Kaddour Arzag[1,*], Zin-Eddine Azzouz[2], Boualem Ghemri[3]

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Lightning Return Stroke Current Analysis Using Electromagnetic Models and the 3D-FDTD Method

The three dimensions finite difference time domain method (3D-FDTD) is employed to calculate lightning return stoke current distributions in a vertical lightning channel. The latter is excited at its bottom by a lumped current source above a flat perfectly conducting ground. In this study four lightning return stroke electromagnetic models are used. The calculating approach, which is based on Taflove formulation of the 3D-FDTD method combined to the UPML boundary conditions, is implemented on Matlab environment. For validation needs, the obtained lightning return stroke space and time distributions are compared with others taken from specialized literature.

Keywords: lightning, lightning return stroke current, Electromagnetic models, 3D-FDTD method, Taflove formulation.

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1. Introduction

In Electromagnetic Compatibility studies, relative to lightning and its effects on various systems, it is necessary to determine the space-time lightning return stroke distribution along the lightning channel to identify the electromagnetic field and its effects on electrical system. Several lightning return stroke distribution models exist. They are currently classified [1] into four types of models namely the physical models, the electromagnetic models, the RLC models, and the engineering models [1]. These models are currently used in lightning return stroke analysis, in lightning electromagnetic environment characterization and in lightning induced effects studies.

Furthermore electromagnetic models which are based on Maxwell's equations are relatively new but effective to analyze the coupling lightning electromagnetic field effects on various systems [5- 6-7].

In the literature, numerous works relative to lightning return stroke electromagnetic models have been performed by the implementation of numerical methods allowing the lightning electromagnetic field calculation. These methods solve Maxwell's equations in order to have the lightning channel current space-time distribution. Among these numerical methods we quote the time domain moment method (MoM) (Van Baricum and Mailler [2]), the moment method (MoM) in frequency domain (Harrington [3]) and the finite difference time domain method (FDTD) (Yee [4]). Indeed, the MoM method, in time domain, was used by Moini *et al*. [5] to have a numerical solution of the electric field integral equation;

[*] Corresponding author: K. Arzag, Faculty of Technology, Electrotechnics Department, University of Saida. BP 138 « Cité En-Nasr », Saida, 20000, Algeria E-mail: ar_kado2006@yahoo.fr, kaddour.arzag@univ-usto.dz.

[1] Faculty of Technology, Electrotechnics Department, University of Saida, Saida 20000, Algeria.

[2] Faculty of Electrical Engineering, Automatic Control Department, University of Sciences and Technology "MB" of Oran, Oran 31000, Algeria.

[3] Faculty of Electrical Engineering, Electrotechnics Department, University of Sciences and Technology "MB" of Oran, Oran 31000, Algeria.

this can give the lightning channel current stroke distribution. Shoory *et al*. [6] employed the frequency domain MoM method with a vertical resistive wire excited by a lumped current source to analyze the lightning current. The FDTD method was used by Baba and Rakov [7-8-9-10] for the specification of the current time-space distribution and the associated electromagnetic field components. The latter employs a simple way to discretize the entire work space into small cubic or rectangular parallelepiped cells for computing the electromagnetic field components and the return stroke current distribution along the lightning channel. This task needs a special lightning channel representation taking into account the current wave propagation velocity knowledge and the excitation sources representation.

The aim of this paper is to contribute in the characterization of lighting by the use of electromagnetic models and the 3D FDTD method. The paper is structured as follows. In the first section, we review some theoretical aspects concerning the Taflove formulation related to the 3D-FDTD method using uniaxial perfectly matched layer (UPML) absorbing boundary conditions [11]. These conditions are employed to solve Maxwell's equation to obtain the current distribution along the lightning channel. At the end of this section we give two formulas linking the space and the time increments which allow us to obtain the computational stability. A review on the lightning channel representations used in electromagnetic models and FDTD calculation is given in the second section ; we also present in this section the channel representations adopted in our analyze. The third section, of this paper, is devoted to the presentation of the simulation results. We also present, in this section, the calculation methodology used in lightning return stroke current analysis and a comparison between four lightning channel representations. For validation reasons, calculated return stroke currents are compared with measured results taken from literature [12].

Moreover, for obtaining the lightning current space-time distribution waveforms, it was necessary for us to develop on Matlab environment a 3D computation code. The latter, is based on the FDTD method including Taflove formulation. Uniaxial perfectly matched layer (UPML) absorbing boundary conditions were implemented in this computation code. Finally conclusions, related to the lightning analysis based on electromagnetic models and the 3D-FDTD method, are given in the last section.

2. 3D-FDTD formulations and boundary conditions

The FDTD method based on Yee algorithm [4] was employed in 2D and 3D by Baba and Rakov associated with the perfectly matched layer (PML) conditions based on Berenger's formulation [13] and the second order Lioas boundary conditions in various works such as in [7], [8], [9] and [10]. This calculation aimed firstly to determine the lightning return stroke current distribution and the associated electromagnetic field components using different lightning channel representations and different excitation sources methods above a perfectly conducting ground and a lossy ground and secondly to study the influence of the strike tall grounded object on lightning electromagnetic field.

The Taflove formulation [11] of the 3D-FDTD method used to analyze lightning current and electromagnetic field in this work has the advantage that it can be used to calculate the electromagnetic components in the entire work space and set uniaxial perfectly matched layer (UPML) absorbing boundary conditions using the same equations of electric and magnetic field associated to their current densities, employing a simple changing into mediums parameters (working space medium and UPML regions) [28, 29, 30].

Equations representing this formulation can be written as follows:

$$D_x^{n+1}\left(i+\frac{1}{2},j,k\right)=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).D_x^{n}\left(i+\frac{1}{2},j,k\right)+\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).$$
$$\left[\frac{H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)-H_z^{n+1/2}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y}\right.$$
$$\left.-\frac{H_y^{n+1/2}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-H_y^{n+1/2}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\right]$$

(1-a)

$$E_x^{n+1}\left(i+\frac{1}{2},j,k\right)=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right).E_x^{n}\left(i+\frac{1}{2},j,k\right)+\left[\frac{1}{(2\varepsilon k_z+\sigma_z\Delta t)\varepsilon}\right].$$
$$\left[(2\varepsilon k_x+\sigma_x\Delta t).D_x^{n+1}\left(i+\frac{1}{2},j,k\right)-(2\varepsilon k_x-\sigma_x\Delta t).D_x^{n}\left(i+\frac{1}{2},j,k\right)\right]$$

(1-b)

$$B_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).B_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)+\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right).$$
$$\left[\frac{E_z^{n+1}\left(i,j+1,k+\frac{1}{2}\right)-E_z^{n+1}\left(i,j,k+\frac{1}{2}\right)}{\Delta y}-\frac{E_y^{n+1}\left(i,j+\frac{1}{2},k+1\right)-E_y^{n+1}\left(i,j+\frac{1}{2},k\right)}{\Delta z}\right]$$

(1-c)

$$H_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right).H_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)+\left[\frac{1}{(2\varepsilon k_z+\sigma_z\Delta t)\mu}\right].$$
$$\left[(2\varepsilon k_x+\sigma_x\Delta t).B_x^{n+3/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)-(2\varepsilon k_x-\sigma_x\Delta t).B_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)\right]$$

(1-d)

Similar expressions can be derived for the remaining electric and magnetic fields components.

With:

$$\sigma_x(x)=\left(\frac{x}{d}\right)^m.\sigma_{max} \quad (2)$$

$$k_x(x)=1+\left(k_{x,max}-1\right).\left(\frac{x}{d}\right)^m \quad (3)$$

$$\sigma_{max}=-\frac{(m+1)ln\left(R(0)\right)}{2\,\eta\,d} \quad (4)$$

$$R(\theta) = e^{-2\,\eta\,cos\theta\,\int_0^d \sigma(x)\,dx} \text{ (Reflection error)} \quad (5)$$

$$\eta=\sqrt{\frac{\mu}{\varepsilon}} \quad (6)$$

d: The PML area thickness.
x: A positive integer number corresponding to the layer's number ($0 < x < d$).

Defining the multiplying coefficients of electric and magnetic fields and current densities permits a unified treatment of both the interior working volume and the UPML area. The parameters σ and k values, in the interior of the working volume, depend on the medium nature. For example in a free space they worth $\sigma = 0$ and $k = 1$. However, in the UPML area the parameters σ and k are assumed to have a polynomial-graded profile given by equations (3) and (4).

For computational stability, it is necessary to satisfy a relation between the space increment Δs and the time increment Δt namely [7]:

$$\frac{\Delta t}{\sqrt{\mu\varepsilon}} \le \frac{\Delta s}{\sqrt{3}} \quad (8)$$

In the same context Noda and Yokoyama [14] proposed the following formula to determine the time step Δt is:

$$\Delta t = \Delta s\sqrt{\frac{\mu\varepsilon}{3}}(1-\alpha) \quad (9)$$

α is a small positive value specified by user in order to prevent the numerical integration instability.

3. Study and result's presentation

3.1 Lightning channel representation

In references [7],[8],[9], and [10], lightning return stroke electromagnetic models are classified into seven types depending on the lightning return stroke channel representations. These types are:

1) a perfectly conducting/resistive wire in air above ground;
2) a wire loaded by a additional distributed series inductance in air above ground;
3) a wire surrounded by a dielectric medium (other than air) that occupies the half space above ground;
4) a wire coated by a dielectric material in air above ground;
5) a wire coated by a fictitious material having high relive permittivity and high relative permeability in air above ground;
6) two parallel wires having additional distributed shunt capacitance in air); and
7) a phase current source array in air above ground.

Among these seven electromagnetic models we have considered, in this work, four models (figure 1):

Model 1: a vertical wire loaded by an additional distributed series inductance and a distributed series resistance (L= 6.57 µH/m, R=$0{,}13\ \Omega/m$).

Model 2: a vertical wire, of 0.2 m radius, surrounded by a dielectric medium of permittivity $\varepsilon_r = 4.12$ (greater than 1) occupying the entire half space above a flat ground.
Model 3: a vertical wire embedded in a parallelepiped of $\varepsilon_r = 4.12$.
Model 4: a vertical wire embedded in a parallelepiped of $\varepsilon_r = 4.12$ and $\mu_r = 4.12$.

Note that these types have been already used by Baba and Rakov [7], [8], [9], and authors of reference [10] in their FDTD computation and analysis of lightning current distribution along the channel and by Moini et al. [5] in their lightning electromagnetic field calculation based using the method of moment (MoM) in time domain.

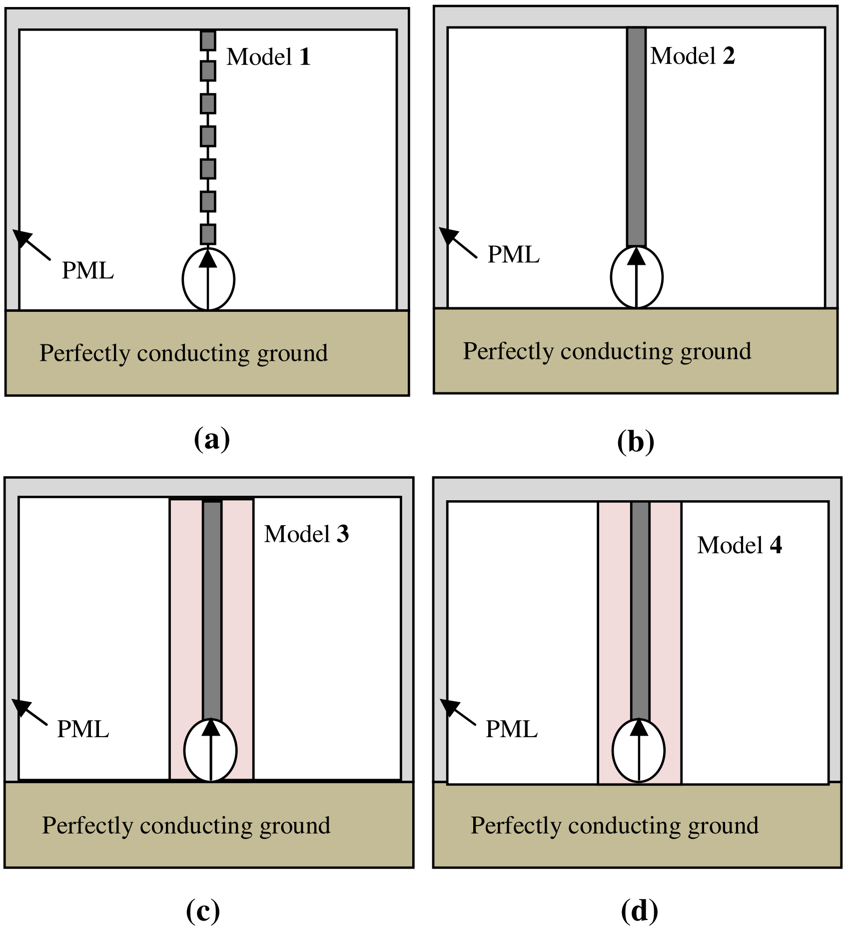


Fig. 1. Two lightning return stroke channel representations with a perfectly conducting ground excited at its bottom by a current source:
(a) Model **1**, **(b)** Model **2,** **(c)** Model **3**, **(d)** Model **4**

3.2 Simulation principal

In this study the lightning return stroke channel is represented by a vertical wire placed at the center of a horizontal perfectly conducting plane and excited at its bottom by a current source. The later is represented by four kinds of electromagnetic models (Figure 1).

In this part of study we analyze the return stroke current space-time distribution along the lightning channel using the electromagnetic model, illustrated in Figure (1) and based on 3D-FDTD formulations presented before.

The wire representing the lightning channel has a length of 4 km; this length value is close enough to reality since the lightning channel length must not exceed 7.5 km. This wire is placed in a working volume of 90 m x 90 m x 4500 m, which is divided into rectangular parallelepiped cells of 1.5 m $\times$ 1.5 m $\times$ 10 m. In this way a vertical wire has an equivalent radius of 0.2 m ($r_{eq} = 0.135 \times \Delta x$, according to Taniguchi *et al* [27]). The time increment was fixed to 2 ns.

Finally, in the second part of this section, we compare our results, consisting of the lightning channel distribution current, with those computed by Izadi et al [12].

3.3 Current time- space distribution along the lightning channel

The vertical wire shown in Figure (1) is excited at its bottom by a lumped current source, this latter produce a current waveform having a peak of 16 kA and a rise time of 0.7 µs. Note that the channel-base current waveform shown at Figure (1), at 0 m, is calculated using Heidler function (equation 10) applied to a subsequent return stroke and using parameters illustrated in table 1 (taken from reference [12] for comparison and validation needs).

$$i(0,t) = \frac{i_{01}}{\eta_1}\frac{(t/\tau_{11})^{n_1}}{1+(t/\tau_{11})^{n_1}} exp\left(\frac{-t}{\tau_{12}}\right) + \frac{i_{02}}{\eta_2}\frac{(t/\tau_{21})^{n_2}}{1+(t/\tau_{21})^{n_2}} exp\left(\frac{-t}{\tau_{22}}\right) \quad (10)$$

Where:

i_{01}, i_{02} are current amplitudes,

τ_{11}, τ_{12} are the front time constants,

τ_{21}, τ_{22} are the decay time constants,

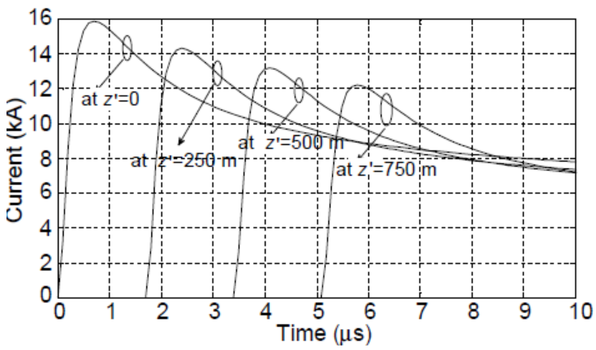
n_1, n_2 are the exponents.

$$\eta_1 = \left[-\left(\frac{\tau_{11}}{\tau_{12}}\right)\left(n_1.\frac{\tau_{12}}{\tau_{11}}\right)^{\frac{1}{n_1}}\right], \quad \eta_2 = \left[-\left(\frac{\tau_{21}}{\tau_{22}}\right)\left(n_2.\frac{\tau_{22}}{\tau_{21}}\right)^{\frac{1}{n_2}}\right]$$

Table 1: Lightning channel base current parameters

i_{01} (kA)	τ_{11} (µs)	τ_{12} (µs)	i_{01} (kA)	τ_{21} (µs)	τ_{22} (µs)	n_1	n_1
14.8	0.244	2.77	6.86	4.18	40.66	2	2

The speed of lightning return stroke current wave is equal to $1.4752.10^8$ m/s [12] (lower to the light velocity 3.10^8 m/s) for this reason we have taken L = 6.57 µH/m, and $\varepsilon_r = 4.12$.



(a)

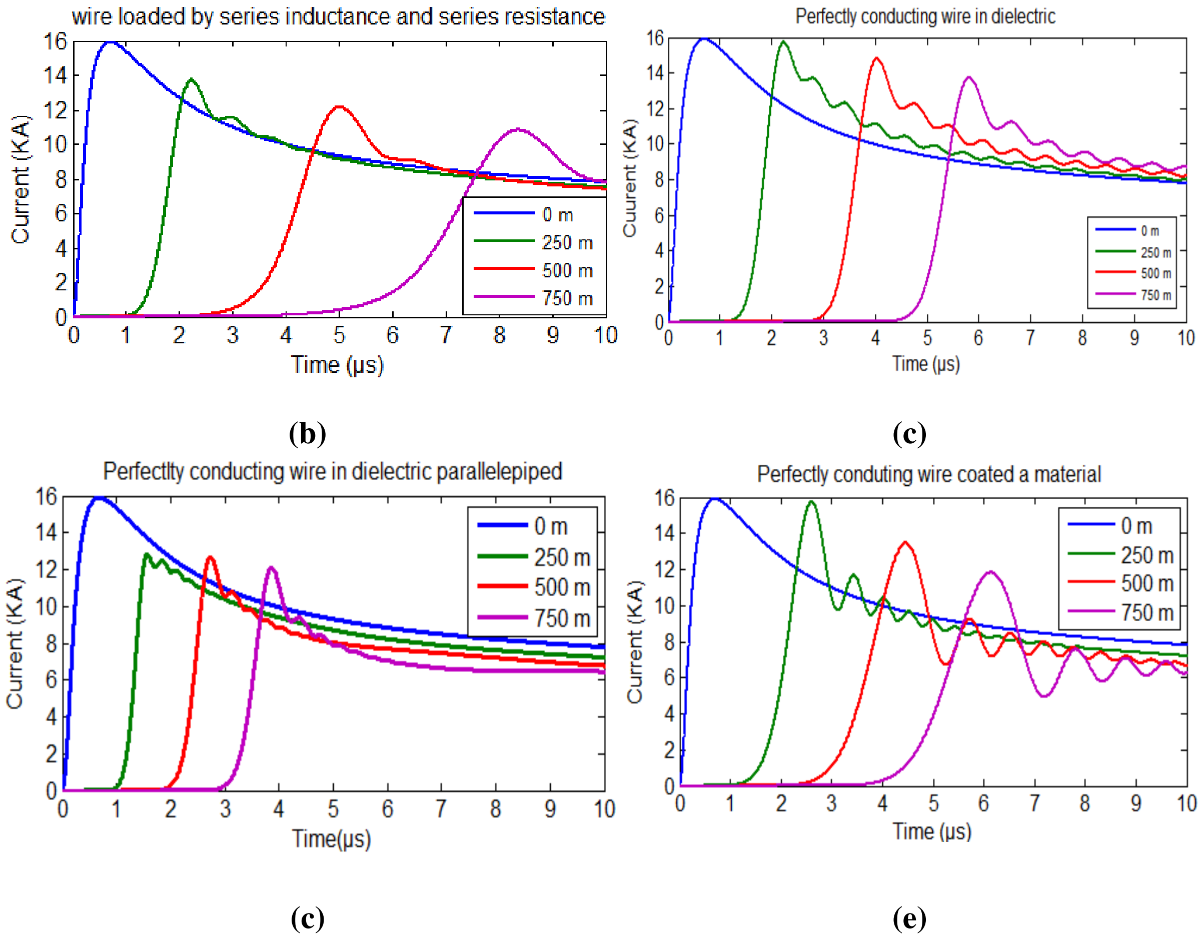


Fig. 2 Current waveforms calculated at different heights using the 3D-FDTD method and EM models:
(a) Results taken from [12],
(b), **(c)**, **(d)** **(e)**: Our results.

Figure (2) present a comparison between results, obtained by the achievement of the 3D-FDTD method and EM models, and those obtained by Izadi et *al* [12] using measured electromagnetic fields. These results consist in current waveforms which are plotted at different lightning channel heights namely: 0 m, 250 m, 500 m, and 750 m. Thus, current waveforms plotted in figure (2-a) are those obtained by Izadi *et al* [12] and curves plotted in figure (2- b-c-d-e) are our results.

4. Discussion

Through the comparison between results calculated using our proposed approach and those taken from reference [12], a good agreement between the two results can be remarked. As seen in figure. (2-a), the current wave suffers both attenuation and dispersion as it propagates along the lightning channel. In all the four electromagnetic models results we can remark that the current waveforms have attenuation, in their magnitude along the lightning channel, and there is also a difference between current speeds obtained for each model. So, in figure (2-b-c) the lightning current speed is equal to $1.47.\ 10^{8}$ m/s, in figure (2-d) this latter is equal to $2.38.\ 10^{8}$ m/s and the speed value taken from figure (2-e) is equal to $1.7857.10^{8}$ m/s. From these lighting return stroke current speeds values it is clearly remarked that all the four electromagnetic models implemented in this calculation give a speed lower than that of the light velocity. The difference between these values is due to the series inductance value (model 1) and the size of the artificial medium. It is also due to the relative permittivity and permeability of this medium (model 2, 3 and 4). Finally, we can notice that there is a difference in the current waveforms attenuation rate along the lightning channel for each model.

In the end, we can say that the use of these four electromagnetic models and the 3D-FDTD method, based on Taflove formulation, has given good results validating thus our computation approach and code, developed on Matlab environment.

5. Conclusion

In this paper, electromagnetic models have been implemented, on Matlab environment, in order to determine the lightning return stroke current space-time distribution along a vertical channel and above a flat perfectly conducting ground. This computation was based on the implementation of the Taflove formulation of the 3D-FDTD method associated to the UPML absorbing boundary conditions. To that effect, four 3D calculation codes based on this numerical method have been developed. The proposed computation approach gives a good agreement between the obtained results and similar results taken from literature. This enables us to validate the proposed approach and at the same time the developed codes.

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