \leq 1, \alpha, \beta \geq 0 \\
0 & \text{otherwise}
\end{cases}$$

(1)

where, $\Gamma$ is gamma function; s is random variable of solar irradiance in kW/m$^2$; $\alpha$ and $\beta$ are the shape parameters of beta distribution function. The cumulative distribution
function \((cdf)\) can be used to estimate the time for which solar irradiances \(\gamma \) is within a certain irradiance interval (e.g. \(s_I\) and \(s_2\)).

\[
f_{\beta}(s) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^t s^{(\alpha-1)}(1-s)^{(\beta-1)} dt
\]

(2)

Probability of solar irradiance being between \(s_1\) and \(s_2\) can be obtained as given below.

\[
f_{\beta}(s_1 < s < s_2) = f_{\beta}(s_2) - f_{\beta}(s_1)
\]

(3)

The power output \(P_o(s)\) of PV cell at any state \(y\) with cell current \(I_y\), cell voltage \(V_y\), \(N\) number of cell and fill factor \(f_f\) is given as below.

\[
P_o(s) = N * f_f * V_y * I_y
\]

(4)

The total expected output power \(P_{SG}\) at any hour can be given as,

\[
P_{SG} = \int_0^\infty P_o(s) * f_\beta(s).ds
\]

(5)

2.3 Wind Power

Wind generation is modeled with Weibull \(pdf\) due to its simplicity. Weibull \(pdf\) \(F_w(v)\) for the wind turbine is as given below [20].

\[
F_w(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} e^{x p \left[ - \left( \frac{v}{c} \right)^k \right]}
\]

(6)

Here, \(k\), \(c\) and \(v\) are the shape index, scale index and wind speed respectively. Rayleigh distribution is a simplified case of the Weibull distribution where \(k\) is assumed as 2 and \(c\) is approximated as \(1.128 v_m\), where \(v_m\) is the mean wind speed. Under the Rayleigh based approach, \(pdf\) and \(cdf\) of wind velocity is given by equation (7) and (8) respectively.

\[
F_w(v) = \left( \frac{2v}{c^2} \right) e^{x p \left[ - \left( \frac{v}{c} \right)^2 \right]}
\]

(7)

\[
f_w(v) = 1 - e^{x p \left[ - \frac{\pi}{4} \left( \frac{v}{v_m} \right) \right]}
\]

(8)

Probability of wind speed being between \(v_1\) and \(v_2\) can be obtained as,

\[
f_w(v)(v_1 < v < v_2) = f_w(v_2) - f_w(v_1)
\]

(9)

The power delivered by a wind turbine \(P_o(w)\) is represented by its power curve that gives a relation between the wind speed and power as below.
\[ P_w(w) = \begin{cases} 
0 & 0 \leq v_{av} \leq v_{ci} \\
0 \left( \frac{v_{av} - v_{ci}}{v_r - v_{ci}} \right) & v_{ci} \leq v_{av} \leq v_r \\
P_r & v_r \leq v_{av} \leq v_{co} \\
0 & v_{co} \leq v_{av} 
\end{cases} \]  

Where, \( v_{ci} \), \( v_{co} \), \( v_r \), \( v_{av} \) and \( P_r \) represent the cut-in speed, cut-off speed, rated speed, average speed and output power of wind turbine respectively. The total expected wind power \( P_{WG} \) at any time interval is given as,

\[ P_{WG} = \int_0^\infty P_w(w) * f_w(v) dv \]  

2.4 Load Modeling

Load considered in this system is hourly peak load, expressed as a percentage of daily peak load. The load profile follows IEEE-RTS system [21]. Hourly loads for three different seasons i.e. summer, monsoon and winter are considered

3. Problem Formulation

3.1. Optimal Sizing and Location of Renewable DG

Once the number of RDGs to provide expected power (i.e., forecasted output power) at optimal location are obtained by optimization, rated optimal size of RDG can be obtained as given below.

3.1.1 Solar DG

The expected \( (P_{SDGE}) \) and rated \( (P_{SDG}) \) optimal size of SDG at optimal location can be given by equations (12) and (13) respectively

\[ P_{SDGE,i} = c_{s,i} P_{SG} \quad \forall i \in D \]  

\[ P_{SDG,i} = c_{s,i} P_{SDGR} \quad \forall i \in D \]  

Where \( C_{s,p} \), \( P_{SG} \), \( P_{SDGR} \) and \( D \) give the integer variables representing number of solar panels at \( i^{th} \) bus, expected solar PV generation, rating of solar PV module and candidate bus respectively

3.1.2 Wind DG

The expected \( (P_{WDGE}) \) and rated \( (P_{WDG}) \) optimal size of WDG at optimal location can be given by equation (14) and (15) respectively.
\[ P_{WDGE,i} = c_{w,i} P_{WG}, \quad \forall i \in D \]  
\[ P_{WDG,i} = c_{w,i} P_{WDGR}, \quad \forall i \in D \]

where, \( c_{w,i} \), \( P_{WG} \), and \( P_{WDGR} \) give the integer variables representing number of wind turbines at \( i \) th bus, expected wind power and rating of wind turbine respectively.

### 3.1.3 Hybrid DG

The expected (\( P_{HDGE} \)) and rated (\( P_{SDGH}, P_{WDGH} \)) optimal size of HDG at optimal location are given by equation (16) and (17-18) respectively.

\[ P_{HDGE,i} = c_{sh,i} P_{SG} + c_{wh,i} P_{WG}, \quad \forall i \in D \]  
\[ P_{SDGH,i} = c_{sh,i} P_{SDGR}, \quad \forall i \in D \]  
\[ P_{WDGH,i} = c_{wh,i} P_{WDGR}, \quad \forall i \in D \]

where, \( c_{sh,i} \) and \( c_{wh,i} \) give the integer variables representing the number of solar panels and wind generators at \( i \)th bus for hybrid combination.

### 3.2 Distribution System Power Flow

The expected hourly generation and load are used for load flow calculation. The power losses \( P_{loss} \) can be calculated with backward/forward (BF) sweep method [22]. The system is assumed balanced and represented on per phase basis. Load current \( I_{Li} \) of node \( i \) can be expressed as,

\[ I_{Li} = \frac{P_i - Q_i}{V_i^*} \]  

Where, \( P_i \) and \( Q_i \) are active and reactive load at node \( i \), respectively. Current through branch \( i \) i.e., \( I_i \) will be load current of node \( i \) plus the branch currents connected to this line,

\[ I_i = \frac{P_i - Q_i}{V_i^*} + \sum_{j \in \beta_i} I_j \]  

Where \( \beta_i \) is the set of branches connected to node \( i \). Thus for calculating branch currents, all branches connected to the node must be determined. Sum of \( I_i^2 r \) losses in each branch gives the total real power loss i.e., \( P_{loss} \) of the system. Considering renewable DG power at node \( i \), active power \( P_i \) gets modified to \( (P_i - P_{RIDGE,i}) \).

### 3.3 Objective Function

The objective is to minimize annual energy losses.

\[ F = \min \left( \sum_{D=1}^{120} \sum_{i=1}^{24} P_{loss,summer,D} + \sum_{D=1}^{120} \sum_{i=1}^{24} P_{loss,monsoon,D} + \sum_{D=1}^{120} \sum_{i=1}^{24} P_{loss,winter,D} \right) \]  

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Here each season has 120 days (i.e., 30 days * 4 months). Annual energy losses are obtained by summation of losses of three seasons.

3.4 Constraints

3.4.1 Active and reactive power balance

Assuming RDG sources are operating at unity power factor and supplying only active power, the active and reactive powers balance are given below.

\[
P_{G1,t} + \sum_{i=1}^{n} P_{RDGEi,t} - \sum_{i=1}^{n} P_{i,t} - \sum_{b=1}^{nb} P_{loss,b,t} = 0 \quad (22)
\]

\[
Q_{G1,t} - \sum_{i=1}^{n} Q_{i,t} - \sum_{b=1}^{nb} Q_{loss,b,t} = 0 \quad (23)
\]

Where, \( P_{G1} \) and \( Q_{G1} \) are injected active and reactive power at substation. \( P_{RDGEi} \) is RDG’s forecasted power. \( P_i \) and \( Q_i \) are total active and reactive loads at \( i^{th} \) node respectively. \( P_{loss,b} \) and \( Q_{loss,b} \) are active and reactive power losses at branch \( b \), respectively. \( nb \) is the total number of branches.

3.4.2 Feeder current

With the placement of renewable DG, feeder current is restricted to maximum feeder capacity.

\[
0 \leq I_{k,ij} \leq I_{ij_{max}} \quad \forall i, j, k \quad (24)
\]

3.4.3 Maximum penetration of renewable DG

RDG penetration is considered as \( k\% \) of system’s total peak load \( (P_{Lmax}) \) for single RDG as well as HDG. Maximum penetration limits for SDG, WDG and HDG are given by equations (25), (26) and (27) respectively. The summation of power injected by all RDGs should be equal to the allowed maximum penetration of RDGs.

\[
\sum_{i=1}^{n} c_{s,d} \times P_{SG} = k \times P_{Lmax} \quad \forall i \in D \quad (25)
\]

\[
\sum_{i=1}^{n} c_{w,d} \times P_{WG} = k \times P_{Lmax} \quad \forall i \in D \quad (26)
\]

\[
\sum_{i=1}^{n} c_{s,d} \times P_{SG} + \sum_{i=1}^{n} c_{w,d} \times P_{WG} = k \times P_{Lmax} \quad \forall i \in D \quad (27)
\]

RDG penetration affects the design and operation of distribution system and may increase cost of distribution system and consumer payments. As per IEA study, 25% to 40% penetration of renewable energy sources put a little additional cost on the system in the long run, and hence is an acceptable penetration limit [23].

4. Optimization algorithm
RDG allocation is a complex, mixed integer, non-linear, constrained optimization problem. Heuristics approaches are efficient in finding global optima with higher success rates for better solutions [24]. Considering the exponentially large search space and complexity of this problem, a robust and high performance algorithm called SOS is proposed in this paper. In SOS, new solutions are generated by imitating the biological interaction between two organisms of the ecosystem i.e., mutualism, commensalism, and parasitism. SOS algorithm is briefly explained here [25].

4.1.1 Mutualism Phase

The mutualism phase of SOS mimics a mutualistic relationships between two organisms e.g. bees and flowers. If $X_i$ is an organism matched to the member of the ecosystem, then organism $X_j$ is randomly selected from the ecosystem that interacts with $X_i$. New candidate solutions for $X_i$ and $X_j$ are calculated based on the mutualistic symbiosis. The benefit factors ($BF_1$, $BF_2$) represent the level of benefit to each organism. A vector called ‘Mutual_Vector’ represents the relationship between organism $X_i$ and $X_j$. The mutualistic efforts for their survival is given by ($X_{best} - \text{Mutual Vector} * BF_1$). $X_{best}$ represents the highest degree of adaptation.

<table>
<thead>
<tr>
<th>Algorithm 1: Symbiotic Organisms Search (SOS)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ecosystem Initialization</strong></td>
</tr>
<tr>
<td><strong>Go to the next iteration</strong></td>
</tr>
<tr>
<td><strong>Identify the best solution $X_{best}$</strong></td>
</tr>
<tr>
<td><strong>Mutualism Phase</strong></td>
</tr>
<tr>
<td>i. Select one organism randomly, $X_j$, where $X_j \neq X_i$</td>
</tr>
<tr>
<td>ii. Determine mutual relationship vector ($\text{Mutual Vector}$) and benefit factor ($BF$)</td>
</tr>
<tr>
<td>iii. Mutual Vector = $(X_i + X_j)_{\geq 2}$</td>
</tr>
<tr>
<td>iv. Modify organism $X_i$ and $X_j$ based on their mutual relationship</td>
</tr>
<tr>
<td>v. $X_{i \text{ new}} = X_i + \text{rand} (0; 1) \cdot (X_{best} - \text{Mutual Vector} \cdot BF_1)$</td>
</tr>
<tr>
<td>$X_{j \text{ new}} = X_j + \text{rand} (0; 1) \cdot (X_{best} - \text{Mutual Vector} \cdot BF_2)$</td>
</tr>
<tr>
<td><strong>Commensalism Phase</strong></td>
</tr>
<tr>
<td>i. Select one organism randomly, $X_j$, where $X_j \neq X_i$</td>
</tr>
<tr>
<td>ii. Modify organism $X_i$ with the assist of organism $X_j$</td>
</tr>
<tr>
<td>iii. $X_{i \text{ new}} = X_i + \text{rand} (-1; 1) \cdot (X_{best} - X_j)$</td>
</tr>
<tr>
<td>iv. Select Fitter organisms as solutions for the next iteration</td>
</tr>
<tr>
<td><strong>Parasitism Phase</strong></td>
</tr>
<tr>
<td>i. Select one organism randomly, $X_j$, where $X_j \neq X_i$</td>
</tr>
<tr>
<td>ii. Create a Parasite ($\text{Parasite Vector}$) from Organism $X_i$</td>
</tr>
<tr>
<td>iii. Select Fitter organisms as solutions</td>
</tr>
<tr>
<td>Go to step 2 if the current $X_i$ is not the last member of the ecosystem otherwise proceed to next step</td>
</tr>
<tr>
<td><strong>Stop if one of the termination criteria is reached otherwise return to step 2 and start the next iteration</strong></td>
</tr>
</tbody>
</table>

4.1.2 Commensalism Phase

This is observed in remora fish and shark. The remora attaches to the shark and eats food leftovers. Remora receives benefit, while shark is unaffected by remora fish. Here an organism $X_j$, is randomly selected to interact with $X_i$. Organism $X_i$ attempts to benefit from the interaction. The organism $X_j$ neither benefits nor suffers. New candidate solution of $X_i$ is calculated according to the commensal symbiosis between them.

4.1.3 Parasitism Phase
This is observed in mosquito and human body. Here a parasite vector is created by duplicating organism $X_i$. The randomly selected organism $X_j$ serves as a host to the parasite vector. Parasite vector tries to replace $X_j$ in the ecosystem. *Parasite* Vector with better fitness kills the organism $X_j$ and fixes its position in the ecosystem. If the fitness value of $X_j$ is better, it will have immunity from the parasite and *Parasite* Vector will be discarded.

![Flow chart for optimal allocation using SOS](image)

**Fig. 1:** Optimal allocation of RDG using SOS

Flow chart for optimal allocation using SOS is shown in Fig.1. Initialization of the algorithm requires system data, bus data, load data, expected generation of sources, no. of organism (*eco_size*) and termination criteria. Fitness of each organism in the ecosystem is checked by load flow, by passing them through all phases of symbiotic relations. The process is repeated for all elements of ecosystem (i.e. *eco_size*) till termination criteria is attained. Similar procedure is applied for the Genetic Algorithm (GA) [26], Particle Swarm Optimization (PSO) [27] and Firefly Algorithm (FFA) [28].

### 5. Economic analysis

Economic analysis is carried out to show the economic benefits of energy loss minimization. The costs includes investment cost and operation & maintenance cost. The benefits are from energy generation and loss minimization due to optimal allocation. The total investment cost $C_I$ and operation & maintenance (O&M) cost $C_{OM}$ is given as below,

$$ C_I = P_{1SG} C_{1S} + P_{1WG} C_{1W} $$

(28)
\[ C_{OM} = \sum_{j=1}^{N_Y} \left( P_{ISG} C_{OMS} + P_{IWG} C_{OMW} \right) \left( \frac{1 + R_{\text{inf}}}{1 + R_{\text{int}}} \right)^j \]  

(29)

Where, \( C_{IS} \) investment cost of SDG in Rs./MW; \( C_{IW} \) investment cost of WDG in Rs./MW; \( P_{ISG} \) total installed capacity of SDG MW; \( P_{IWG} \) total installed capacity of WDG MW; \( C_{OMS} \) O&M cost of SDG Rs./MW; \( C_{OMW} \) O&M cost of WDG Rs./MW; \( N_Y \) total number of years; \( R_{\text{inf}} \) inflation rate; \( R_{\text{int}} \) interest rate or discount rate; Revenue can be obtained by the generation of renewable energy. The total benefit by the production of renewable generation can be obtained as below,

\[ B_{EG} = \sum_{j=1}^{N_Y} \left( P_{SG} C_{SE} + P_{WG} C_{WE} \right) \left( \frac{1 + R_{\text{inf}}}{1 + R_{\text{int}}} \right)^j \]  

(30)

Where, \( C_{SE} \) cost of solar energy Rs./kW h; \( C_{WE} \) cost of wind energy Rs./kW h; With the optimal allocation of RDG significant loss minimization is obtained. The cost of this saved energy due to loss minimization can be given as below.

\[ B_{EL} = \sum_{j=1}^{N_Y} \left( P_{LS} C_{SEL} + P_{LW} C_{WEL} \right) \left( \frac{1 + R_{\text{inf}}}{1 + R_{\text{int}}} \right)^j \]  

(31)

Where, \( P_{LS} \) annual energy loss minimization by SDG M W h; \( P_{LW} \) annual energy loss minimization by WDG M W h; \( C_{SEL} \) cost of energy losses for SDG Rs./kW h; \( C_{WEL} \) cost of energy losses for WDG Rs./kW h;

The various indices used for economic analysis of RDGs are mainly net present value (NPV), aggregate benefit cost ratio (ABCR) and discounted payback period (DPBP). NPV is the net value of all benefits (i.e. cash inflows) and costs (i.e. cash outflows) of the project, discounted back to the beginning of the investment.

\[ NPV = \text{cash inflows} - \text{cash outflows} \]

The project with higher NPV is most preferable.

ABCR is the ratio of the accumulated present value of all the benefits to the accumulated present value of all costs, including the initial investment.

\[ ABCR = \frac{\text{Present value of benefits}}{\text{Present value of costs}} \]  

(33)

PBP indicates the minimum period over which the investment for the project is recovered considering the time value of money. At payback period,

\[ \text{Present value of costs} = \text{present value of benefits} \]  

(34)

The project with lower pay back period is most preferable.
6. Results and discussions

6.1 System under Study

The proposed methodology is applied to a 34 bus test system shown in Fig.2 [29]. The system load is modelled as specified in IEEE-RTS system. Hourly solar and wind data of 5 years is taken for Satara (Longitude:74.05 E Latitude:17.75 N) Maharashtra state, India.

![Fig. 2: 34 bus system](image)

To get hourly predicted RDG generation more accurately, the solar irradiance data and the wind speed data is analyzed on seasonal basis. The whole year is segmented into three seasons and hence each season has 4 months. Thus in a 5 years span, each season has a total of 20 months (i.e. 5 years * 4 months each season) and 600 days (i.e. 20 months * 30 days per month). Considering a typical hour of each day (say 9 am), we have 600 solar irradiance and wind speed data for this typical hour (i.e. we get 600 time slots of 9 am and hence 600 corresponding data). The solar PV module used is 325 W [30] and wind turbine is 100 kW [31]. These specifications of SPV module and wind turbine are shown in Appendix A. The cdfs are generated selecting different states per hour for $\beta$ cdfs and Rayleigh cdfs respectively. The states are selected based on the maximum solar irradiance and maximum wind speed. The values of various parameters, probability and output for different states of a typical hour (i.e. 8 am) is shown in Appendix B. Solar and wind power is available for 11 and 24 Hrs respectively. Total system peak load is 5.0 MVA. RDG penetration is considered as 2 MW i.e., 40% of system’s total peak load ($P_{Lmax}$) at unity power factor.

6.2 Optimal location and sizing

The possible candidate bases are randomly selected and the size of RDG can be increased in a discrete step of 100 kW. Maximum 5 RDGs (i.e. 500 kW) can be placed optimally on any of the candidate buses. This maximum number of RDG at any bus can be limited by the maximum current capacity of the feeder. Here the maximum current capacity is 50 A. The selected ten candidate buses are {12,15,18,22,25,27,28,29,30,32}.
In practice, candidate buses are selected depending upon physical availability of space for RDG and potential for RDG generation. Thereafter, optimal locations on selected buses can be found by optimization. Optimal allocation of RDG is discussed in the following sub-sections.

6.2.1 Solar DG

Table 1. shows the optimal number of SDG with its rating, optimal locations, annual energy losses and percentage loss minimization. System’s annual energy losses without any RDG (i.e. base case) are 1078.20 MWh. SDG’s minimum size is considered as 100 kW. A SDG of 100 kW requires 308 rated solar modules. Thus 20 SDGs have to be optimally allocated to obtain 2 MW penetration. About 10-11% loss minimization is obtained by optimal SDG placement. GA and FFA places RDG on nine candidate buses, SOS on 8 and PSO only on 5 candidate buses.

<table>
<thead>
<tr>
<th>Location (Bus no)</th>
<th>SOS</th>
<th>PSO</th>
<th>GA</th>
<th>FFA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DG Size (kW)</td>
<td>DG Size (kW)</td>
<td>DG Size (kW)</td>
<td>DG Size (kW)</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
<td>500</td>
<td>300</td>
<td>200</td>
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<td>15</td>
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<td>100</td>
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<td>22</td>
<td>300</td>
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<td>25</td>
<td>400</td>
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<td>200</td>
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<tr>
<td>32</td>
<td>500</td>
<td>500</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

| Losses (MWh) | 958.97 | 959.40 | 963.40 | 963.78 |

Fig. 3: Convergence plot for solar DG allocation

SOS provides improved loss minimization as compared to other optimization methods. SOS places SDG of rating (kW) 200, 100, 300, 400, 300, 100, 100 and 500 on buses \{12, 18, 22, 25, 27, 29, 30, 32\} respectively. Annual energy losses obtained by SOS are 958.97 MWh and loss minimization is 11.06%. PSO locates SDG of rating (kW) 500, 200, 300, 500 and 500 on buses \{12, 22, 25, 27, 32\}. GA and FFA offer similar placement and sizing with each other as shown in Table I. Convergence plot for SDG is shown in Fig. 3. For all algorithms, the population size and maximum iteration count was fixed to 30 and 250 respectively. SOS offers best fitness value (i.e. loss in kW) with slow
convergence as compared to all other algorithms. FFA offers faster convergence but weaker fitness value, as compared to other algorithms. GA and PSO offer convergence and fitness value intermediate to FFA and SOS.

6.2.2 Wind DG

The minimum rating of wind turbine is considered as 100 kW, hence 20 wind turbines need to be optimally allocated. SOS offers optimal result by placing WDG on eight buses \{12,18,22,25,27,28,29,30\}. FFA gives least loss minimization of 28.68%, by allocating WDG on all buses. Loss minimization by GA is 29.18 % and by PSO is 29.46 %. WDG allocation by GA and PSO is as shown in Table II. The convergence plot of the algorithms for wind DG allocation are as shown in Fig. 4. SOS offers best fitness value though converges slowly. FFA has faster convergence but weaker fitness. GA and PSO have convergence and fitness value intermediate to FFA and SOS.

### TABLE II: Optimal allocation of Wind DG

<table>
<thead>
<tr>
<th>Location (Bus no)</th>
<th>SOS</th>
<th>PSO</th>
<th>GA</th>
<th>FFA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DG nos.</td>
<td>DG Size (kW)</td>
<td>DG nos.</td>
<td>DG Size (kW)</td>
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<td>2</td>
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<tr>
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</tr>
<tr>
<td>22</td>
<td>2</td>
<td>200</td>
<td>2</td>
<td>200</td>
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<tr>
<td>25</td>
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<td>100</td>
<td>3</td>
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</tr>
<tr>
<td>27</td>
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</tr>
<tr>
<td>29</td>
<td>2</td>
<td>200</td>
<td>–</td>
<td>–</td>
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<tr>
<td>30</td>
<td>–</td>
<td>–</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>32</td>
<td>5</td>
<td>500</td>
<td>5</td>
<td>500</td>
</tr>
</tbody>
</table>

| Losses (MWh) | 759.3 | 760.6 | 763.55 | 768.97 |

Fig. 4: Convergence plot for wind DG allocation

6.2.3 Hybrid DG

The combined maximum penetration for HDG is considered as 2 MW. Thus SDG or WDG can contribute maximum 50 % penetration. Maximum 10 SDG and 10 WDG can be optimally allocated on the candidate buses. Optimal allocation results of Table III indicates that a maximum of 22.43% loss minimization is obtained by SOS by placing 40% SDGs and 60% WDGs. Thus it allocates total 800 kW of SDG on buses \{12,25,30,32\} and 1200 kW of WDG on buses \{12,25,27,29,32\}. PSO, GA and FFA optimally place 35 % of SDG and 65% of WDG. GA and FFA place WDG on all candidate buses. Convergence plot of algorithms for HDG optimization are shown as Fig. 5. Fitness value obtained by SOS is better, though its convergence is slow as compared to others.
GA gives faster convergence. FFA and PSO has convergence value in between the GA and SOS.

Above results shows that the optimal solutions of loss minimization by SOS provides significant economical benefits over the life span of the RDG. Also optimization results of SOS are found to be more consistent. Hence SOS is more suitable for these type of optimization problems though it has a slow convergence. Optimal results were obtained by SOS after varying the candidate buses also (e.g. 15 buses, 20 buses and 25 buses).

**TABLE III: Optimal allocation of hybrid RDG**

<table>
<thead>
<tr>
<th>Location (Bus no)</th>
<th>SOS Solar (kW)</th>
<th>Wind (kW)</th>
<th>PSO Solar (kW)</th>
<th>Wind (kW)</th>
<th>GA Solar (kW)</th>
<th>Wind (kW)</th>
<th>FFA Solar (kW)</th>
<th>Wind (kW)</th>
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<tr>
<td>12</td>
<td>300</td>
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<td>–</td>
<td>–</td>
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<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>18</td>
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<td>–</td>
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<td>200</td>
<td>–</td>
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<td>–</td>
<td>200</td>
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</tr>
<tr>
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<td>500</td>
<td>200</td>
<td>–</td>
<td>100</td>
<td>100</td>
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<td>200</td>
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<tr>
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<td>100</td>
<td>100</td>
</tr>
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<td>200</td>
<td>100</td>
</tr>
<tr>
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<td>200</td>
<td>–</td>
<td>300</td>
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<td>100</td>
<td>–</td>
<td>200</td>
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<table>
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<tr>
<th>% share of source</th>
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<th>35</th>
<th>65</th>
<th>35</th>
<th>65</th>
<th>35</th>
<th>65</th>
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</thead>
<tbody>
<tr>
<td>Losses (MWh)</td>
<td>836.35</td>
<td>837.33</td>
<td>842.62</td>
<td>850.81</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Fig. 5: Convergence plot for hybrid DG allocation

Thus SOS provide better solution for RDG allocation as compared to FFA, GA and PSO irrespective of the number of buses in the investigated system.

6.3 Economic Study

Economic benefits of SDG, WDG and HDG is analyzed using NPV, ABCR and DPBP. The investment cost for SDG and WDG in Rs./MWh is 58733000 and 61916000 respectively and the O&M cost in Rs./MWh is 1300000 and 1063000 respectively. The inflation rate and discount rate is considered as Rs. 6.10 and Rs. 10.81. The cost of energy for SDG and WDG in Rs./kWh is 6.86 and 6.58 respectively [32]. The cost of energy for HDG is taken as average of costs of SDG and WDG.
TABLE IV: Economic Analysis

<table>
<thead>
<tr>
<th>DG</th>
<th>Loss reduction benefit (Rs.)</th>
<th>NPV</th>
<th>ABCR</th>
<th>DPBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar</td>
<td>12204507</td>
<td>156530715</td>
<td>2.0017</td>
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</tr>
<tr>
<td>Wind</td>
<td>31310571</td>
<td>732363343</td>
<td>5.7081</td>
<td>2.7</td>
</tr>
<tr>
<td>Hybrid</td>
<td>24250792</td>
<td>502532492</td>
<td>4.2232</td>
<td>3.9</td>
</tr>
</tbody>
</table>

The cost of energy losses are 6.86 Rs./kWh for SDG, 6.58 Rs./kWh for WDG and 6.72 Rs./kWh for HDG. Table 4. shows the NPV, ABCR and DPBP for SDG, WDG and HDG. The useful life of the renewable projects are considered as 20 years. Higher NPV Rs. 732363343, higher ABCR 5.7081 and lowest DPBP 2.7 years is obtained for WDG. Thus WDG is more economical than SDG and HDG. After observing NPV, ABCR and DPBP for renewable DG, we can conclude that energy saving by loss minimization over the life period of renewable DG is worth to cover capital and running cost as shown in Table IV.

7. Conclusion

The proposed methodology provides optimal solutions for allocation of single DG as well as hybrid DG. Variability of renewable sources and load is modeled to obtain deterministic generation-load models. Optimal allocation of WDG achieves better loss minimization. SOS provides a better solution than other algorithms, i.e., FFA, GA, and PSO. Thus, SOS is suitable for this non-linear constrained RDG allocation problem. Significant economic benefits are obtained by loss minimization with optimal allocation over the life span of RDG. This optimal allocation technique can be applied to the planning of RDGs.

References


[31] www.windenergysolutions.nl/wes100.

APPENDIX A: Specifications of Solar Panel and Wind Turbine

TABLE A1: KD325GX-LFB solar panel

<table>
<thead>
<tr>
<th>STC</th>
<th>NOCT</th>
<th>Temp.Coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{MPPT}(V)$</td>
<td>40.3</td>
<td>36.2</td>
</tr>
<tr>
<td>$I_{MPPT}(A)$</td>
<td>8.07</td>
<td>6.47</td>
</tr>
<tr>
<td>$V_{oc}(V)$</td>
<td>49.7</td>
<td>45.5</td>
</tr>
<tr>
<td>$I_{sc}(A)$</td>
<td>8.69</td>
<td>7.04</td>
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TABLE A2: Wind turbine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>100 kW</td>
</tr>
<tr>
<td>Cut-in wind speed</td>
<td>3 m/s</td>
</tr>
<tr>
<td>Rated wind speed</td>
<td>13 m/s</td>
</tr>
<tr>
<td>Cut-off wind speed</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Survival wind speed</td>
<td>60 m/s</td>
</tr>
</tbody>
</table>

APPENDIX B: Various parameters, probability and output of states

TABLE B1: Values of various parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solar PV generation</th>
<th>Wind generation</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>0.378</td>
<td>2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.077</td>
<td>8.6380</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>33.217</td>
<td>9.7437</td>
</tr>
<tr>
<td>$\beta$</td>
<td>54.584</td>
<td></td>
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</table>

TABLE B2: Probability and output power of states for solar DG

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S</th>
<th>$f_{\beta}(s)$</th>
<th>$P_{o}(s)$</th>
<th>$P(s)$</th>
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<td>0</td>
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<td>0.2</td>
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<td>0.06111</td>
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<td>0.32201</td>
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TABLE B3: Probability and output power of states for wind DG

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<tr>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$v$</th>
<th>$f_{w}(u)$</th>
<th>$P_{o}(\omega)$</th>
<th>$P(\omega)$</th>
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<tbody>
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