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Optimal/flatness based-control of stand-alone power systems using fuel cells, batteries and supercapacitors

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In this work, an optimal control (under constraints) based on the Pontryagin's maximum principle is used to optimally manage energy flows in a basic PEM (Proton Exchange Membrane) fuel cells system associated to lithium-ion batteries and supercapacitors through a common DC bus having a voltage to stabilize using the differential flatness approach. The adaptation of voltage levels between different sources and load is ensured by use of three DC-DC converters, one boost connected to the PEM fuel cells, while the two others are buck/boost and connected to the lithium-ion batteries and supercapacitors. The aim of this paper is to develop an energy management strategy that is able to satisfy the following objectives: - Impose the power requested by a habitat (representing the load) according to a proposed daily consumption profile, - Keep fuel cells working at optimal power delivery conditions, - Maintain constant voltage across the common DC bus, - Stabilize the batteries voltage and stored quantity of charge at desired values given by the optimal control. Results obtained under MATLAB/Simulink environment prove that the cited objectives are satisfied, validating then, effectiveness and complementarity between the optimal and flatness concepts proposed for energy management. Note that this study is currently in experimentally validation within MSE Laboratory.

Keywords: Stand-alone power system; PEM fuel cell; lithium-ion battery; supercapacitor; energy management; optimal control; flatness control.

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1. Introduction

Reduction of the environmental pollution is the main objective in innovations of clean energy generation devices. Due to their efficiency and reliability, fuel cells are considered as one of the most promising technologies. However, to achieve an economic use of hydrogen (which represents the fuel), several essential technological issues arise and must be solved in the case of real applications. More specifically, the operating characteristics of systems using fuel cells are closely related to their control and diagnosis. The control problems are not limited in regulating temperature and pressure, because there are still a variety of indices and performance variables to optimize, which are object of current intense researches whose this work is part.

This work deals with a multi-sources system composed with PEM fuel cells considered as main source that provide power demands, lithium-ion batteries used as secondary source which supply the lacked power between fuel cells and load, while supercapacitors considered as power source which guarantee regulation of the common DC bus voltage by absorbing or providing necessary energy. For this, the present paper is focused on an optimal/flatness control strategy applied to manage energy flows in stand-alone power

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systems using fuel cells, batteries and supercapacitors, in order to limit power provided by fuel cells, and maintains the batteries voltage and stored quantity of charge in an acceptable interval.

2. Opted control strategies

2.1. The optimal control

The control theory analyses properties of a dynamic system in which is possible to act by means of control laws. The goal is to bring the system of a given initial state to a desired final state, and determine optimal solutions under a certain optimization criterion and constraints. The modern theory of optimal control began in the 1950s, with formulation of the Pontryagin's maximum principle that generalizes the Euler-Lagrange equation [1-3]. Among optimization methods presented in literature, those with a necessary knowledge of the consumption profile are found; in this case, use of global optimization algorithms is possible. These algorithms are used to calculate at every moment, the best power distribution between energy sources of the hybrid system.

The global optimization problem can be solved either by using dynamic programming, or the optimal control that can be applied when it is possible to evaluate variation of the criterion in function of variation of the control value [1]. These methods are based on the Hamilton-Jacobi-Bellman equation (for the dynamic programming), and Pontryagin's maximum principle (for the optimal control) [1, 4-6].

From [2, 5], some benefits to application of the optimal control comparing with dynamic programming, can be concluded:
- It represents an effective technic for energy management,
- The solution can be formed into a command depending on the state,
- Easy implementation,
- It has the advantage of a relatively fast calculation, and a low space memory.

2.1. The nonlinear flatness control

In previous works, interest of control strategies based on the differential flatness concept was shown [7-19]. This property allows characterizing the state trajectory of a nonlinear system through use of flat outputs variables and their successive derivatives. The property of differential flatness is a concept that has been proposed and developed by M. Fliess et al. [20-21].

A system of ordinary differential equations is called "differentially flat" if there are variables x, u and y, satisfying:

$$\dot{x} = f(x, y) \tag{1}$$

$$x = [x_1, x_2, \ldots, x_n]^T \tag{2}$$

$$u = [u_1, u_2, \ldots, u_n]^T \tag{3}$$

$$y = [y_1, y_2, \ldots, y_m]^T \tag{4}$$

x, u, and y represent vectors of state, control, and flat outputs variables, and $(n, m) \in N$.

a) The vector y can be written as a function of x and u as follows:

$$y = \left(x, u, \dot{u}, ..., u^{(s)}\right) \tag{5}$$

b) The vectors x and u can be expressed as a function of the output vector y and a finite number of its derivatives by:

$$\begin{cases} x = \varphi\left(y, \dot{y}, ..., y^{(r)}\right) \\ u = \psi\left(y, \dot{y}, ..., y^{(r+1)}\right) \end{cases} \tag{6}$$

c) There is no differential equation in the form:

$$0 = \zeta\left(y, \dot{y}, ..., y^{(k)}\right) \tag{7}$$

The flatness study passes by the following three steps [22]:

- Verification that the system belongs to the class of "differentially flat": check flatness conditions expressed in relations (5) and (6),
- Planning of the desired reference trajectories dedicated to the flat outputs variables,
- Synthesis of control laws opted for regulation of the flat outputs variables to their desired reference trajectories.

3. Verification of the proposed optimal/flatness based-control

The problem of energy management in hybrid systems consists essentially to develop algorithms and control strategies, whose role is to find in every moment the best power distribution between various energy sources.

3.1. Description of the fuel cells/batteries hybrid source

Architecture of the studied hybrid power system, shown in figure 1, has double advantage: a very high specific energy, and a very high specific power. He is composed with:

- A common DC bus (C_{dc}) having a low voltage v_{dc},
- PEM fuel cells (FCs), considered as the main energy source, having a voltage v_{fc}, providing a power P_{fc}, and connected to the DC bus through a non-isolated DC-DC boost converter (CVS Boost),
- Lithium-ion batteries (BATs) used as the secondary energy source, having a voltage v_{ba}, a power P_{ba}, and connected to the DC bus by a non-isolated DC-DC buck/boost converter (CVS Buck/Boost),
- Supercapacitors (SCs) considered as the power source, having a voltage v_{sc}, supplying and absorbing a power P_{sc}, and connected to the DC bus through a non-isolated DC-DC buck/boost converter (CVS Buck/Boost),
- A variable resistor (RV) representing the load.

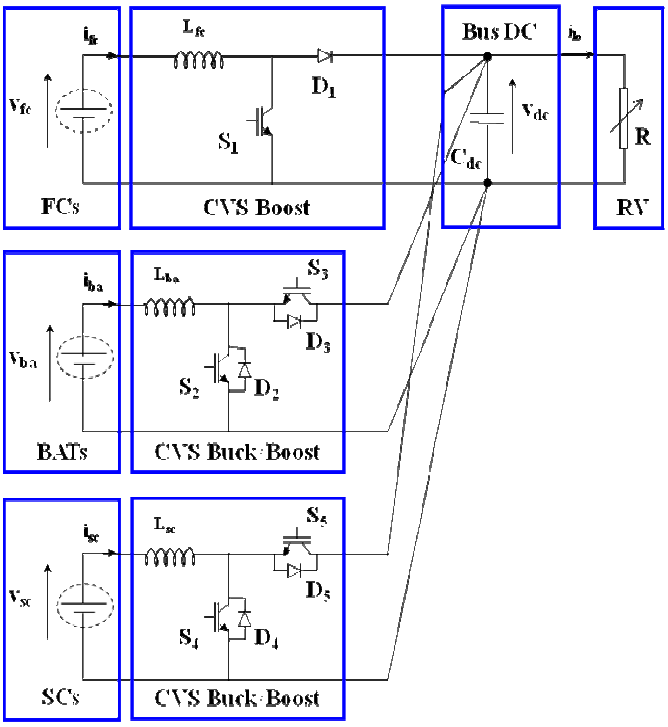

Fig. 1. Architecture of the studied hybrid power system.

The aim of optimizing energy of the fuel cells/batteries/ supercapacitors hybrid source is to minimize the electric power P_{fc} provided by fuel cells to the load during the period of system operation. This power should be limited, so, the batteries provide the difference in power between fuel cells and load, verifying that:

$$P_{fc}(t)+P_{ba}(t)=P_{lo}(t) \tag{8}$$

In this case, supercapacitors are designed to provide or absorb the needed energy assuring that power exchanges between different elements are through a common DC voltage having a controlled and stabilized voltage.

In summary, the opted energy management strategy must satisfy in every moment the power conservation equation:

$$P_{fc}(t)+P_{ba}(t)+P_{sc}(t)=P_{lo}(t)+P_{dc}(t) \tag{9}$$

3.2. Application of the optimal control based on the technique of Euler-Lagrange

In this section, the objective is to solve problem of optimizing electric power in the hybrid fuel cells system by using an optimal control (under constraints) based on the maximum Pontryagin's method founded on the Euler-Lagrange equation.

Considering that:

$$x(t)=Q_{ba}(t) \tag{10}$$

$$\dot{x}(t)=-i_{ba}(t) \tag{11}$$

$$x_{ref}=Q_{baref} \tag{12}$$

$$u(t) = i_{ba}(t) \tag{13}$$

$$l(t, x, u) = P_{lo}(t) - P_{ba}(t) \tag{14}$$

$$d_f\left(t_f, x(t_f)\right) = \left(x\left(t_f\right) - x_{ref}\right)^2 \tag{15}$$

$$f(t, x, u) = -u \tag{16}$$

The reference current i_{baref} (t) that minimizes energy provided by fuel cells, taking into account constraints on fuel cells power and batteries final state of charge, is given by:

$$i_{baref}(t) = \frac{-2C_{ba}Q_{baref} + Q_{ba}(t_0)\left(1 + 2C_{ba}\right)}{2R_{ba}C_{ba} + \left(1 + 2C_{ba}\right)\left(t_f - t_0\right)}$$

if: (17)

$$P_{fc\min} \le P_{fcref}(t) \le P_{fc\max}$$

$$i_{baref}(t) = \frac{\frac{Q_{ba}(t)}{C_{ba}} - \sqrt{\left(\frac{Q_{ba}(t)}{C_{ba}}\right)^2 - 4R_{ba}\left(P_{lo}(t) - P_{fc\max}\right)}}{2R_{ba}} \tag{18}$$

if:

$$P_{fcref}(t) \succ P_{fc\max}$$

$$i_{baref}(t) = \frac{\frac{Q_{ba}(t)}{C_{ba}} - \sqrt{\left(\frac{Q_{ba}(t)}{C_{ba}}\right)^2 - 4R_{ba}\left(P_{lo}(t) - P_{fc\min}\right)}}{2R_{ba}} \tag{19}$$

if:

$$P_{fc\min} \succ P_{fcref}(t)$$

To preserve batteries for other operating cycles, their voltage and quantity of charge stored at the end of each cycle should be returned to final values that satisfy relations (20) and (21):

$$V_{ba}(t_f) = \frac{2C_{ba}}{2C_{ba} + 1} Q_{ba}(t_0) \tag{20}$$

$$Q_{ba}(t_f) = \frac{2}{2C_{ba} + 1} Q_{ba}(t_0) \tag{21}$$

With:

P_{fcref} (t): reference power of the fuel cells.

P_{fcmax}: maximum power of the fuel cells.

P_{fcmin}: minimum power of the fuel cells.

3.3. Application of the flatness control

3.3.1. Flatness verification

To demonstrate flatness, it is necessary to verify that it is still possible to express all state and control variables in function of the flat output and a finite number of its successive derivatives. For this, v_{dc}, y_{dc}, and P_{sc} are defined as: state, flat output, and control variables.

- ➢ The voltage v_{dc} can be written as follows:

$$v_{dc}(t) = \sqrt{\frac{2y_{dc}(t)}{C_{dc}}} = f_{v_{dc}}\left(y_{dc}(t)\right) \tag{22}$$

- ➢ The power P_{sc} is given by:

$$P_{sc}(t) = \sqrt{\frac{2y_{dc}(t)}{C_{dc}}}.i_{lo}(t) + \dot{y}_{dc}(t) \tag{23}$$

$$-P_{fc}(t) - P_{ba}(t) = h_{P_{sc}}\left(y_{dc}(t), \dot{y}_{dc}(t)\right)$$

✓ $v_{dc}(t) = f_{v_{dc}}\left(y_{dc}(t)\right)$ and $P_{sc}(t) = h_{P_{sc}}\left(y_{dc}(t), \dot{y}_{dc}(t)\right)$, then, the system is considered as «differentially flat».

3.3.2. Planning of the desired reference trajectory

The desired reference trajectory of the electrostatic energy stored in the common DC bus is given by:

$$y_{dcref}(t) = \frac{1}{2} C_{dc} V_{dcref}(t)^2 \tag{24}$$

3.3.3. Synthesis of the control laws

A control law governing the error evolution (in energy) permits to have an asymptotic convergence of the error to zero is used and described by:

$$(\dot{y}_{dc}(t) - \dot{y}_{dcref}(t)) + k_{11}(y_{dc}(t) - y_{dcref}(t)) + \tag{25}$$

$$k_{12}\int_0^t (y_{dc}(t) - y_{dcref}(t))dt = 0$$

The integral part is introduced to ensure zero static error, and compensate modeling errors and/or errors associated with parametric uncertainties. Choice of the coefficients k_{11} and k_{12} is done by studying roots of the following characteristic equation (roots placement method):

$$s^2 + k_{11}s + k_{12} = 0 \tag{26}$$

With:

$$\begin{cases} k_{11} = 2\xi w_n \\ k_{12} = W_n{}^2 \end{cases} \tag{27}$$

ξ and w_n represent the desired dominant damping ratio and the natural frequency.

3.4. Results of the proposed tests

In this part, and using MATLAB/Simulink environment, performances of the proposed optimal/flatness hybrid control are evaluated. Thus, three testes are offered depending on the batteries initial state of charge.

a) First test: lithium-ion batteries charged at their nominal voltage of 28.8V.

b) Second test: lithium-ion batteries charged at 75%: V_{ba} (t = 0) = 30V.

c) Third test: lithium-ion batteries fully charged: V_{ba} (t = 0) = 31.2V.

To verify performances of the proposed energy management strategy applied to fuel cells/batteries/supercapacitors hybrid source, a daily consumption profile is imposed as shown in figure 2.

The Table 1 summarizes parameters of the studied hybrid source.

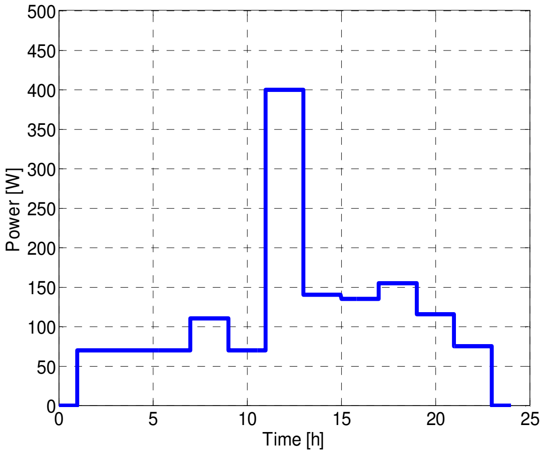

Fig. 2. Consumption profile.

Table 1. Parameters of the studied hybrid source

Symbol	Name	Value
	PEM fuel cells	
V_{fcn}	Nominal voltage [V]	38.2
P_{fcmax}	Maximum power [W]	300
P_{fcmin}	Minimum power [W]	35
	Lithium-ion batteries	
V_{ban}	Nominal voltage [V]	28.8
C_{ban}	Nominal capacity [Ah]	73
	Supercapacitors	
V_{scn}	Nominal voltage [V]	32.4
C_{sc}	Capacity [F]	25
	Common DC bus	
V_{dcref}	Desired reference voltage [V]	48
C_{dc}	Capacity [F]	0.0036

3.4.1. First test: lithium-ion batteries charged at their nominal voltage

The next figures show respectively:

- Power transfers,
- Common DC bus voltage,
- Batteries voltage,
- Quantity of charge stored in batteries.

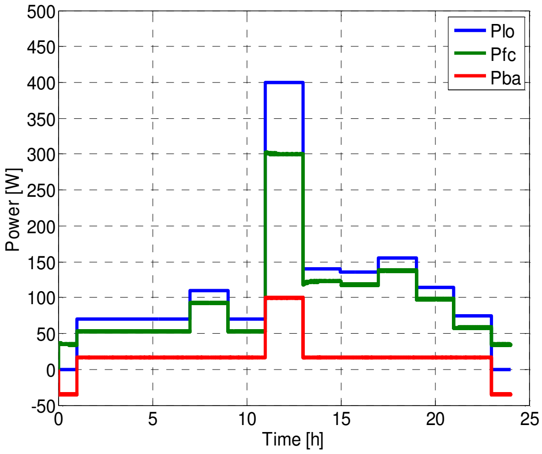

Fig. 3. Power transfers.

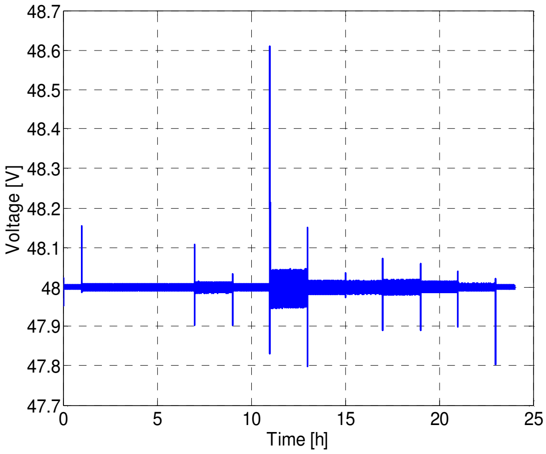

Fig. 4. Common DC bus voltage.

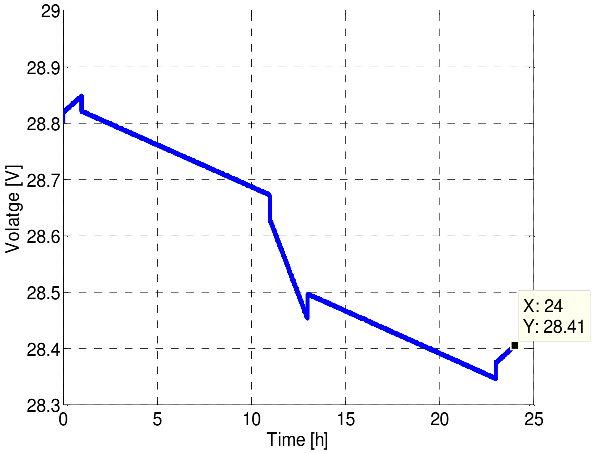

Fig. 5. Batteries voltage.

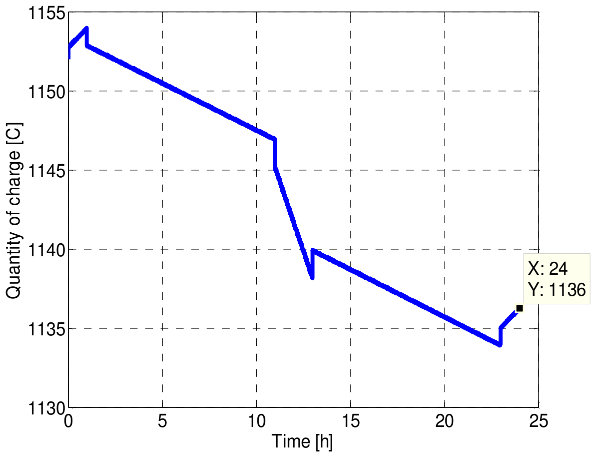

Fig. 6. Quantity of charge stored in batteries.

At startup (t = 0), for a voltage V_{ba} of 28.8V and a stored quantity of charge Q_{ba} of 1152C, it is seen at the end of the operation period ($t_f = 24$) that:

- The load power requirements are satisfied (see figure 3),
- The common DC bus voltage is stabilized at its reference value of 48V, with the observation of very small overruns during power changes (see figure 4),
- The power supplied by the fuel cells varies in the interval [P_{fcmin} = 35W, P_{fcmax} = 300W], and is always lower than power consumed by load (see figure 3),
- The batteries supply or absorb the difference in power (see figure 3) between fuel cells and load.
- The batteries voltage and stored quantity of charge are stabilized at their desired values (see figures 5 and 6). These last two values satisfy relations (20) and (21) given by the optimal control:

$$V_{ba}(t_f) = \frac{2}{2C_b + 1} Q_{ba}(0) = \frac{2}{2 \times 40 + 1} 1152 = 28.4\text{V}$$

$$Q_{ba}(t_f) = \frac{2C_b}{2C_b + 1} Q_{ba}(0) = \frac{2 \times 40}{2 \times 40 + 1} 1152 = 1136\text{C}$$

These conclusions explain how the control is optimal when the objective is to minimize the fuel cell electric power.

3.4.2. Second test: lithium-ion batteries charged at 75%

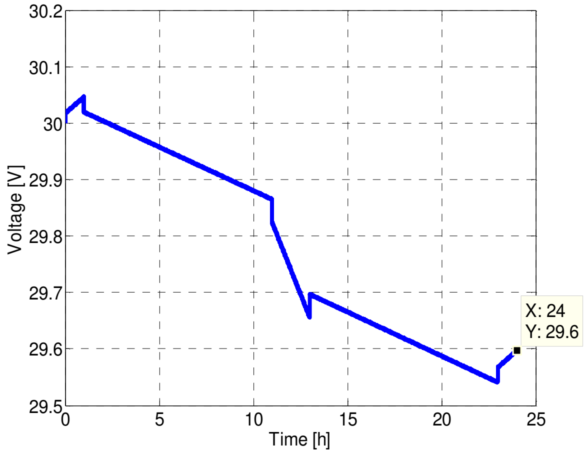

Fig. 7. Batteries voltage.

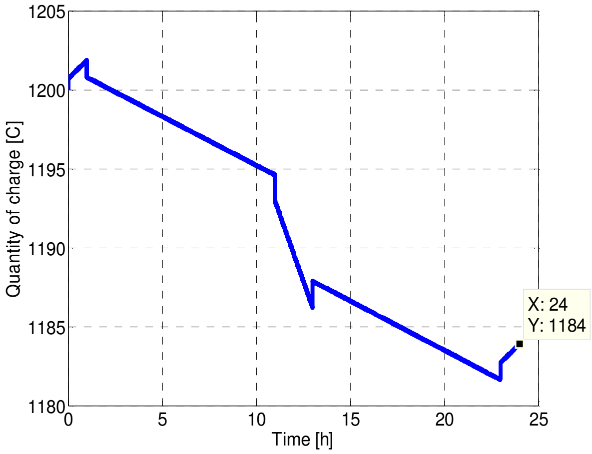

Fig. 8. Quantity of charge stored in batteries.

By keeping the same consumption profile, v_{ba} and Q_{ba} are stabilized at $t_f = 24$ to: $V_{ba}(t_f) = 29.6V$ and $Q_{ba}(t_f) = 1184C$ for an initial voltage $V_{ba}(0) = 30V$ corresponding to a stored quantity of charge $Q_{ba}(0) = 1200C$. It can be said that V_{ba} and Q_{ba} satisfy relations (20) and (21) determined by the optimal control:

$$V_{ba}(t_f) = \frac{2}{2C_b + 1} Q_{ba}(0) = \frac{2}{2 \times 40 + 1} 1200 = 29.6V$$

$$Q_{ba}(t_f) = \frac{2C_b}{2C_b + 1} Q_{ba}(0) = \frac{2 \times 40}{2 \times 40 + 1} 1200 = 1184C$$

3.4.3. Third test: lithium-ion batteries fully charged

The third simulation performed for a voltage $V_{ba}(0) = 31.2V$ and a quantity $Q_{ba}(0) = 1248C$, proves that whatever the batteries initial voltage and stored quantity of charge, these values satisfy equations obtained by applying the optimal control (see figures 9 and 10). For this test:

$$V_{ba}(t_f) = \frac{2}{2C_b + 1} Q_{ba}(0) = \frac{2}{2\times 40 + 1} 1248 = 30.8V$$

$$Q_{ba}(t_f) = \frac{2C_b}{2C_b + 1} Q_{ba}(0) = \frac{2\times 40}{2\times 40 + 1} 1248 = 1232C$$

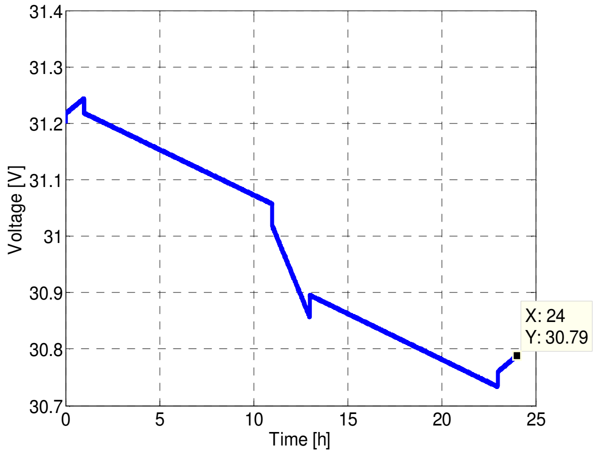

Fig. 9. Batteries voltage

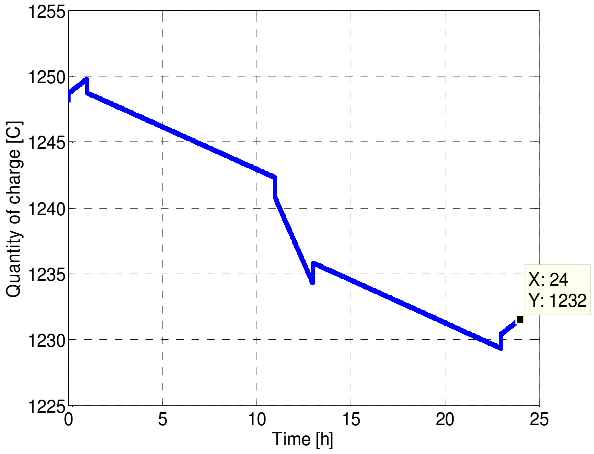

Fig. 10. Quantity of charge stored in batteries.

4. Conclusion

For a domestic application, the main objective of this work was to manage and optimize operation of a hybrid power system composed of three sources: one main represented by PEM fuel cells, the two others secondaries, and use lithium-ion batteries and supercapacitors.

By opting an optimization hybrid strategy (under constraints) based on the Pontryagin's maximum principle which uses the Euler-Lagrange equation, and other control using properties of the differential flatness, obtained results proved effectiveness and complementarity between the opted strategies whatever variations in requested power and batteries initial state of charge.

The fuel cells power was limited, and the batteries voltage and stored quantity of charge were kept in a limited interval: consequently, lifetime of the hybrid source was, theoretically, increased.

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